Beyond the IMME

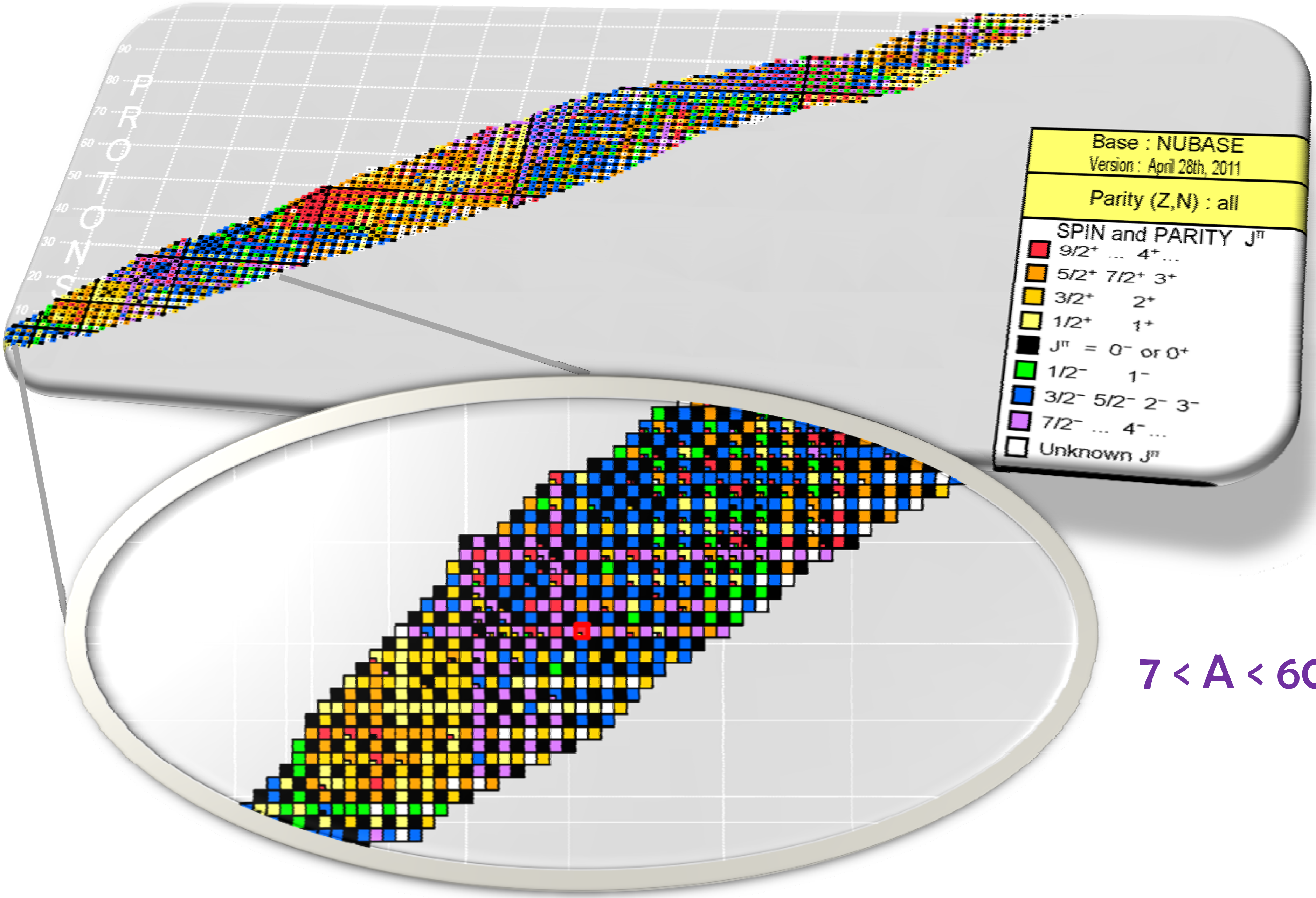
(Isobaric Multiplet Mass Equation)

Marion MacCormick

Institut de Physique Nucléaire

Orsay, FRANCE

Experimental Isobaric Analogue States



7 < A < 60

Experimental Isobaric Analogue States (IAS)

Mass, M of each nuclear configuration is fully defined by

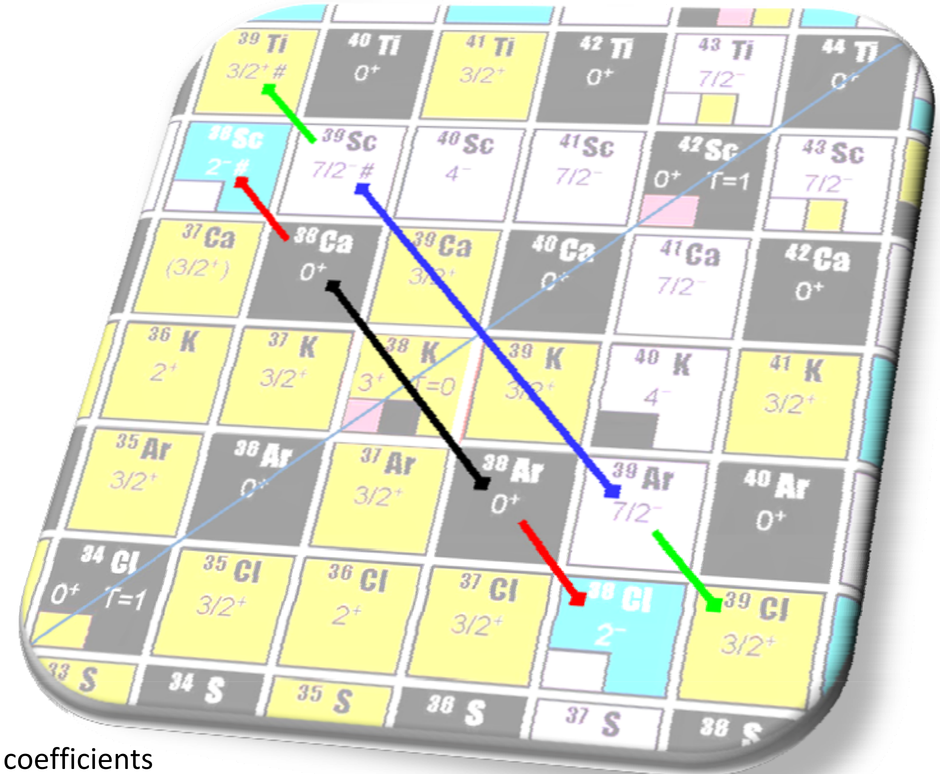
$$M(\text{space}, \text{spin} - \text{parity}, \text{isospin}) = M\left(A^{1/3}, J^{\pi}, \frac{N-Z}{2}\right)$$

Isospin projection (good quantum number)

$$T_Z = \sum_{i=1}^{A} t_{z,i} = \frac{N-Z}{2}$$

Isospin

$$\left|\frac{N-Z}{2}\right| \leq T \leq \frac{N+Z}{2} \quad \text{and} \quad T \geq T_Z$$



Evaluated experimental isobaric analogue states from T = 1/2 to T = 3 and associated IMME coefficients

M. MacCormick, G. Audi, **Nuclear Physics A 925 (2014) 61–95**

Isobaric Multiplet Mass Equation (IMME)

IMME $M\,(T, T_Z) = a + bT_Z + cT_Z^2$ theoretical

$a = M_0 + E_c^{(0)} - T(T+1)E_c^{(2)}$ $b = \Delta_{nH} - E_c^{(1)}$ $c = 3E_c^{(2)}$

$E_c^{(0)}$ isoscalar, $E_c^{(1)}$ isovector, $E_c^{(2)}$ isotensor Coulomb energies

Δ_{nH} = neutron - ^{1}H mass difference

→ coefficients a, b, c extracted from quadratic fits to evaluated experimental data (AME2012, NUBASE2012, ENSDF)

Evaluated IMME coefficients

$$M\,(T, T_Z) = a + bT_Z + cT_Z^2 + dT_Z^3$$

a "bulk" mass, 10^4 keV scale

b $\sim 10^3$ keV scale

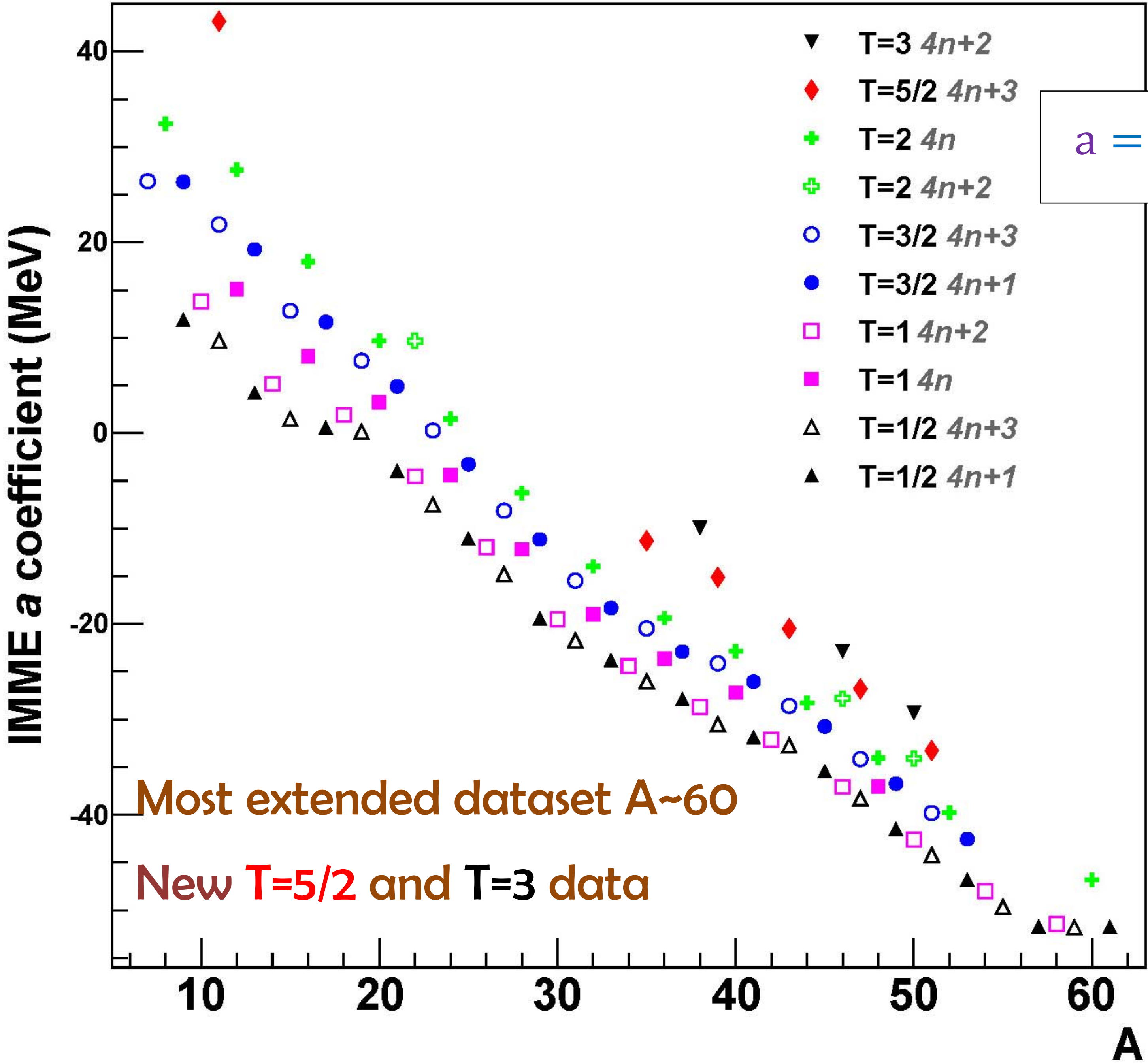
c $\sim 10^2$ keV scale

d $\sim 10^1$ keV scale

Main focus

T=1 and T=3/2 datasets

Multiplets subdivided in 4n groups



Most extended dataset A~60

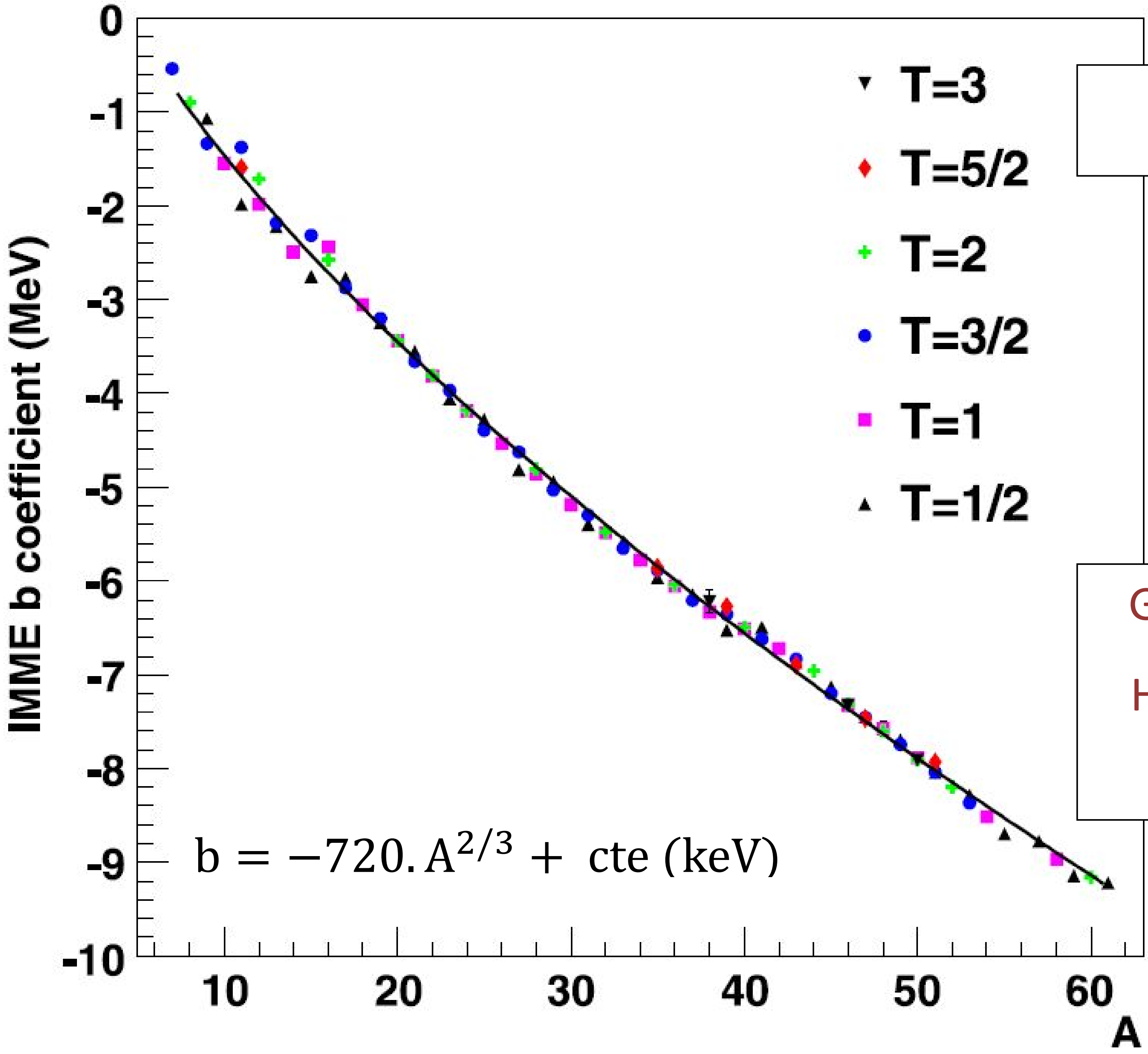
New T=5/2 and T=3 data

$$a = M_0 + E_c^{(0)} - T(T+1)E_c^{(2)}$$

First evaluation linked in to the AME

global function

→ mass predicitions



$$b = \Delta_{nH} - E_c^{(1)}$$

General smooth function

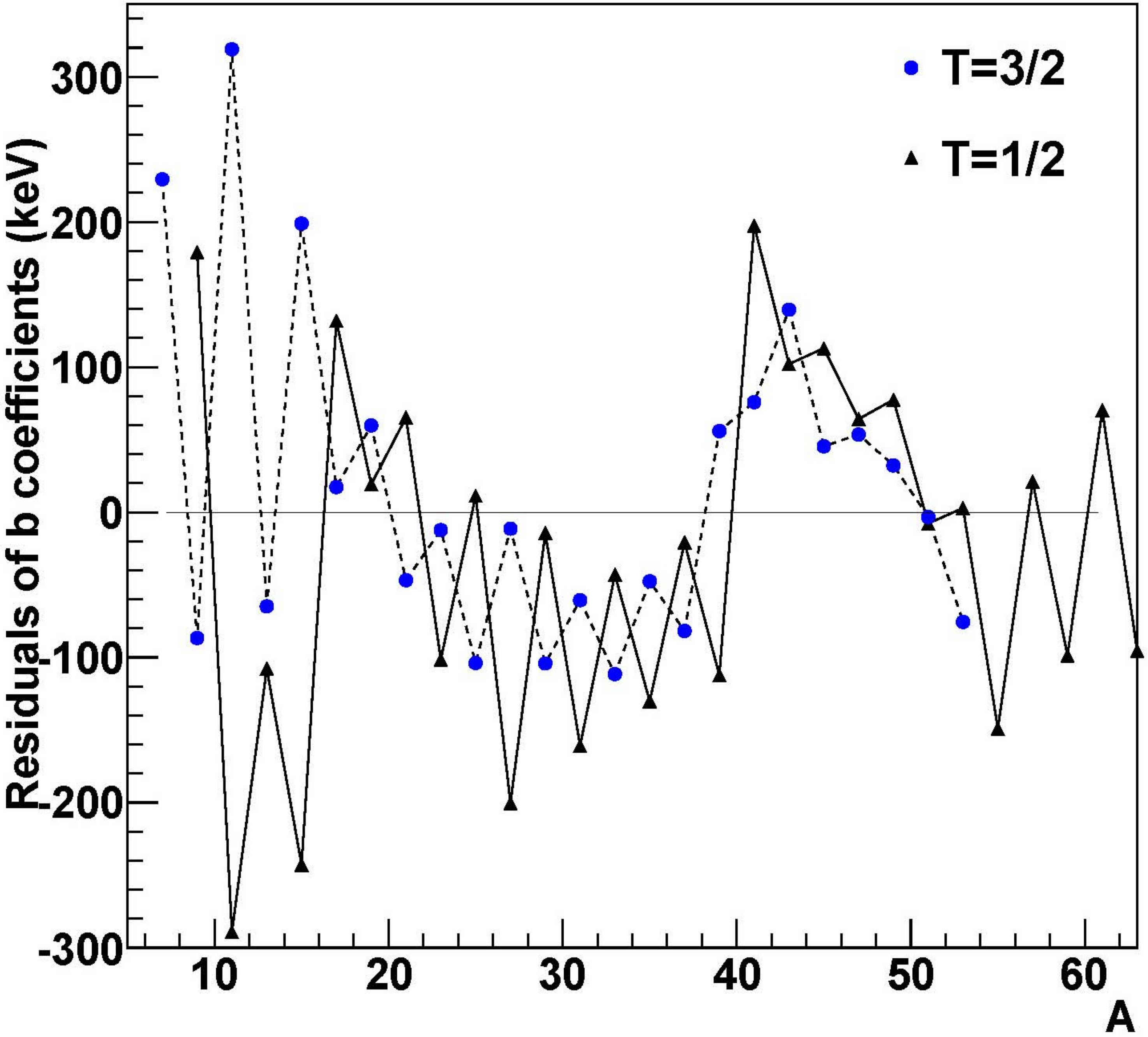
Homogeneously charged spherical nucleus

Staggering

A < 40

Progressive attenuation beyond A=40

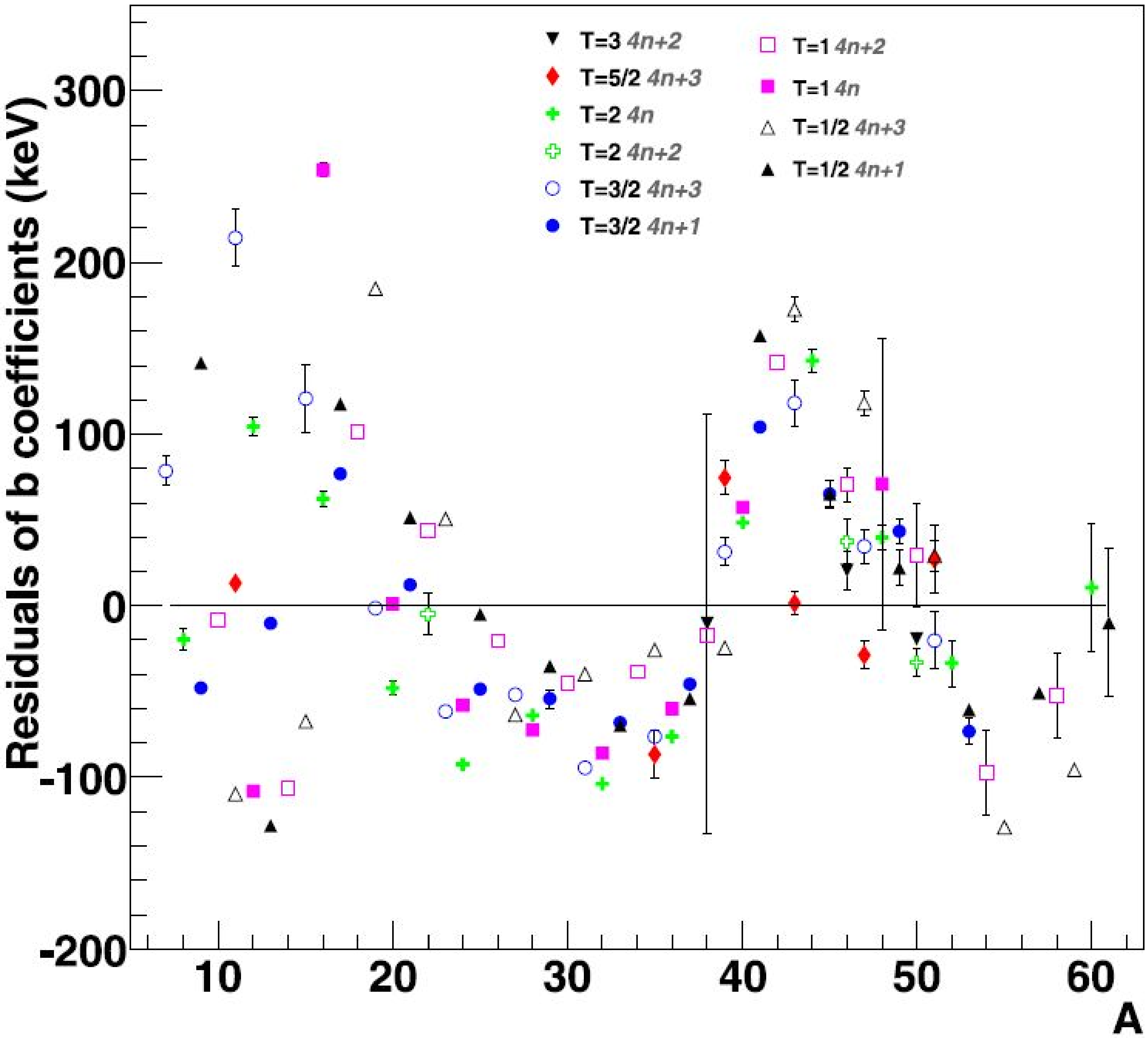
T=1/2 and 3/2 out of phase

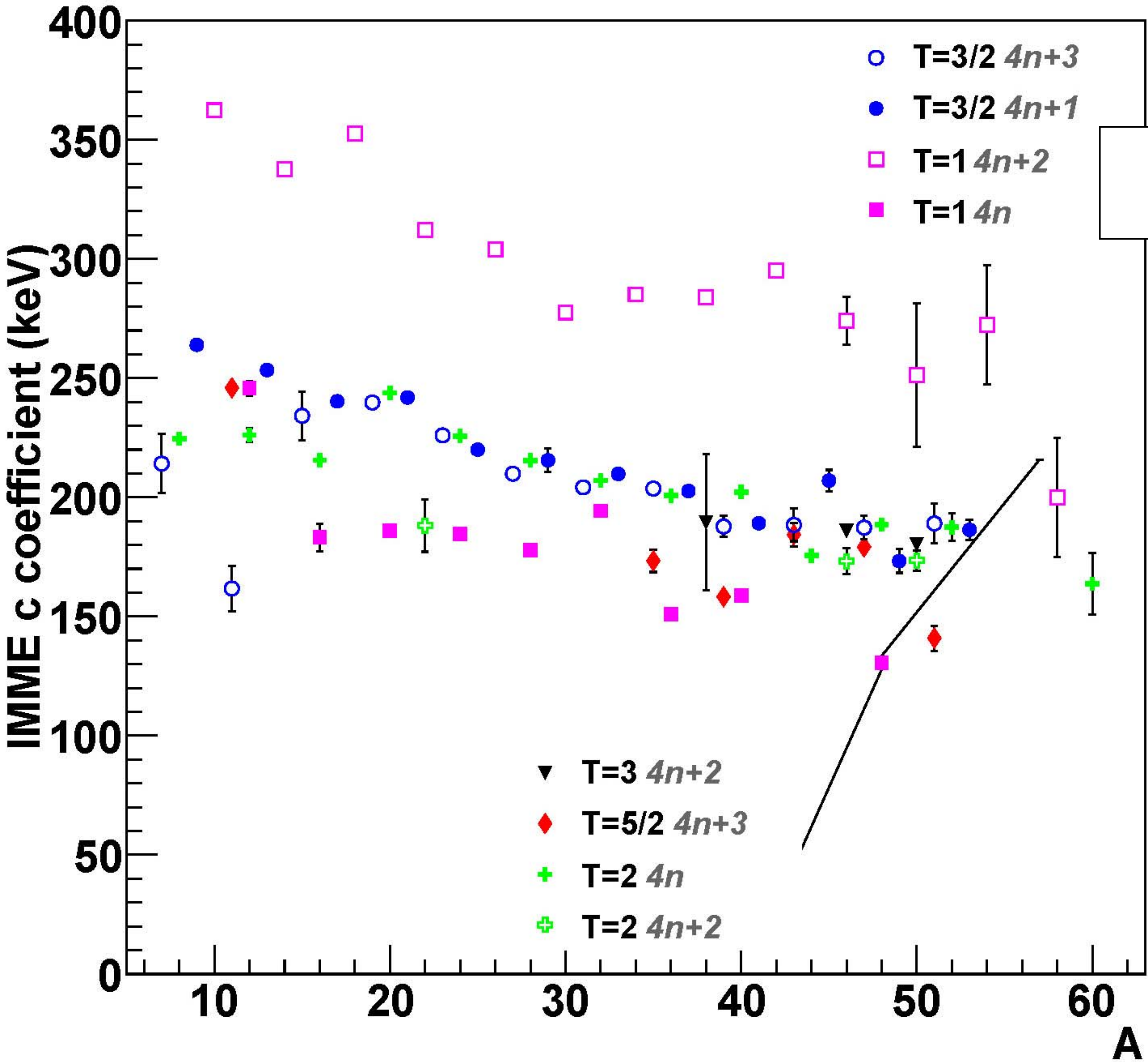


4n groups

Spherical model reference

Variations of +200 to -100 keV dominate the experimental resolution




IMME c coefficient (keV)
A
400
350
300
250
200
150
100
50
0
10
20
30
40
50
60
T=3/2 4n+3
T=3/2 4n+1
T=1 4n+2
T=1 4n
T=3 4n+2
T=5/2 4n+3
T=2 4n
T=2 4n+2

$$c = 3E_c^{(2)}$$

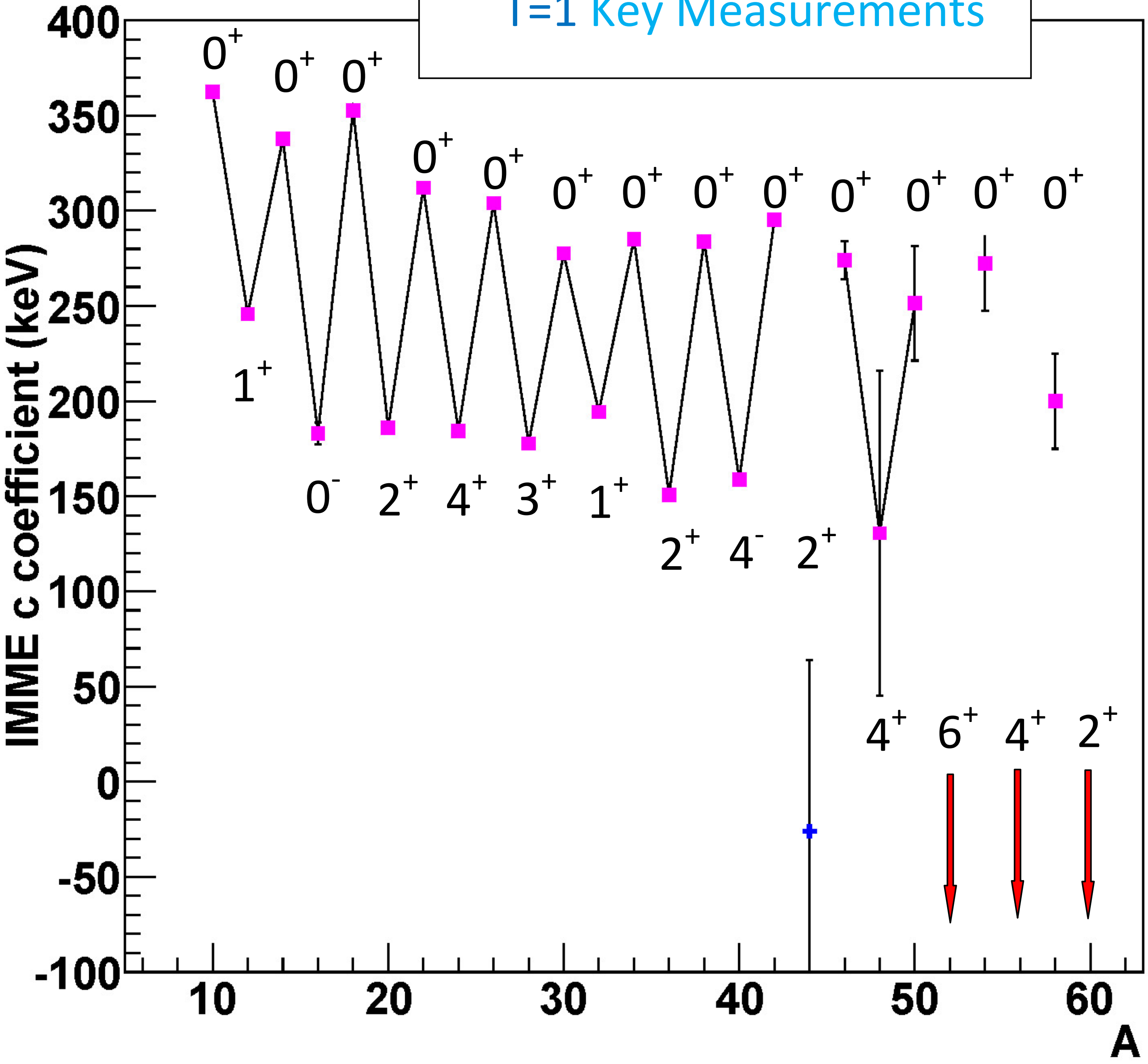
T=1 Key Measurements

A=44

^{44}Sc – ^{44}Ti^{i} – ^{44}V

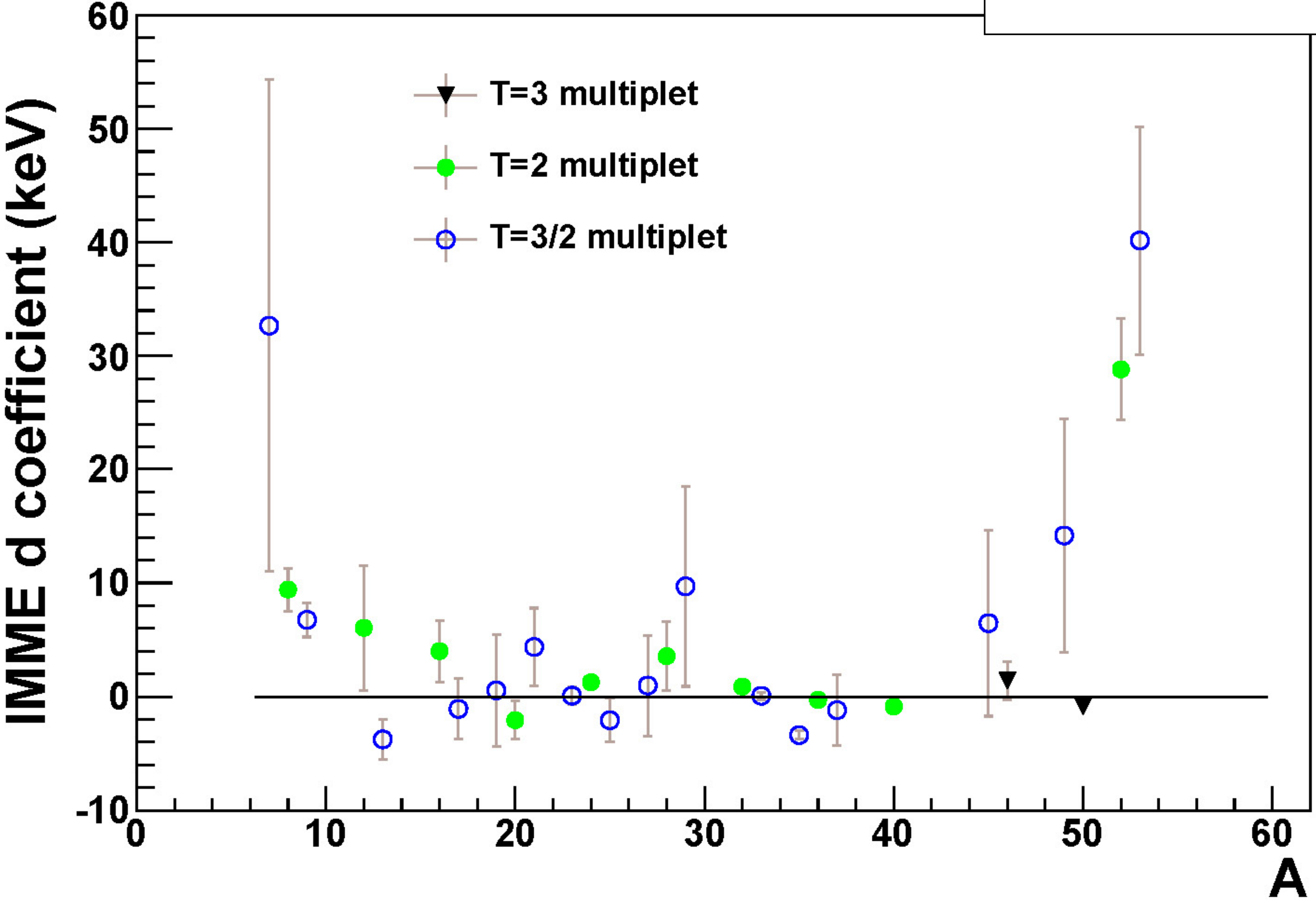
^{44}V g.s. is isomer contaminated

No experimental data for ^{52}Co, ^{56}Cu and ^{60}Ga ground states



Beyond the quadratic IMME

$$\text{Mass} = a + bT_z + cT_Z^2 + \color{red}{d}T_Z^3$$



$\pm 1\sigma$ (68 %)

error bars

$\pm 3\sigma$ (99.7%)

A=8, 32, 52

A=9, 35, 53

A=33

Year 2000

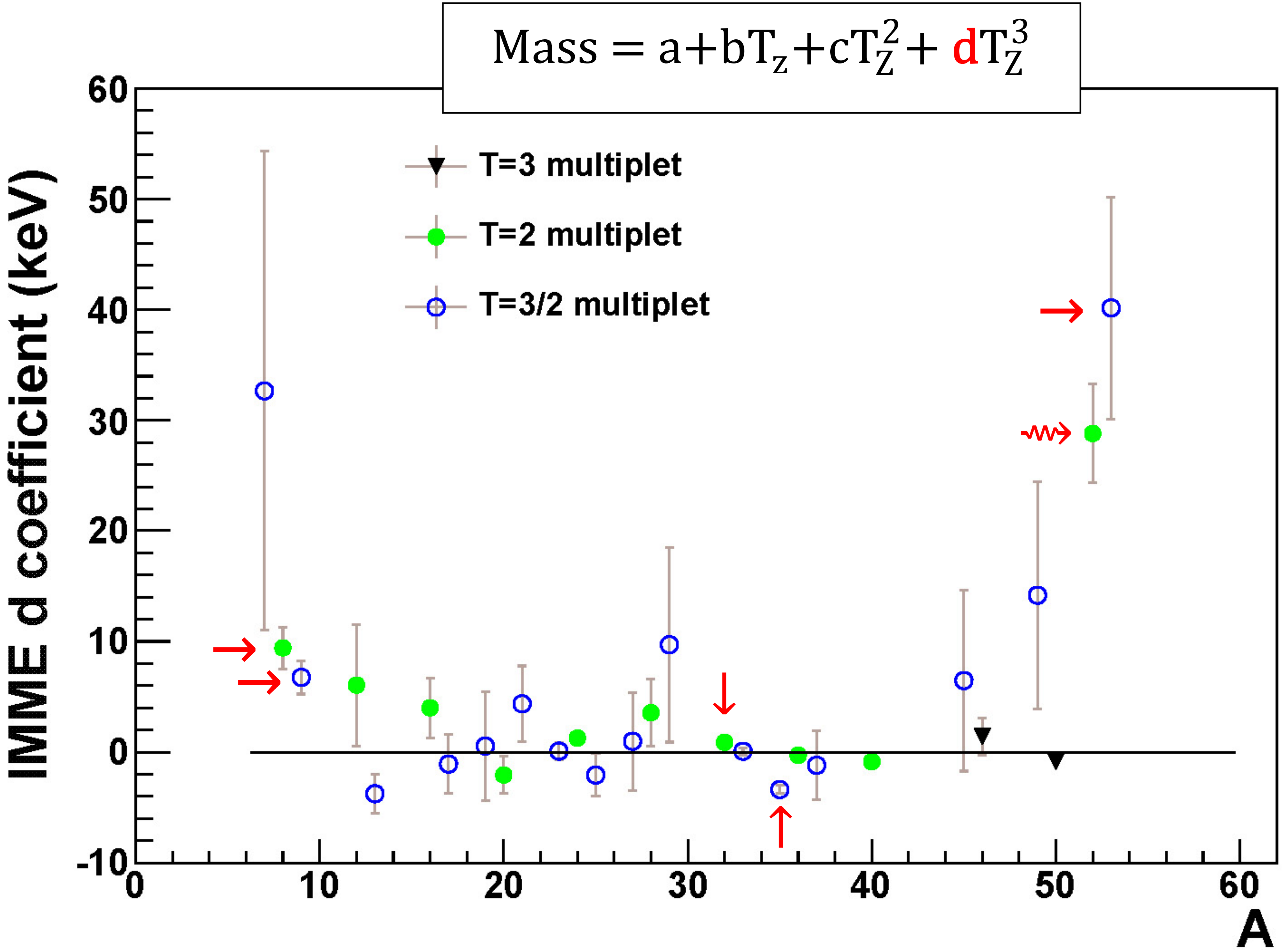
$d = -2.95 \pm 0.9$ keV

By 2012

$^{33}Ar \rightarrow +3$ keV

$^{33}Cl^{i} \rightarrow +6$ keV

$d = 0.08 \pm 0.32$ keV



d-coefficients

Precision

For T=3/2, in theory:

$$d = \frac{1}{6}\left(M_{T_Z=-\frac{3}{2}} - M_{T_Z=+\frac{3}{2}} - 3M_{T_Z=-\frac{1}{2}} + 3\,M_{T_Z=+\frac{1}{2}}\right)$$

d-coefficients appear through basic symmetry

The precision is *crucial* to establishing non-zero d-coefficients

How does the d-coefficient react (δd) to small experimental mass shifts (δM)?

The d-coefficient is 3 times more sensitive to $T_Z = \pm\frac{1}{2}$ mass shifts

How do the c-coefficients react to imposing d≅0?

24 datasets, 16 with all four IAS levels measured

Which high precision measurements would have the greatest impact?

Effect of *single* measurements on global datatset.

T=3/2 Key Measurements   Sensitivity based priority

A	Element	E* (keV)	Jπ	Most recent data	Evaluated Precision (keV)	Mass Shift for d→0 (keV)
45	Cr	g.s.	(7/2-)	2012 Zhang	± 40	−39
45	V	4791	7/2-	2007 Dossat	± 12	+13
29	S	g.s.	(5/2+)	1973 MSU $^{32}S(^3He,^6He)^{29}S$	± 50	−80
21	Mg	g.s.	5/2+	2014	± 16	+26.1
49	Mn	4833	(7/2-)	2007 Dossat	± 18	+30
21	Na	8975	5/2+	1973 Sextro	± 4	−8.7
53	Co	4395	7/2-	2007 Dossat	± 18	+80
49	Fe	g.s.	(7/2-)	2012 Zhang	± 24	−120
9	B	14654.7	3/2-	1974 MSU $^{11}B(p,t)^{9}B^{i}$	± 2.5	−13.5
53	Ni	g.s	(7/2-)	2012 Zhang	± 25	−240
9	C	g.s.	(3/2-)	1971	± 2.1	+40.5

Precision

Accuracy

T=3/2 Unidentified IAS (all are $T_Z = -\frac{1}{2}$)

A	Element	Jπ	Estimated E* (keV)	Nearest in ENSDF (keV)	observations
11	C^i	$\frac{1}{2}+$	11952 ± 10	12160 ± 40 (IAS)	$\Gamma = 270 \pm 50$ keV
15	O^i	$\frac{1}{2}+$	11173 ± 10	11151 ± 7	100% p-decay
39	Ca^i	$\frac{7}{2}-$	6378 ± 10	6450 ± 10	$(\frac{7}{2}-)$
43	Ti^i	$\frac{7}{2}-$	4195 ± 10	Data stops @ 3220	none
47	Cr^i	$\frac{5}{2}-$	4161 ± 50	4169 ± 12	No J^π
51	Fe^i	$\frac{7}{2}-$	4447 ± 50	4456 ± 13	No J^π

Hypothesis: the other measured multiplet states are accurate.

Summary and Conclusions

- AME2012 and NUBASE2012 IAS highlights
  - 107 new IAS states linked in to g.s. masses
  - Most precise and complete set of IAS experimental data
  - New data sets for T=5/2 and T=3
- Fitted Isobaric Multiplet Mass Equation (IMME) coefficients
- Clear global trends observed in a-, b- and c-coefficients
- d-coefficients for 4-point datasets
- Key T=1 experimental measurements
- Key T=3/2 experimental measurements
- Unidentified T=3/2 IAS

References and further reading

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Isobaric mass equation for A=1-45 and systematics of Coulomb displacement energies.
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and also

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Evaluated experimental isobaric analogue states from T = 1/2 to T = 3 and associated IMME coefficients
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NUBASE 2012 and AME2012

The NUBASE2012 evaluation of nuclear properties
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The AME2012 atomic mass evaluation (I) Evaluation of input data, adjustment procedures
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The AME2012 atomic mass evaluation (II) Tables, graphs and references
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