Effect of fluctuations of quadrupole deformation and neutron-proton correlations on double-beta decay nuclear matrix element

Nobuo Hinohara*
Jon Engel

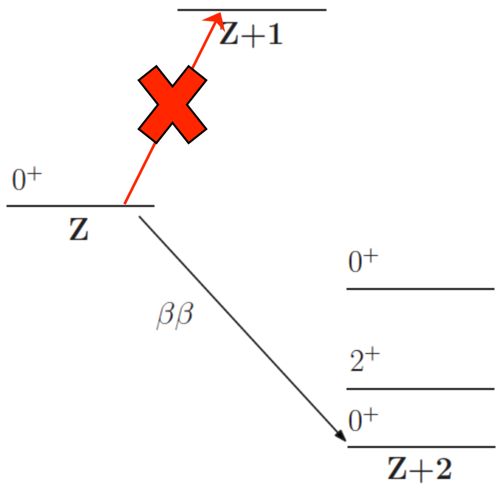
University of North Carolina at Chapel Hill, USA
* University of Tsukuba, Japan (from 6/1/2014)

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

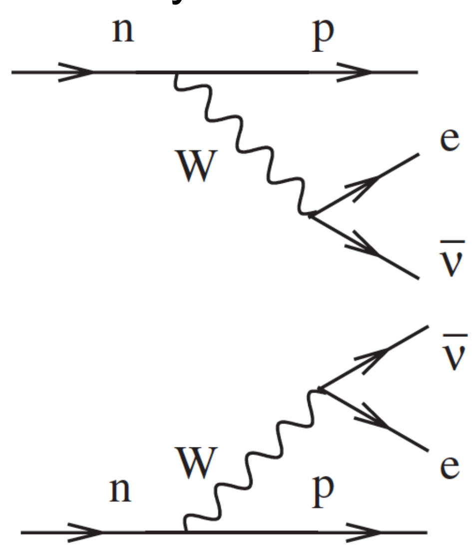
Jun.2, 2014

Advances in Radioactive Isotope Science (ARIS2014)

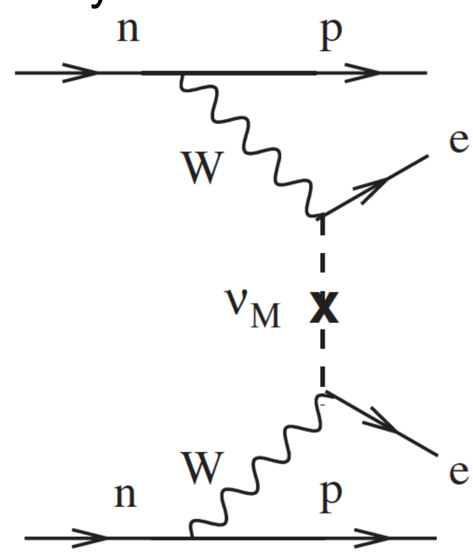
Double-beta decay



2ν decay



0ν decay



- single β-decay forbidden
- two modes (2ν and 0ν)
- 2ν decay measured (half-lives: order of $10^{19\sim21}$ yr)
- 0ν is possible if the neutrino is Majorana particle
- 2ν measured nuclei: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Ba, ^{150}Nd, ^{238}U

half life of 0ν ββ decay

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$

effective mass of Majorana neutrino

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_k m_k U_{ek}^2\right|$$

Review: Avignone, et al., Rev. Mod. Phys. **80**, 481 (2008)

Nuclear Matrix Element

2v and 0v half lives

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}(Q_{\beta\beta}, Z)|M_{2\nu}|^2$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

nuclear matrix element in closure approximation

$$M_{0\nu} = \frac{2R}{\pi g_A^2}\int_0^\infty q dq \langle f|\sum_{ab}\frac{j_0(qr_{ab})[h_{\mathrm{F}}(q) + h_{\mathrm{GT}}(q)\vec{\sigma}_a\cdot\vec{\sigma}_b]}{q + \bar{E} - (E_i + E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

$$M_{0\nu} \approx M_{0\nu}^{\mathrm{GT}} - \frac{g_V^2}{g_A^2}M_{0\nu}^F$$

$$M_{0\nu}^F = \langle f|\sum_{a,b} H(r_{ab}, \bar{E})\tau_a^+\tau_b^+|i\rangle \qquad M_{0\nu}^{GT} = \langle f|\sum_{a,b} H(r_{ab}, \bar{E})\vec{\sigma}_a\cdot\vec{\sigma}_b\tau_a^+\tau_b^+|i\rangle$$

H: neutrino potential

nuclear structure theories for nuclear matrix element

- ☐ shell model
- ☐ proton-neutron QRPA
- ☐ generator coordinate method
- ☐ IBM

Importance of pn correlations: pnQRPA

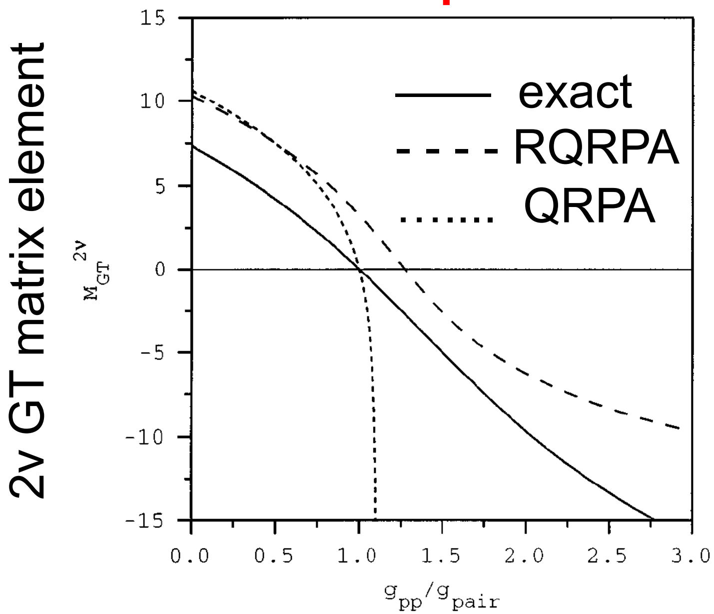
Advantages

- large single-particle model space
- odd-odd intermediate states as one-phonon excitation
- pn pairing quenches the matrix element

Limitation

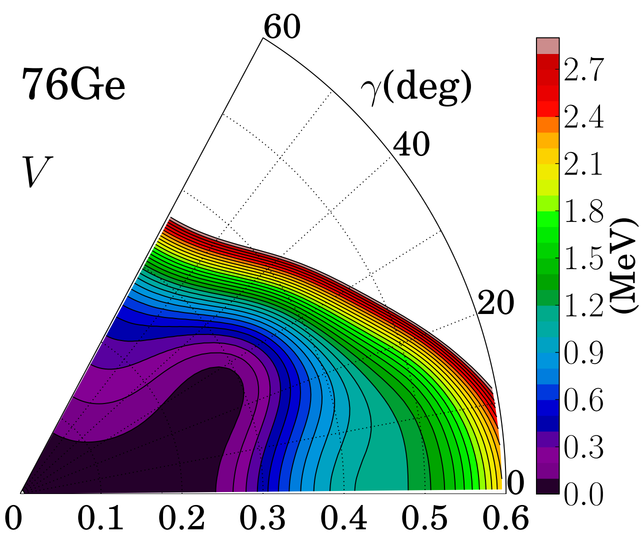
- small-amplitude approximation:
  not reliable near (and after) the phase transition (isovector→isoscalar)
- based on a single mean field
  cannot handle the large-amplitude fluctuation of the mean field

isovector-isoscalar phase transition



Engel et al. PRC55,1781(1997)

quadrupole shape fluctuation (^{76}Ge)

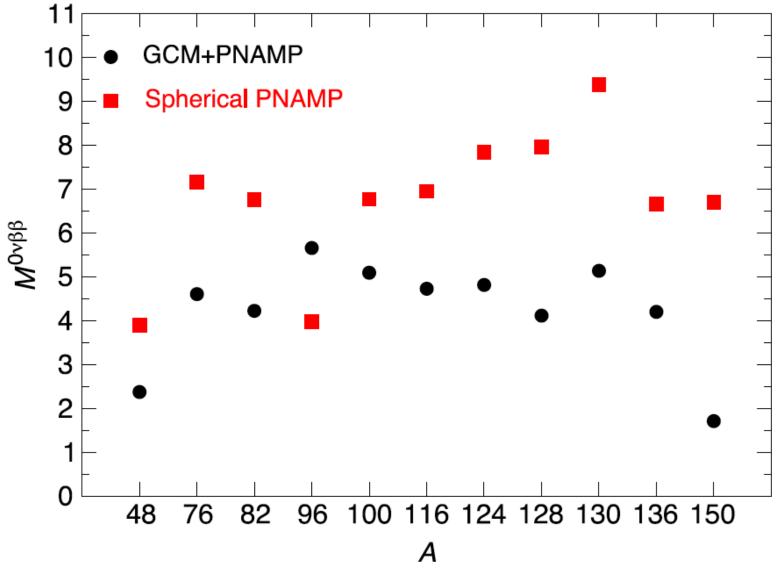


single mean field may not be a good approximation of the ground state

Going beyond mean field (GCM)

Rodriguez and Martinez-Pinedo, Prog. Part. Nucl. Phys. **66**, 436 (2011)
Vaquero et al. Phys. Rev. Lett. **111**, 142501 (2013)

Generator coordinate method: (Gogny D1S)



$$|I^{+\sigma}_{i/f}\rangle = \sum_{\beta_2,\delta} g^{I\sigma}_{i/f}(\beta_2,\delta)|\Psi^{I}_{i/f}(\beta_2,\delta)\rangle$$

$$|\Psi^{I}_{i/f}(\beta_2,\delta)\rangle = P^{N_{i/f}} P^{Z_{i/f}} P^{I} |\phi(\beta_2,\delta)\rangle$$

deformation and like-particle pairing -constrained mean fields

- mean fields with different deformation and pairing: large-amplitude fluctuation
- fluctuation of deformation decreases (and pp and nn pairing increases) matrix element
- no neutron-proton residual correlations considered

Goal

to compute the nuclear matrix elements including large-amplitude fluctuations of
- quadrupole deformation
- neutron-proton correlations
- using generator coordinate method (no other pn-GCM calculations ever)

Our approach: GCM with pn degrees of freedom

Generalized HB (3D harmonic oscillator basis)

neutron and proton mixed quasiparticles

$$\hat{a}_k^\dagger = \sum_l \left(U_{lk}^{(n)} \hat{c}_l^{(n)\dagger} + V_{lk}^{(n)} \hat{c}_k^{(n)} + U_{lk}^{(p)} \hat{c}_l^{(p)\dagger} + V_{lk}^{(p)} \hat{c}_k^{(p)} \right)$$

Constrained HB: q (generator coordinates): $a_k|\phi(q)\rangle = 0$

- ☐ axial quadrupole deformation Q_{20}
- ☐ T=1, S=0 Isovector (np) pairing ← for Fermi Matrix element
- ☐ T=0, S=1 Isoscalar pairing ← for Gamow-Teller

Projections

isoscalar pairing condensation breaks both particle number conservation and rotational symmetry

$$|\phi^{N,Z}_{I=0,M=0}(q)\rangle = \hat{P}^N \hat{P}^Z \hat{P}^{I=0}_{M=0K=0}|\phi(q)\rangle$$

Superposition of projected mean fields (GCM)

$$|\Psi(N,Z,I=0,M=0)\rangle = \int dq f_k(q) |\phi^{N,Z}_{I=0,M=0}(q)\rangle$$

^{76}Ge→^{76}Se ββ decay

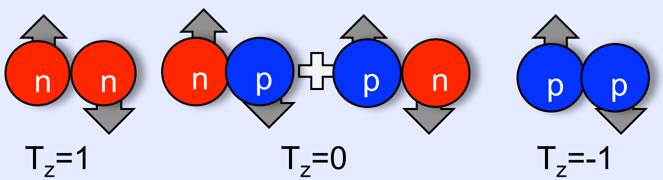
Hamiltonian

$$H = h_0 - \sum_{\mu=-1}^{1} g_\mu^{T=1}\, S_\mu^\dagger S_\mu - \frac{\chi}{2}\sum_{K=-2}^{2} Q_{2K}^\dagger Q_{2K} - g^{T=0}\sum_{\nu=-1}^{1} P_\nu^\dagger P_\nu + g_{ph}\sum_{\mu,\nu=-1}^{1} F_\nu^{\mu\dagger} F_\nu^{\mu}$$

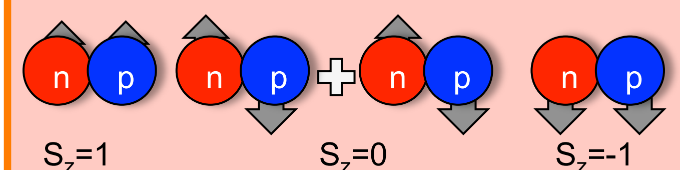
s.p.energy | isovector pairing | quadrupole interaction | isoscalar pairing | Gamow-Teller interaction

s.p. model space: full pf + sdg shells

Isovector (T=1, S=0) pairings



Isoscalar (T=0, S=1) pairings



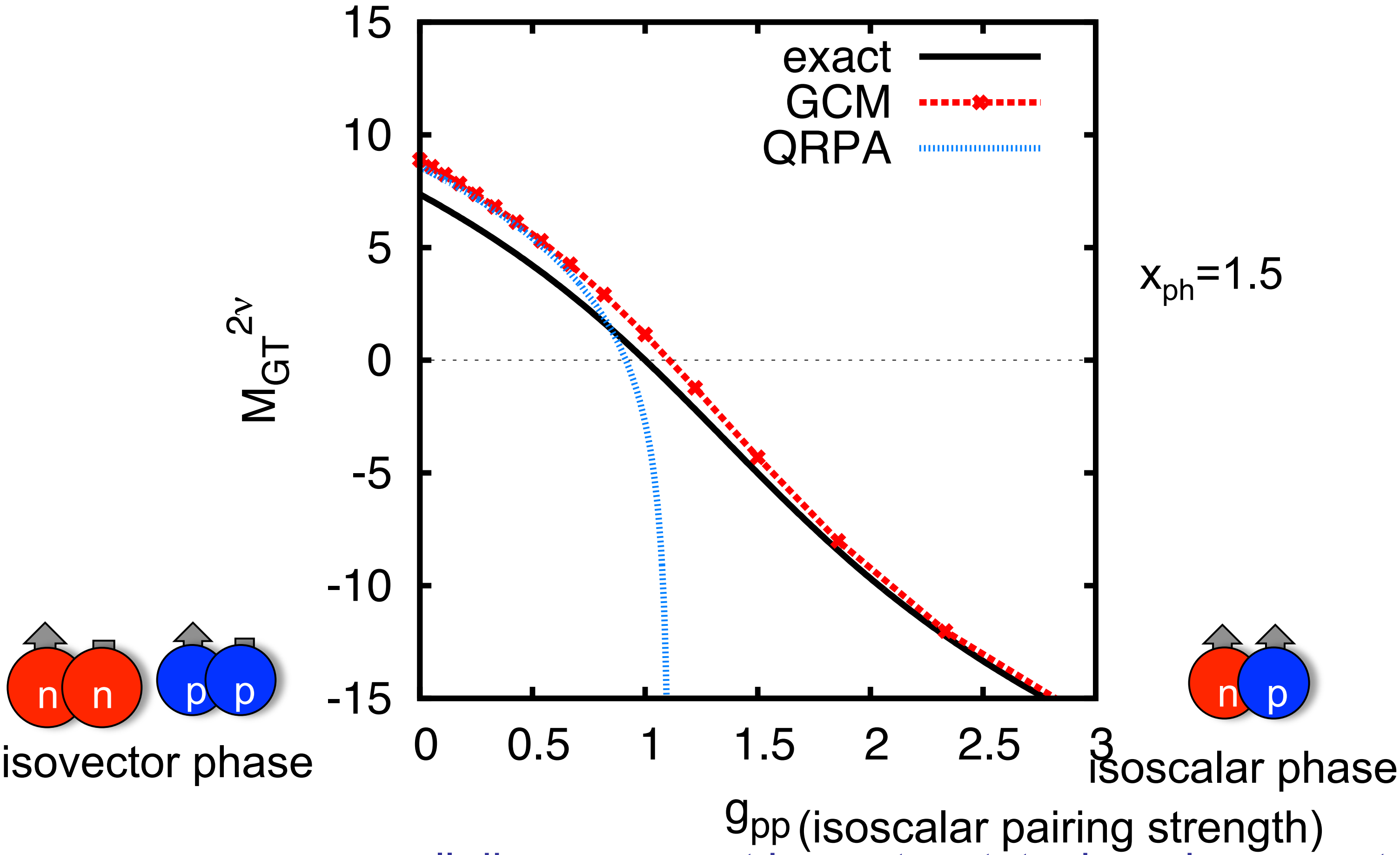
στ (Gamow-Teller) particle-hole (T=1, S=1)

parameter:

- ☐ s.p.energy, pp and nn pairings, quadrupole strength: from Skyrme HFB (SkO' and SkM*)
- ☐ T=1 pn pairing: value which vanishes 2ν closure matrix element
- ☐ Gamow-Teller interaction: ^{76}Ge GT- resonance peak from Skyrme-QRPA
- ☐ T=0 pn pairing: total β+ strength of ^{76}Se

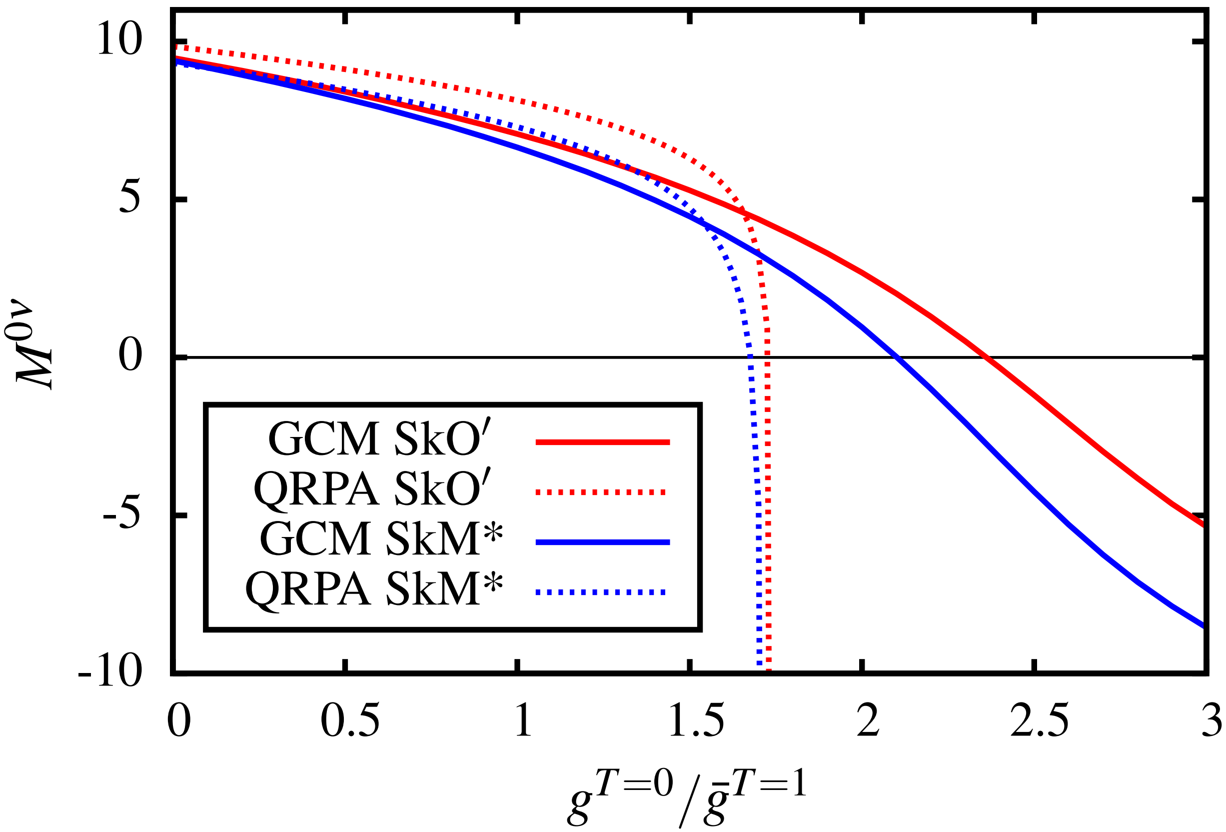
Test calculation in solvable SO(8) model

SO(8): solvable version of the previous Hamiltonian
(w/o sp energy, quadrupole int)
GCM with isoscalar pairing coordinate
2ν GT (closure) matrix element of T=4→T=2



small discrepancy at isovector state: isospin symmetry breaking

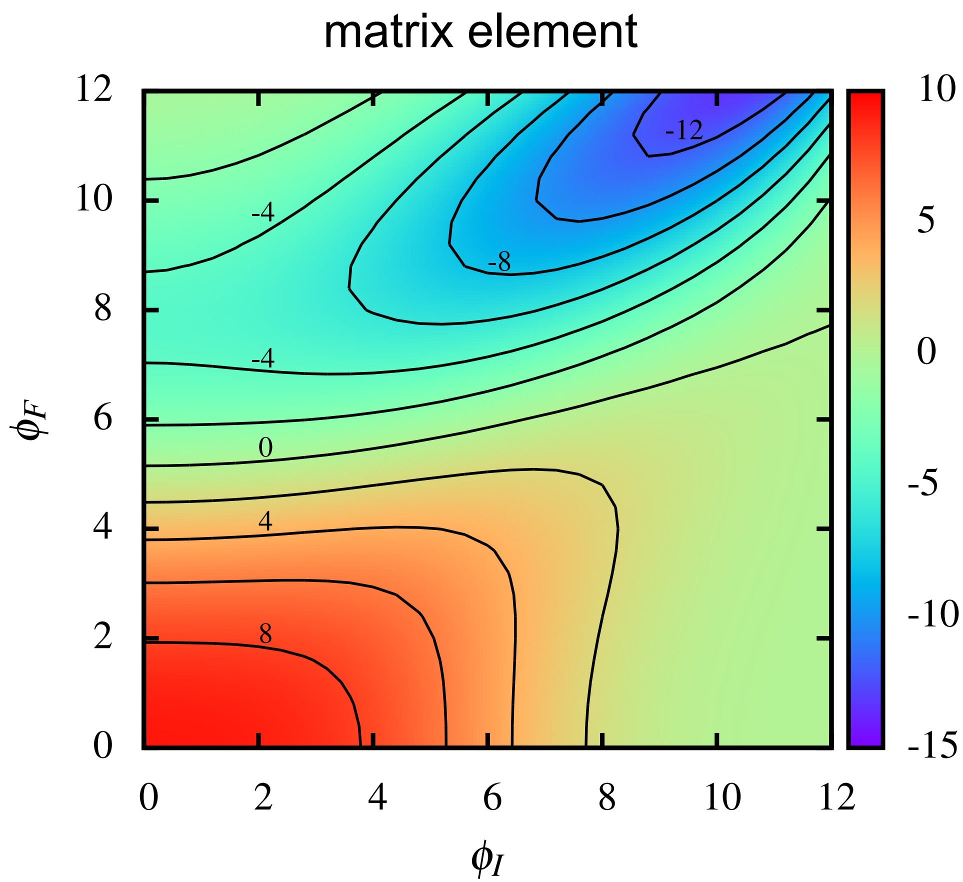
$^{76}Ge \rightarrow {}^{76}Se$ 0v matrix element

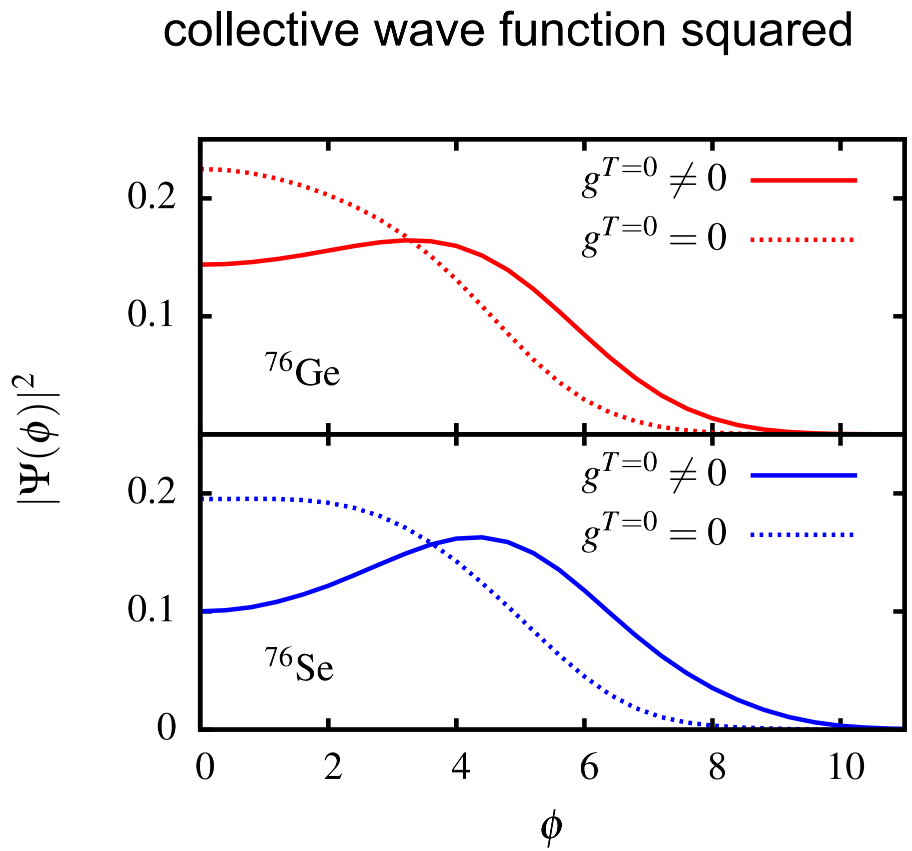


QRPA: collapse near the phase transition $g^{T=0}/g^{T=1} \sim 1.6$
GCM: smooth dependence on isoscalar pairing

Skyrme	no gph/ $g^{T=0}$	no $g^{T=0}$	1D full	QRPA
SkO'	14.0	9.5	5.4	5.6
SkM*	11.8	9.4	4.1	3.5

^{76}Ge→^{76}Se 0ν matrix element



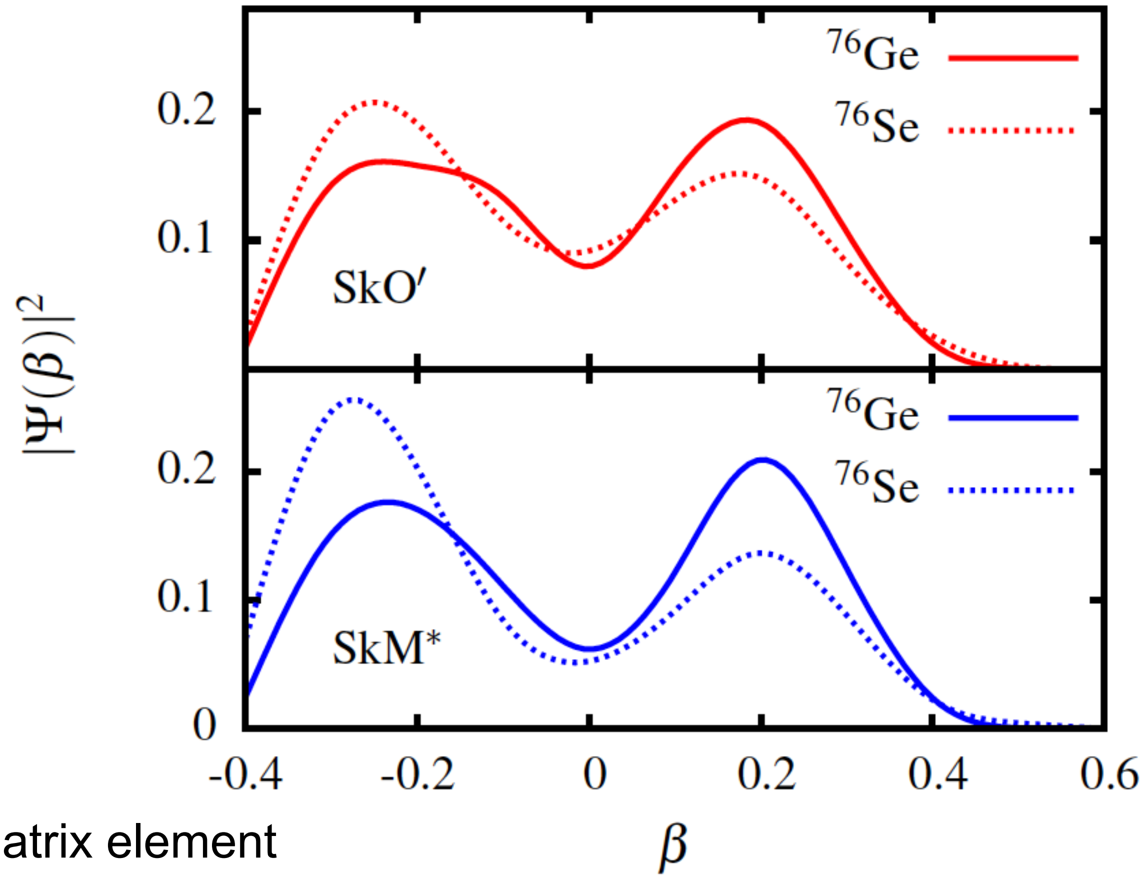


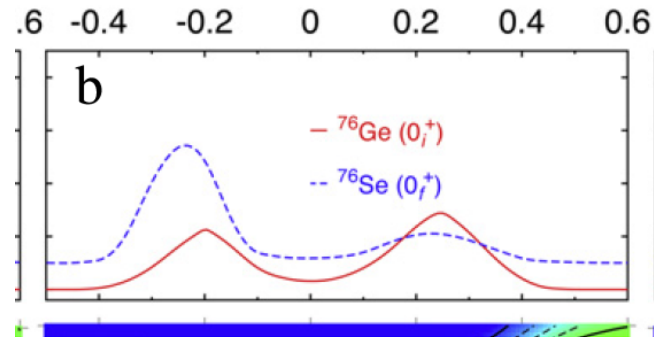
generator coordinate: $$\phi = \frac{\langle P_0 + P_0^\dagger \rangle}{2}$$

negative region at large isoscalar paring of final state
isoscalar pairing shifts the wave function to isoscalar region

Inclusion of quadrupole deformation (2D GCM)

collective wave function (isoscalar pairing d.o.f. integrated out)



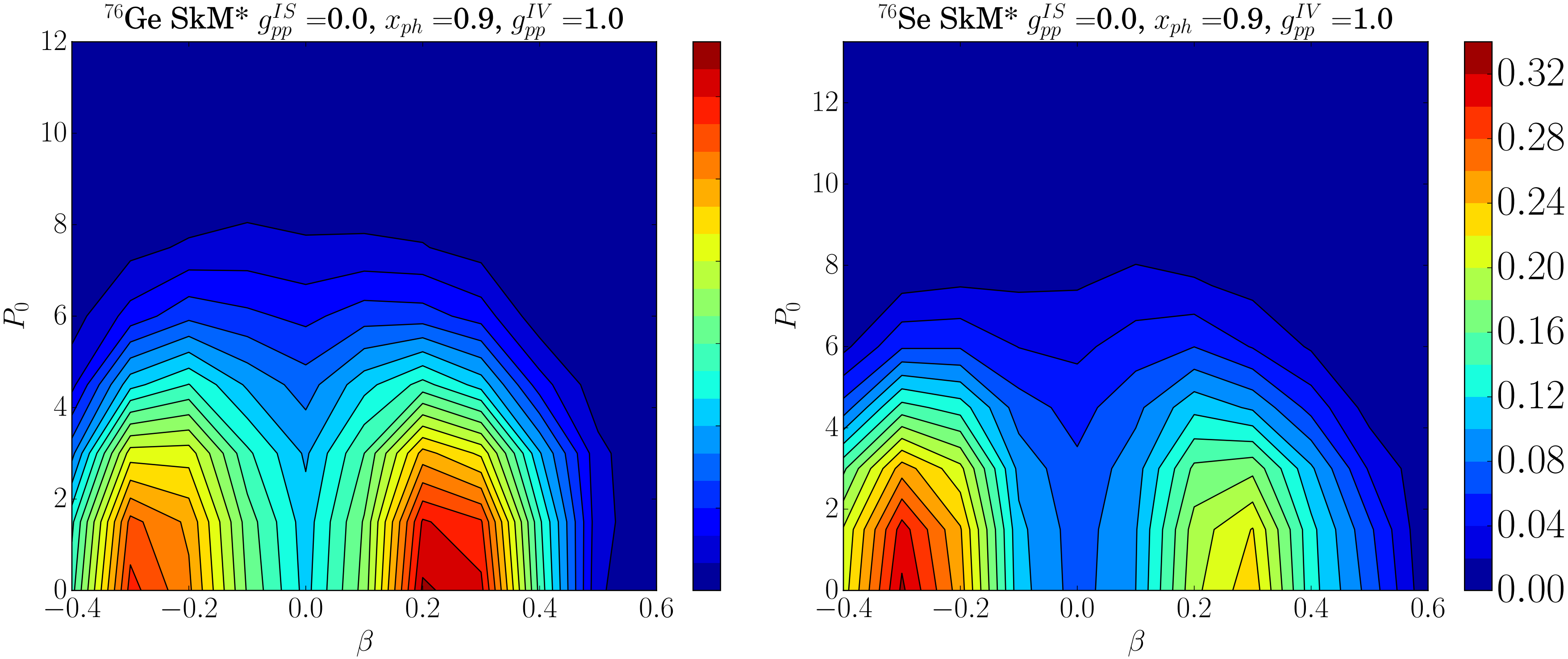


Rodríguez and Martinez-Pinedo
Prog. Part. Nucl. Phys. **66** (2011) 436.

matrix element

Skyrme	1D full	2D full	spherical QRPA
SkO’	5.4	4.7	5.6
SkM*	4.1	4.7	3.5

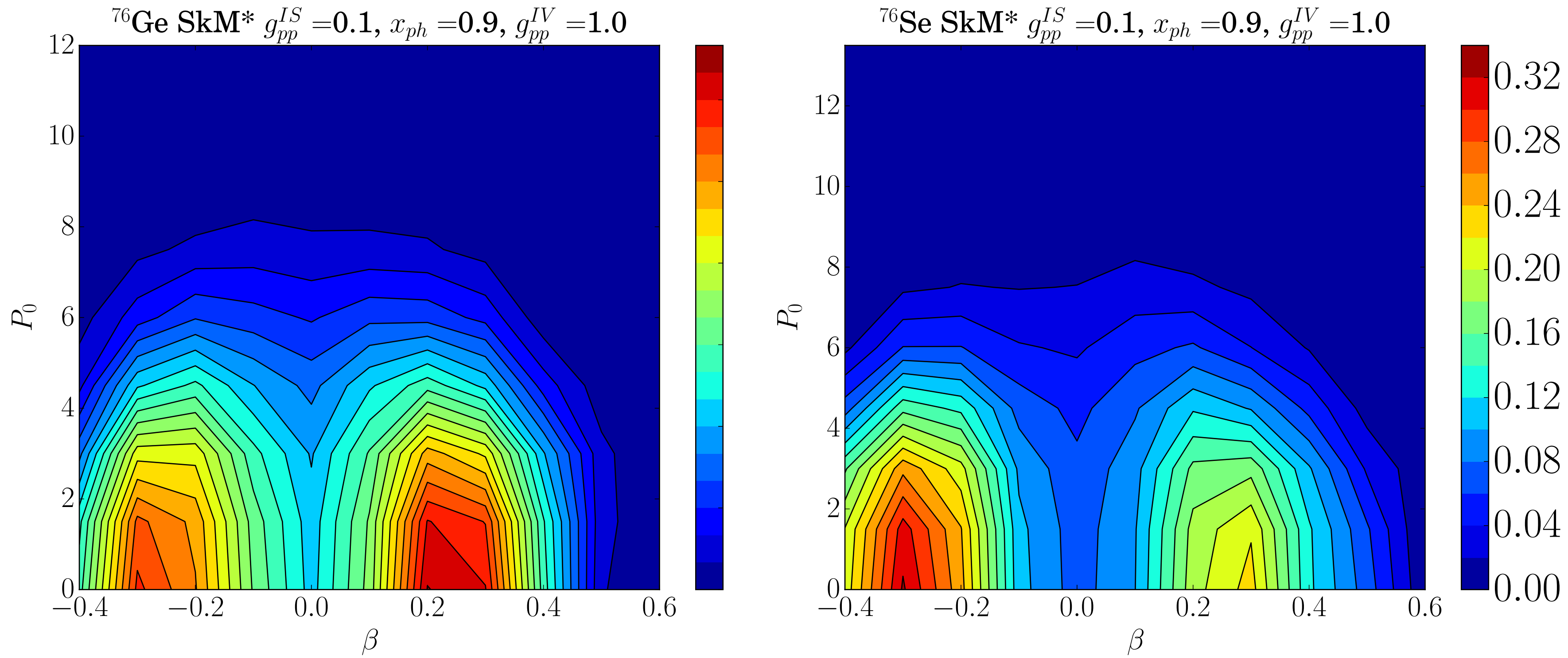
Gogny beta-GCM: 4.6
PRL105,252503(2010)
Gogny beta+delta GCM: 5.6
PRL111,142501(2013)
Skyrme pnQRPA SkM*: 5.1
PRC87, 064302(2013)



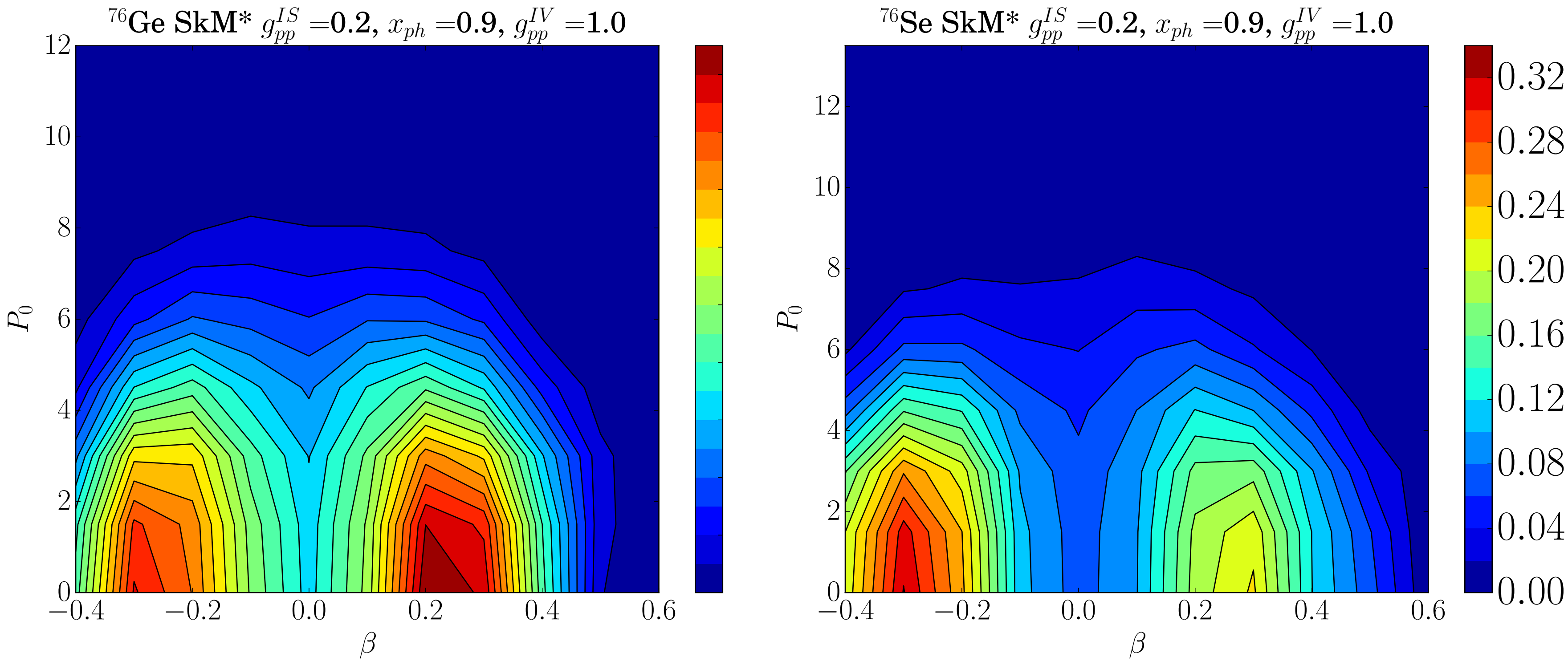
^{76}Ge ^{76}Se

$$g^{T=0}/g^{T=1} = 0.0$$

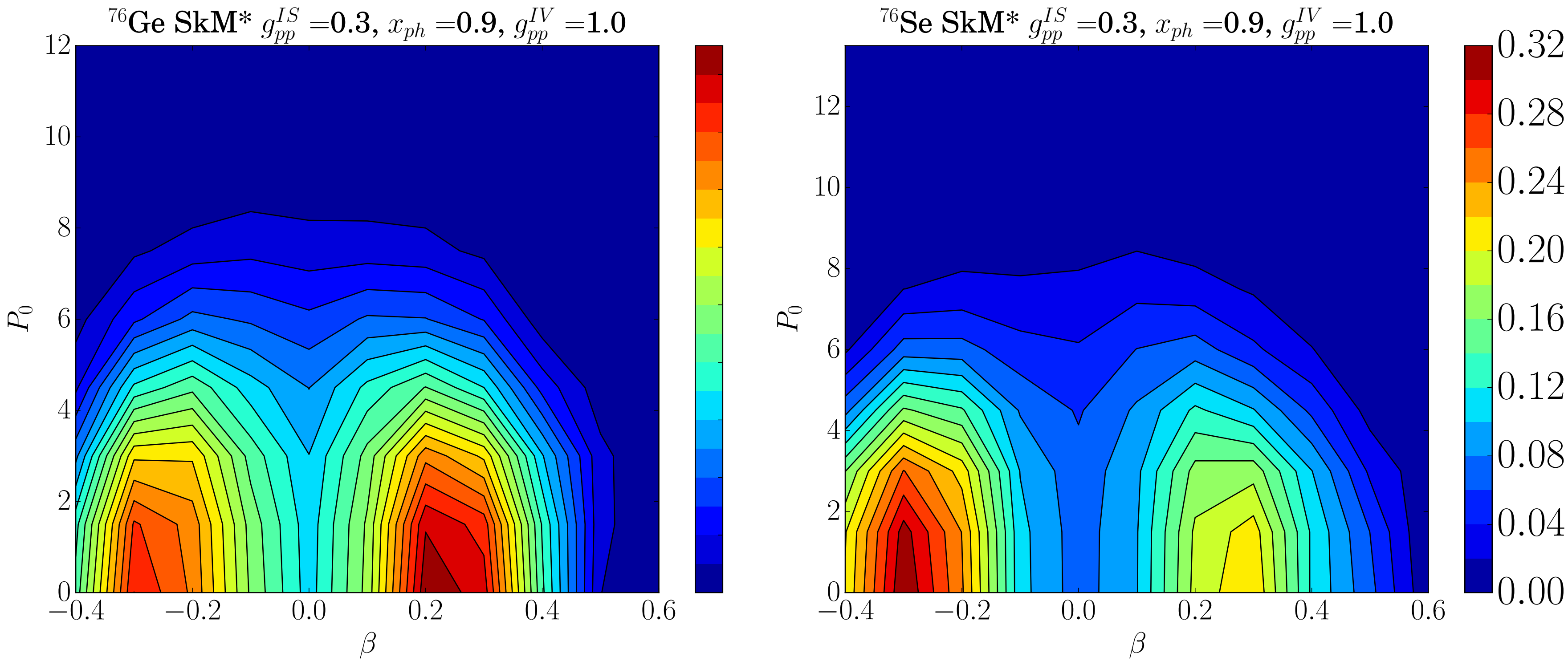
SkM* collective wave function $\Phi(\beta, P_0)$



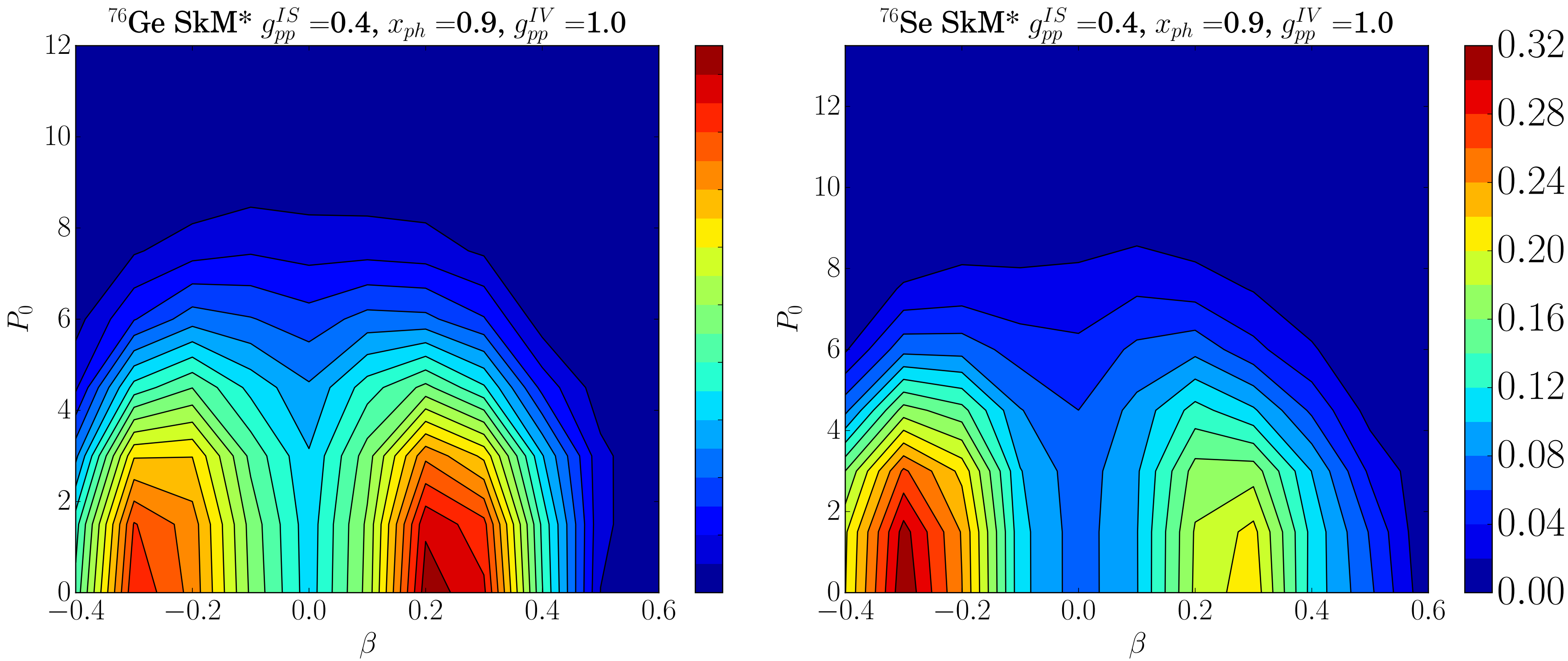
^{76}Ge SkM* g_{pp}^{IS} =0.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0 2 4 6 8 10 12
−0.4 −0.2 0.0 0.2 0.4 0.6
0.00 0.04 0.08 0.12 0.16 0.20 0.24 0.28 0.32



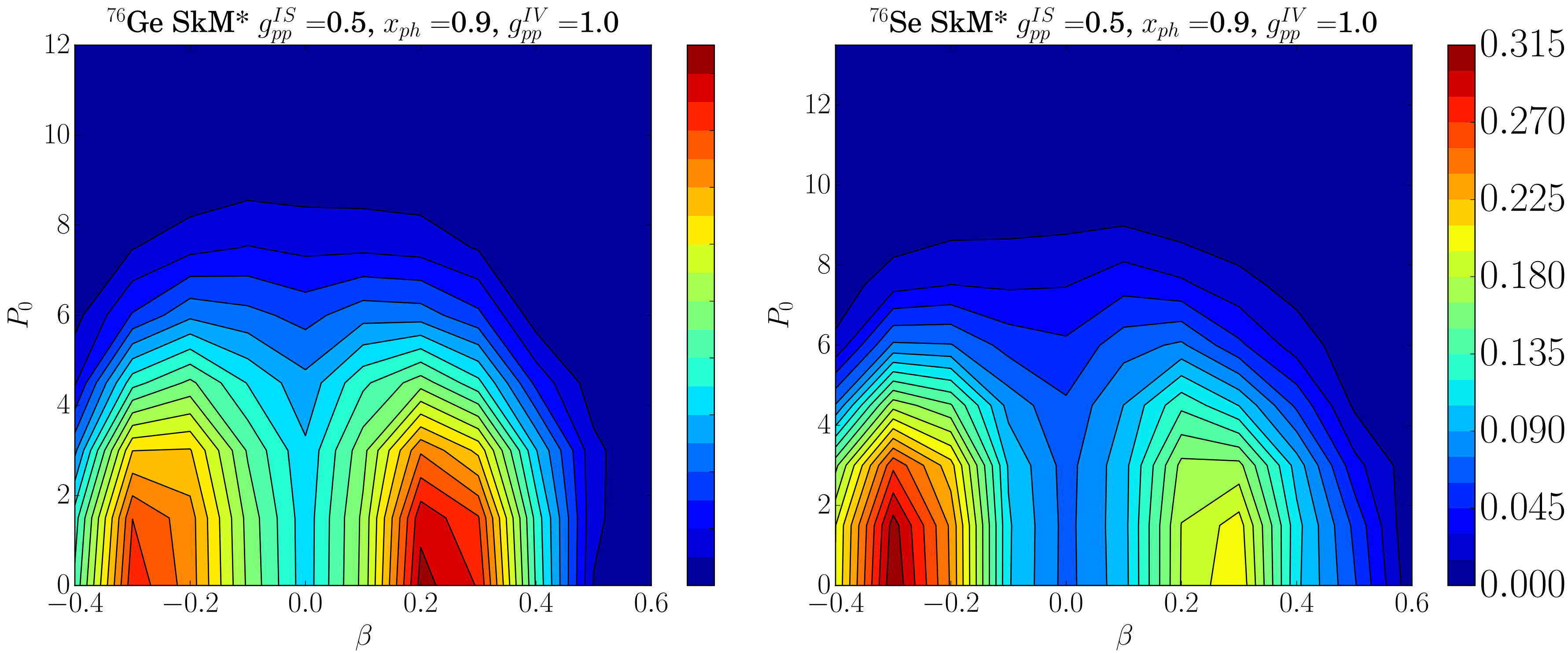
^{76}Ge SkM* g_{pp}^{IS} =0.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32



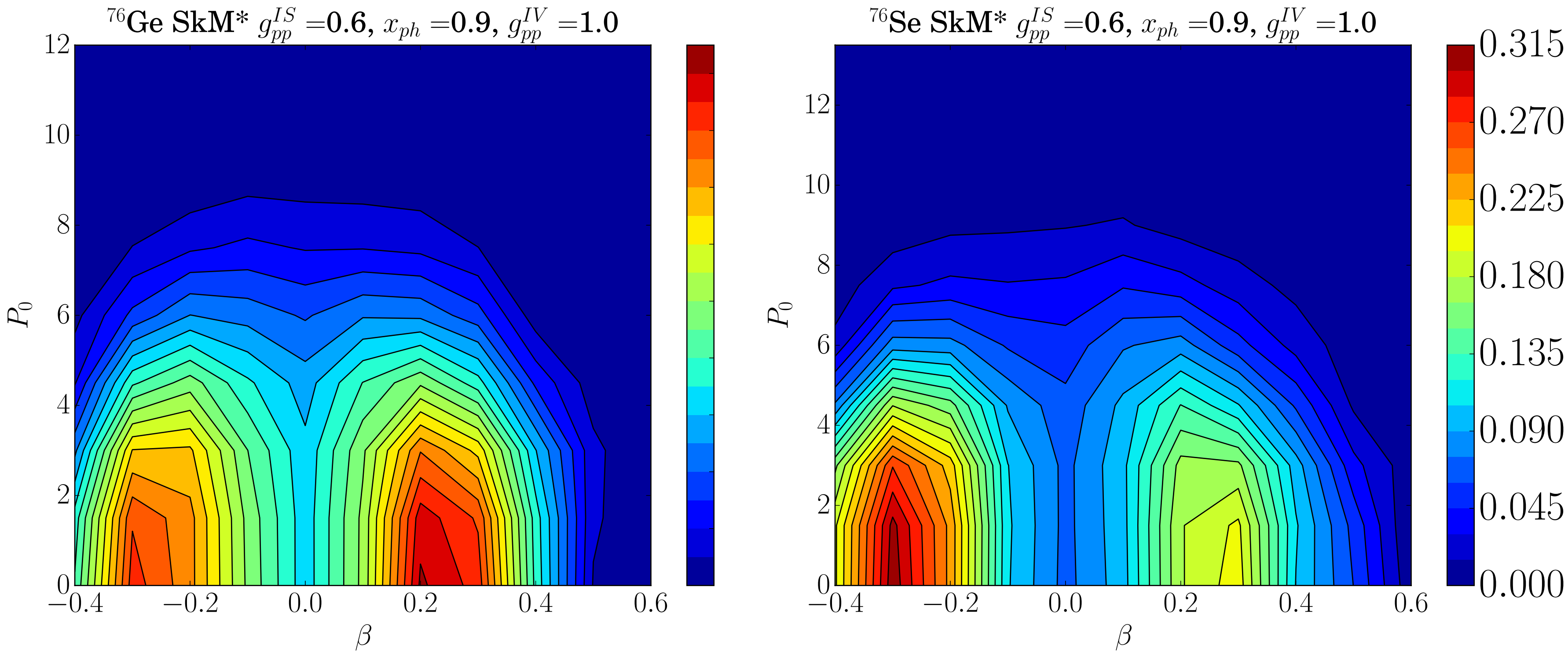
^{76}Ge SkM* g_{pp}^{IS} =0.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32



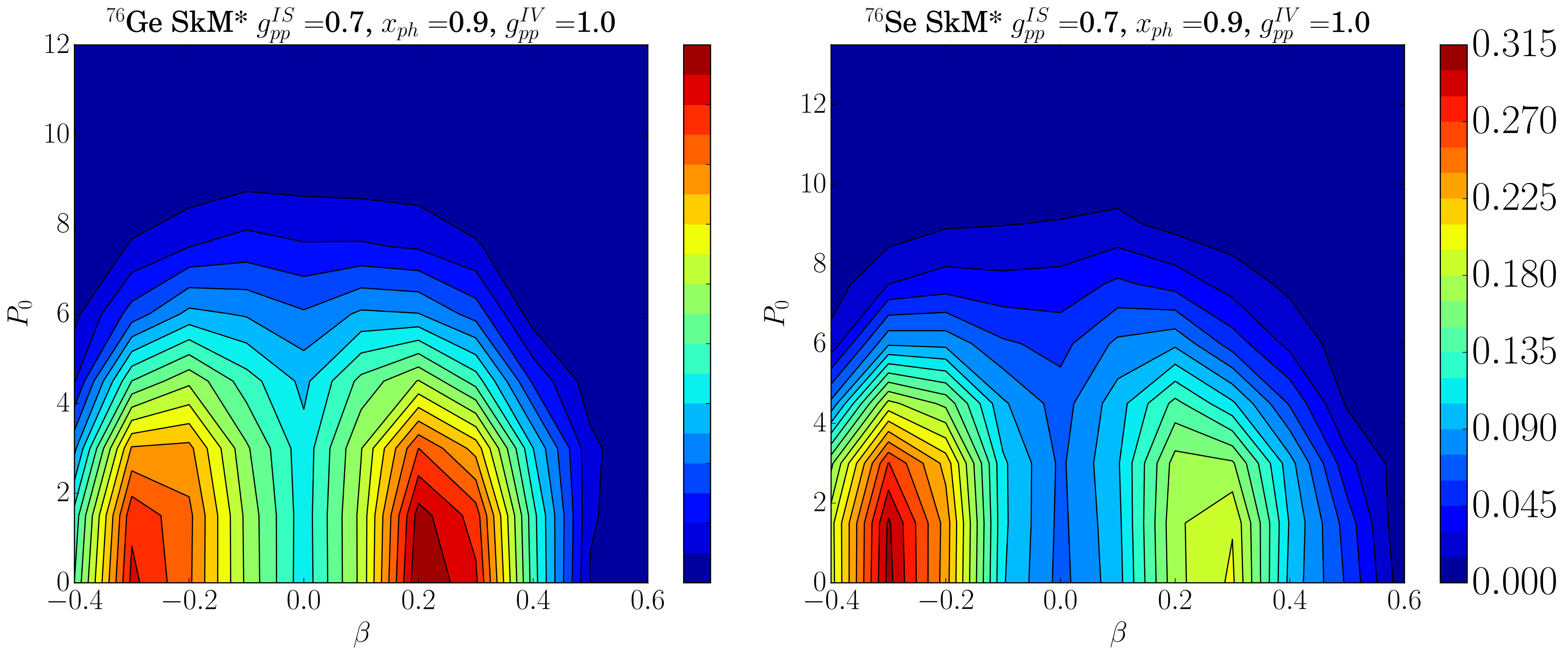
^{76}Ge SkM* g_{pp}^{IS} =0.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0.00



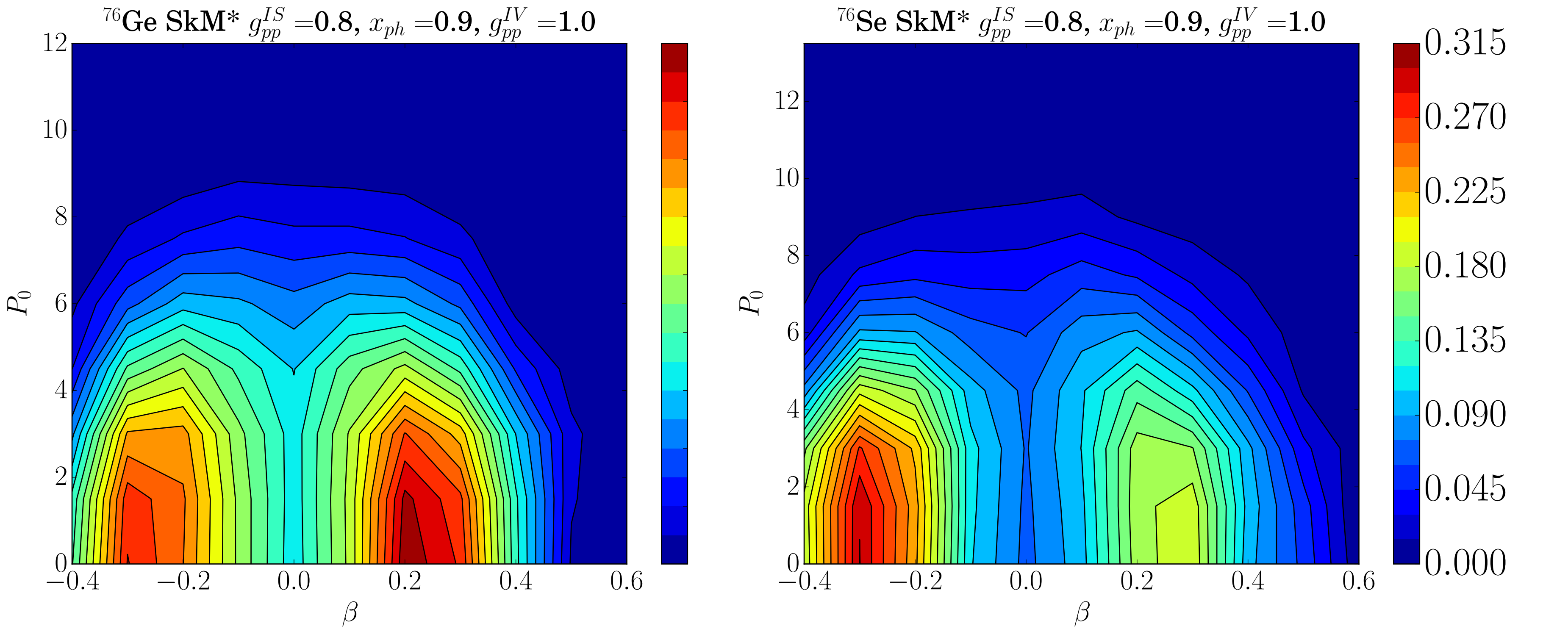
^{76}Ge SkM* g_{pp}^{IS} =0.5, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.5, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.315
0.270
0.225
0.180
0.135
0.090
0.045
0.000

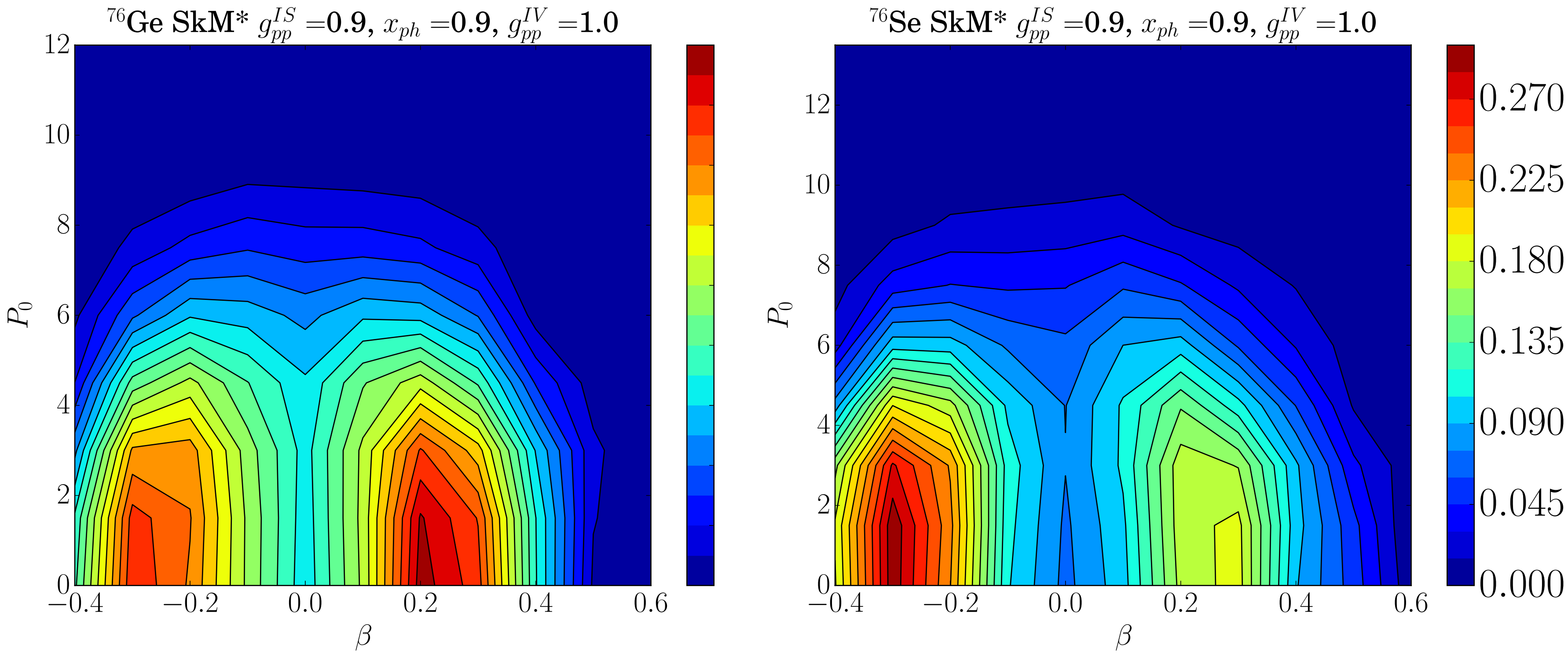


^{76}Ge SkM* g_{pp}^{IS} =0.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0 2 4 6 8 10 12
−0.4 −0.2 0.0 0.2 0.4 0.6
0.000 0.045 0.090 0.135 0.180 0.225 0.270 0.315

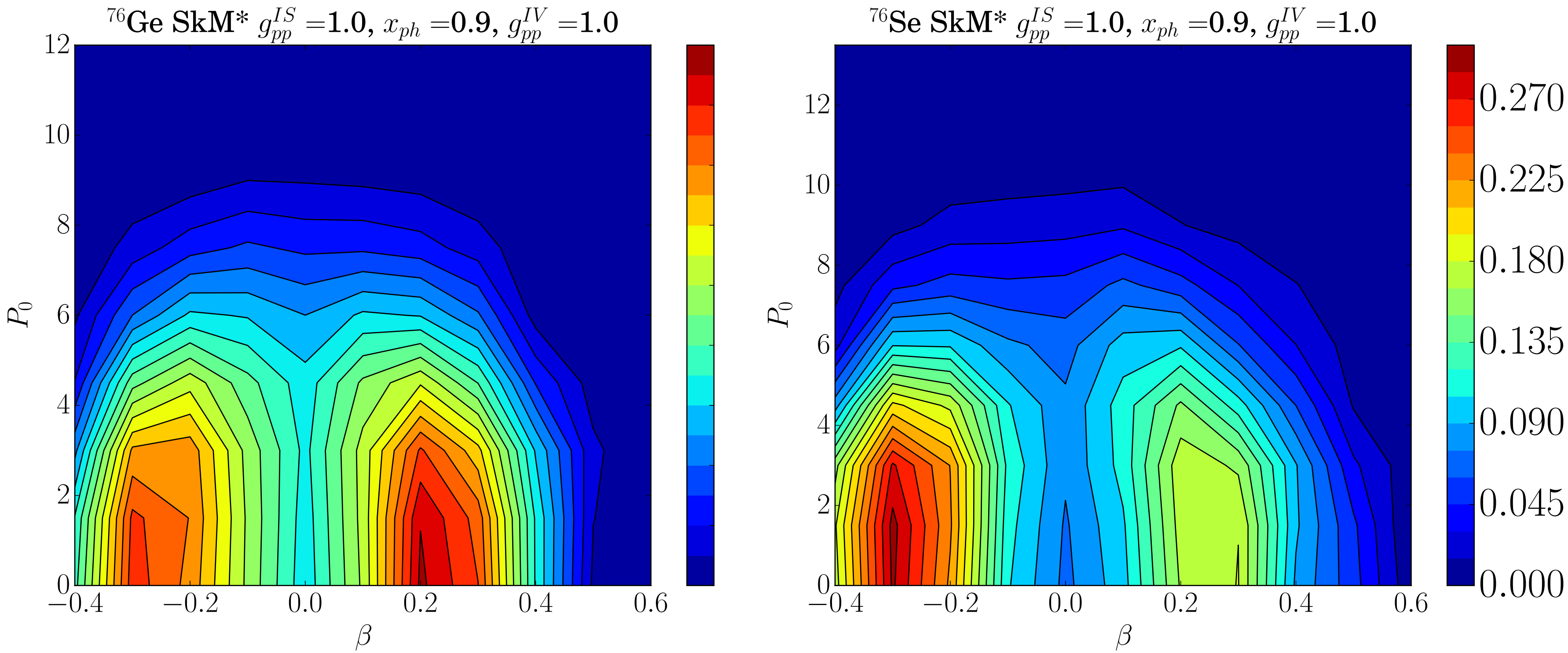


^{76}Ge SkM* g_{pp}^{IS} =0.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.315
0.270
0.225
0.180
0.135
0.090
0.045
0.000


^{76}Ge SkM* g_{pp}^{IS} =0.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.000
0.045
0.090
0.135
0.180
0.225
0.270
0.315

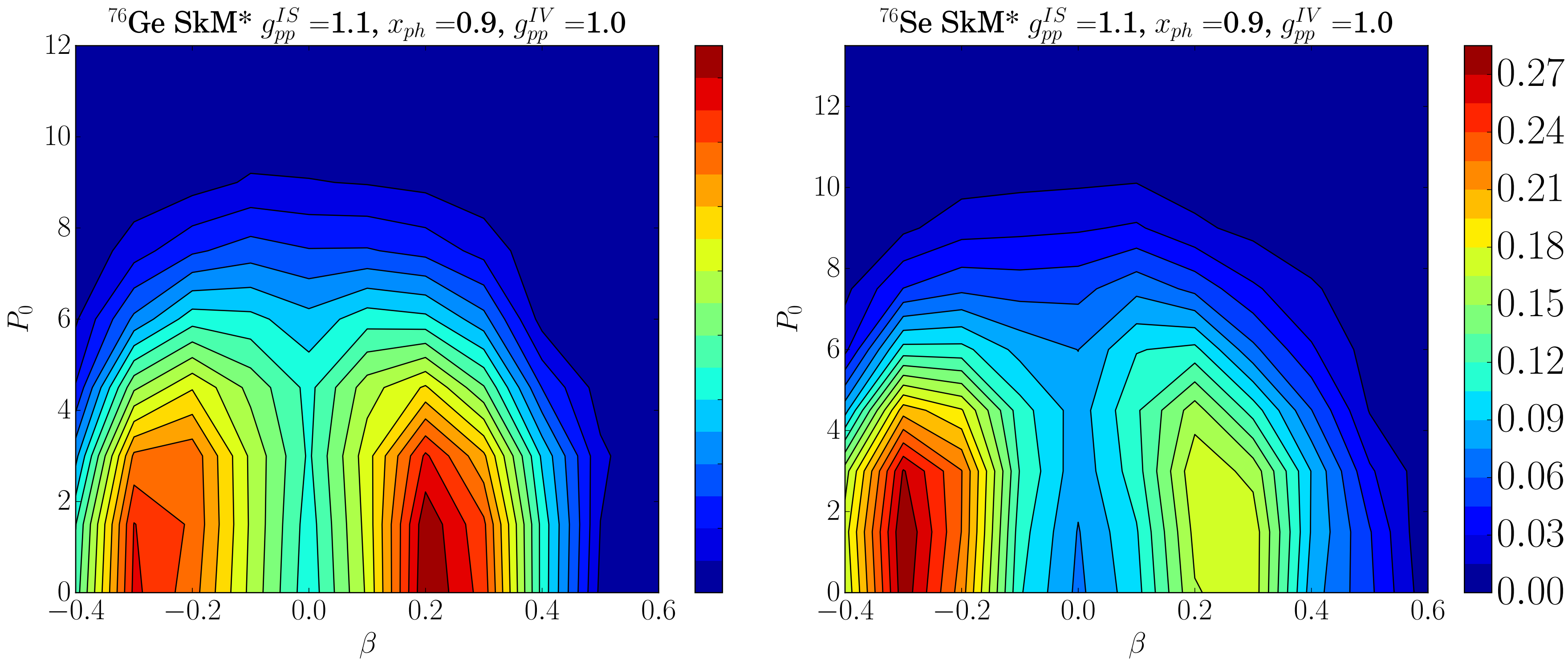


^{76}Ge SkM* g_{pp}^{IS} =0.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =0.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.270
0.225
0.180
0.135
0.090
0.045
0.000

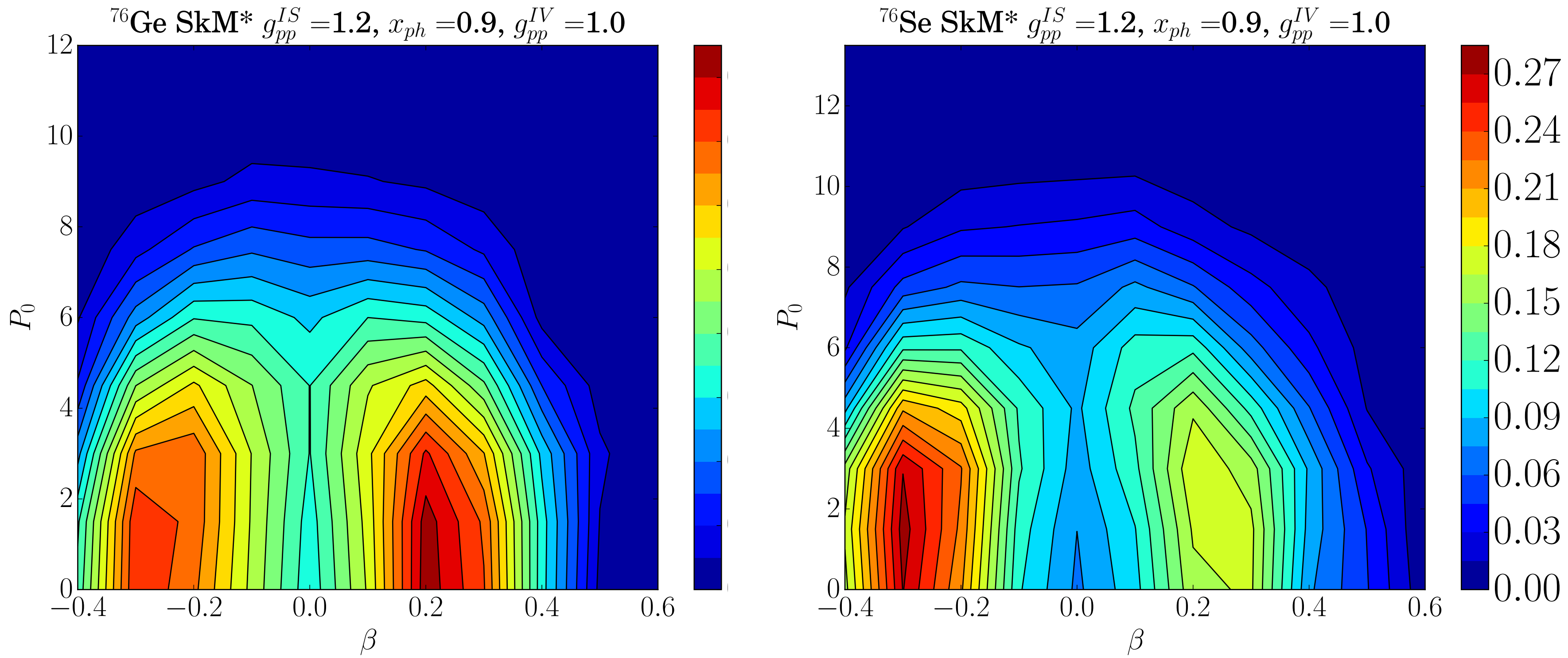


^{76}Ge SkM* g_{pp}^{IS} =1.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.270
0.225
0.180
0.135
0.090
0.045
0.000

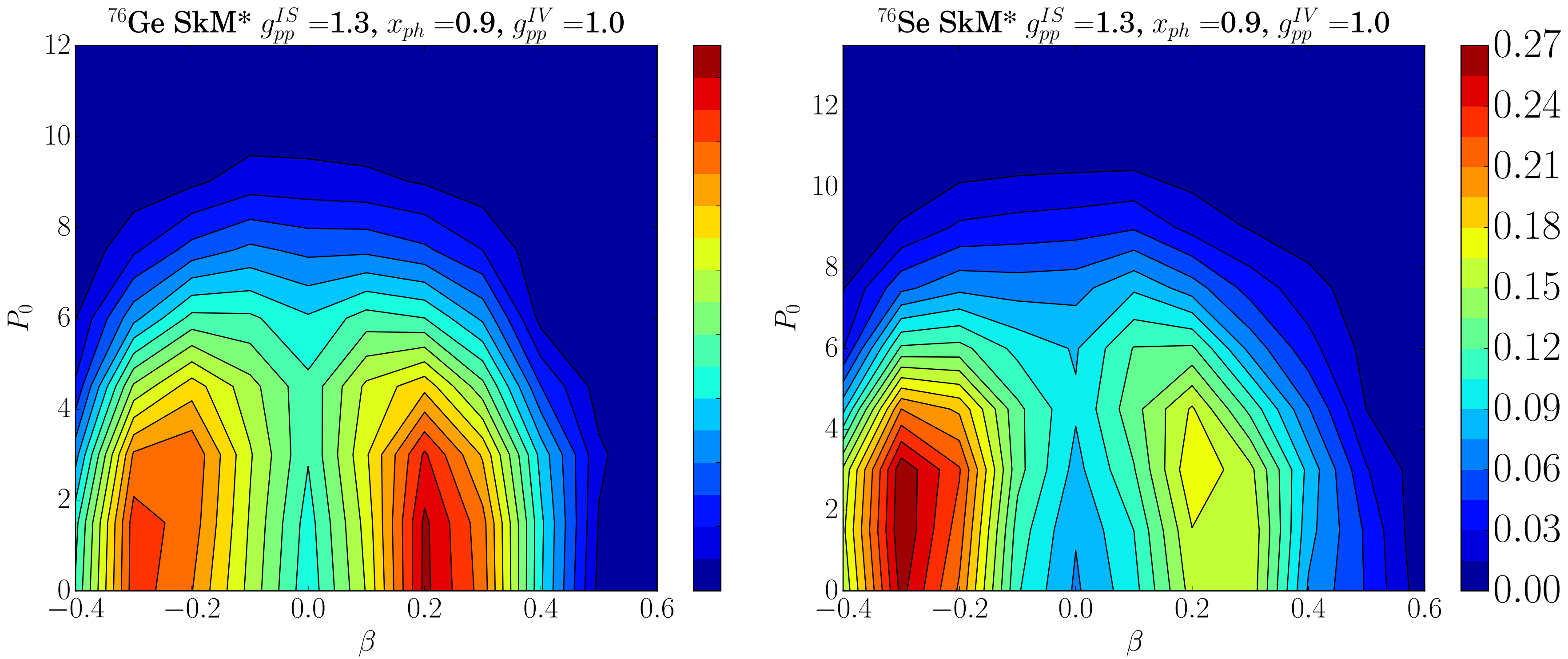
$g^{T=0}/g^{T=1} = 1.0$



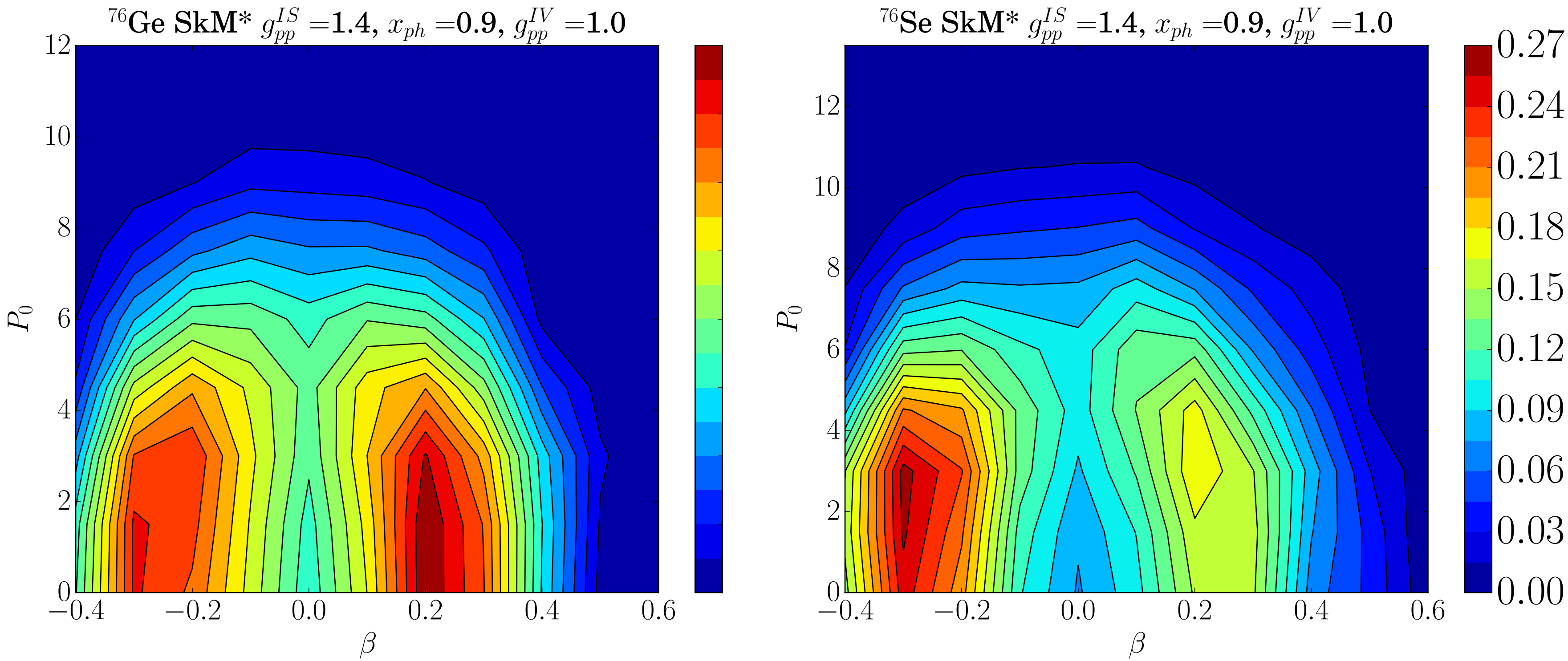
^{76}Ge SkM* g_{pp}^{IS} =1.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0 2 4 6 8 10 12
−0.4 −0.2 0.0 0.2 0.4 0.6
0.00 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24 0.27

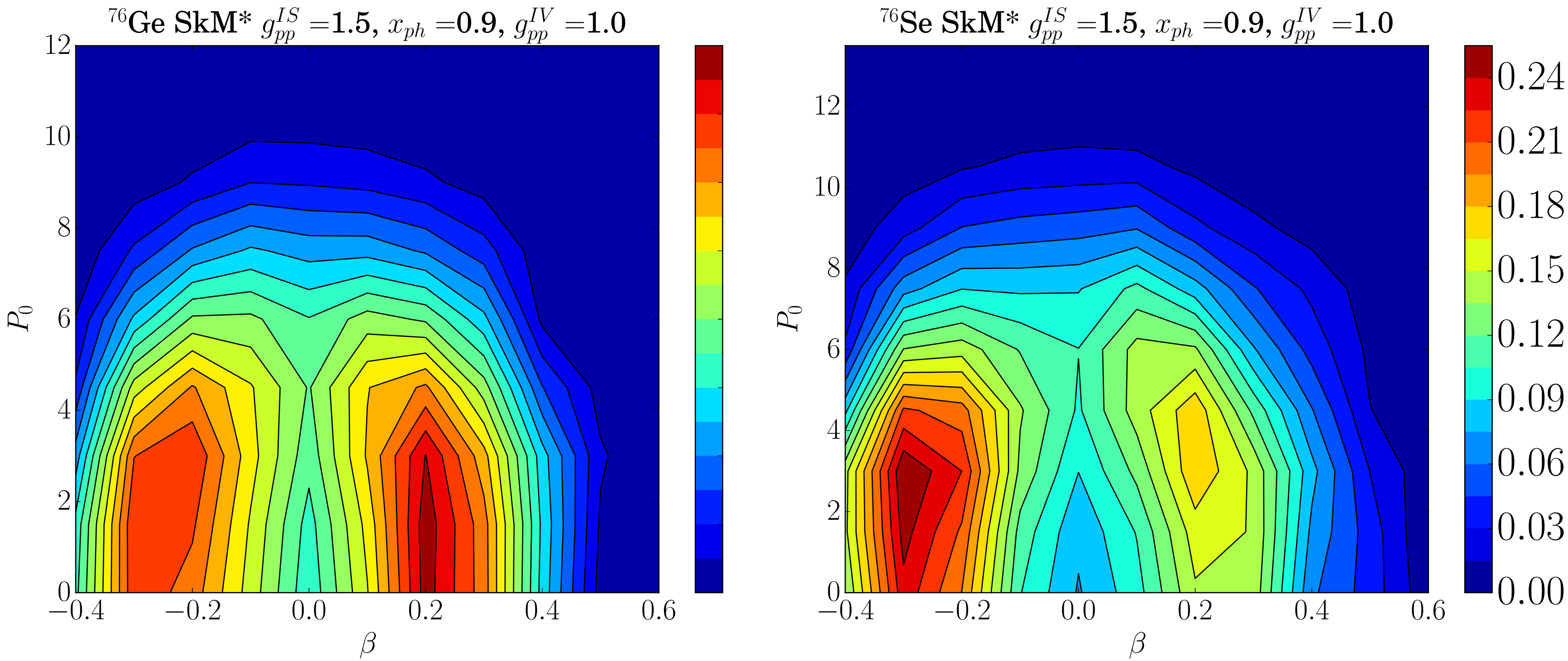


^{76}Ge SkM* g_{pp}^{IS} =1.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
0.27



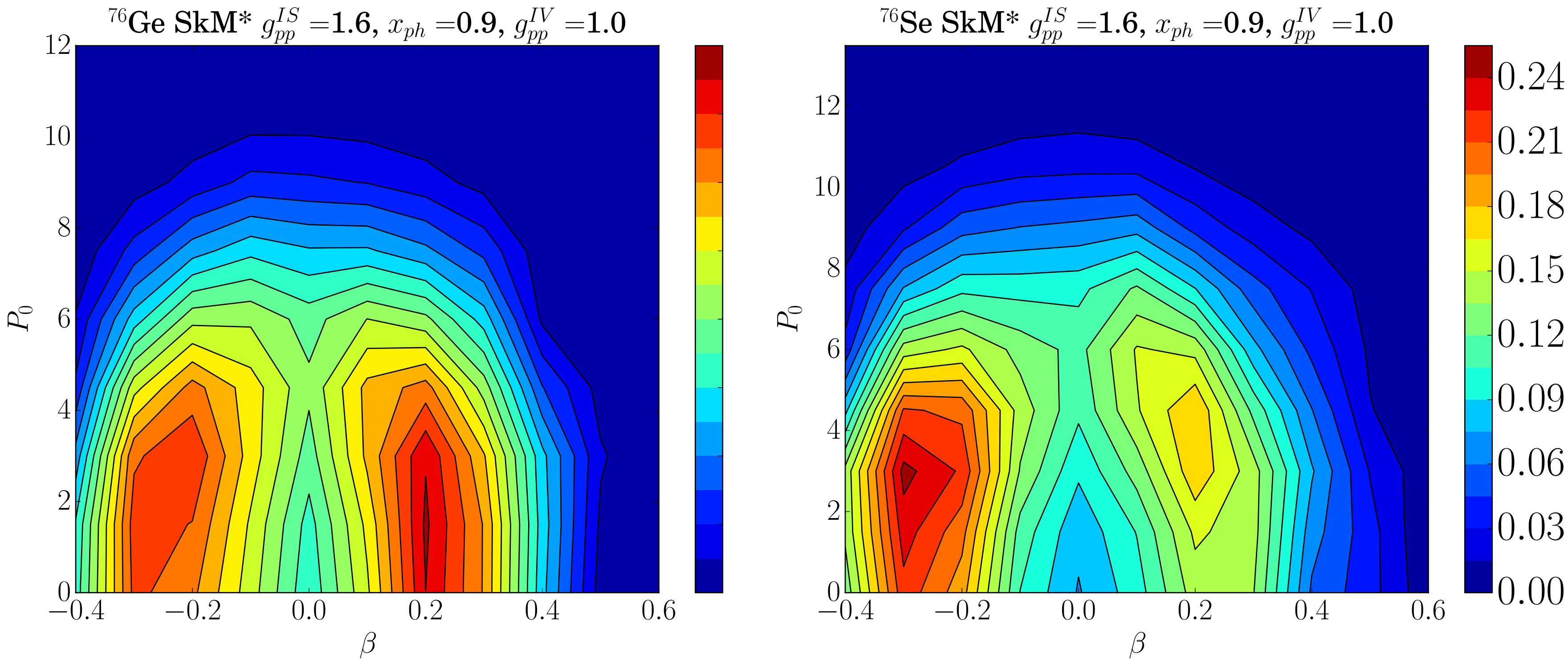
^{76}Ge SkM* g_{pp}^{IS} =1.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
0.27


^{76}Ge SkM* g_{pp}^{IS} =1.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.27
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

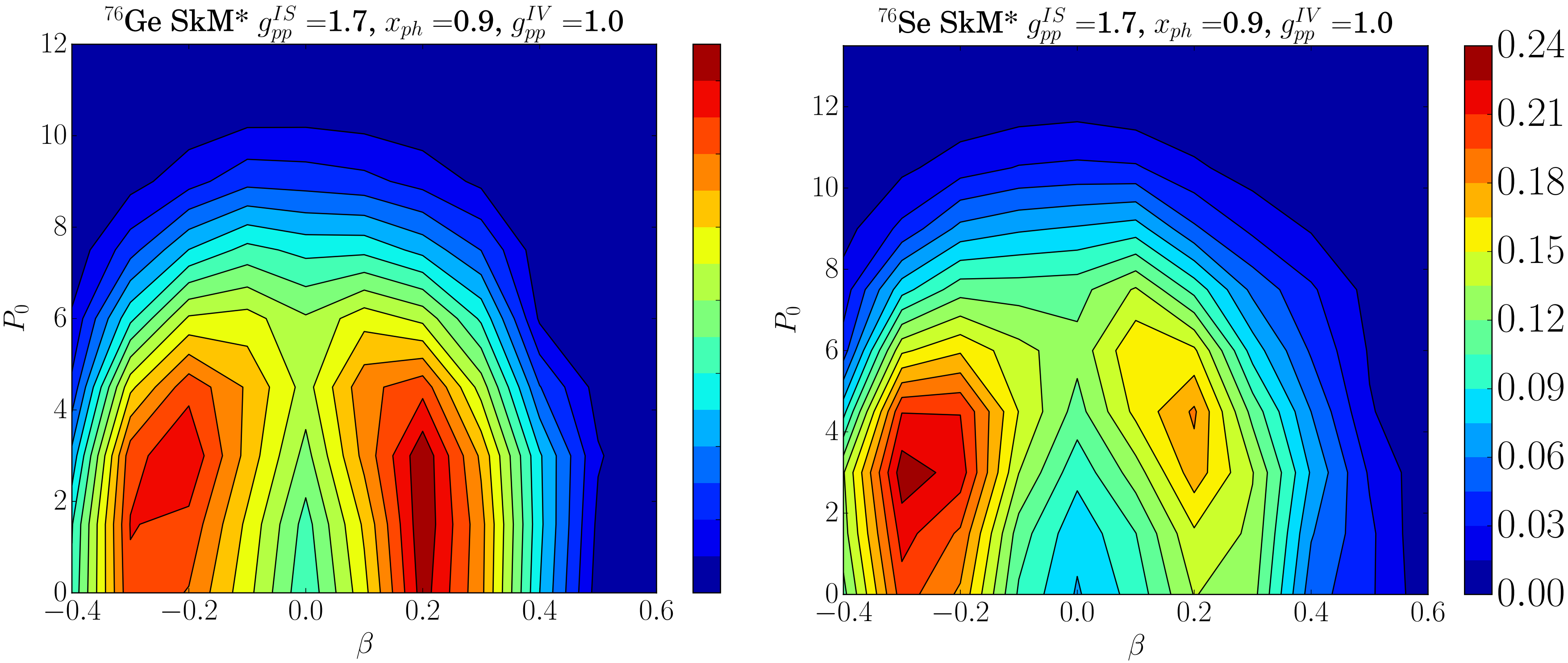


76Ge SkM* gppIS =1.5, xph =0.9, gppIV =1.0
76Se SkM* gppIS =1.5, xph =0.9, gppIV =1.0
P0
β
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

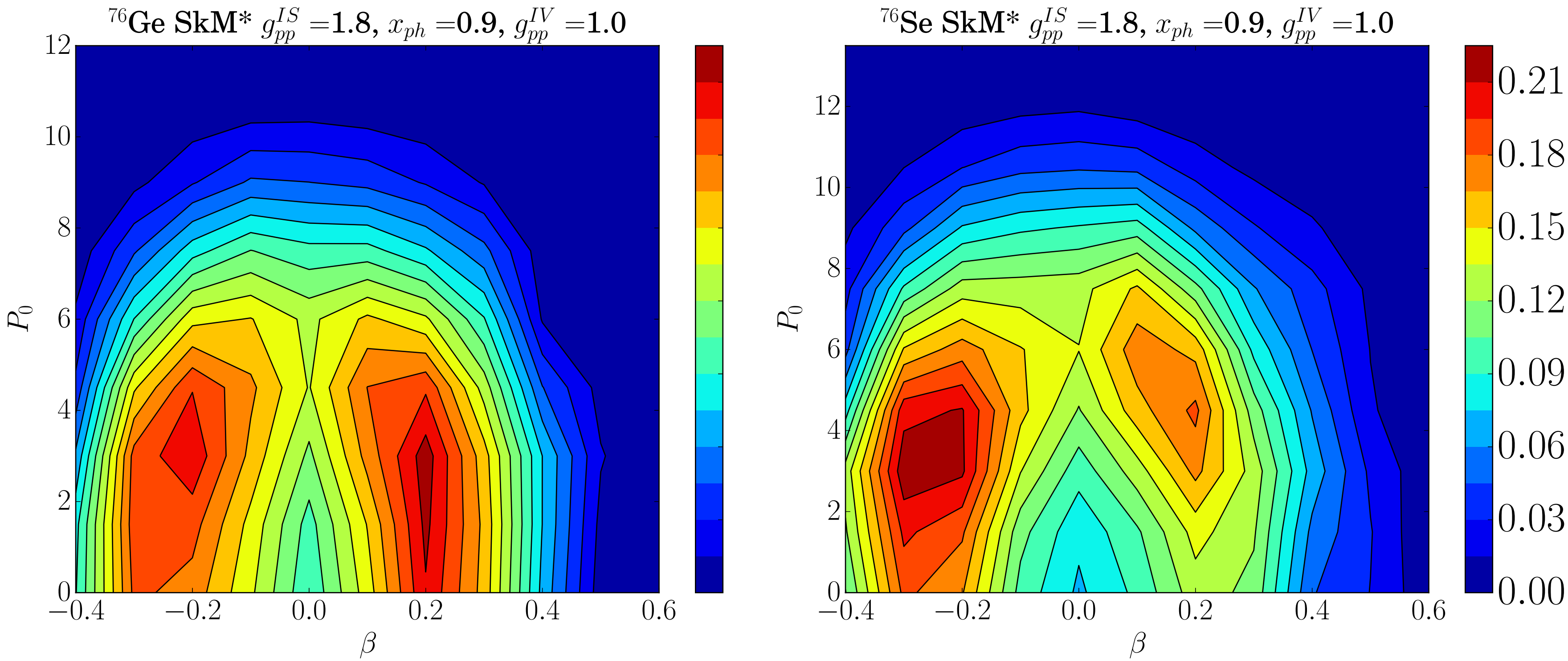
$g^{T=0}/g^{T=1} = 1.5$



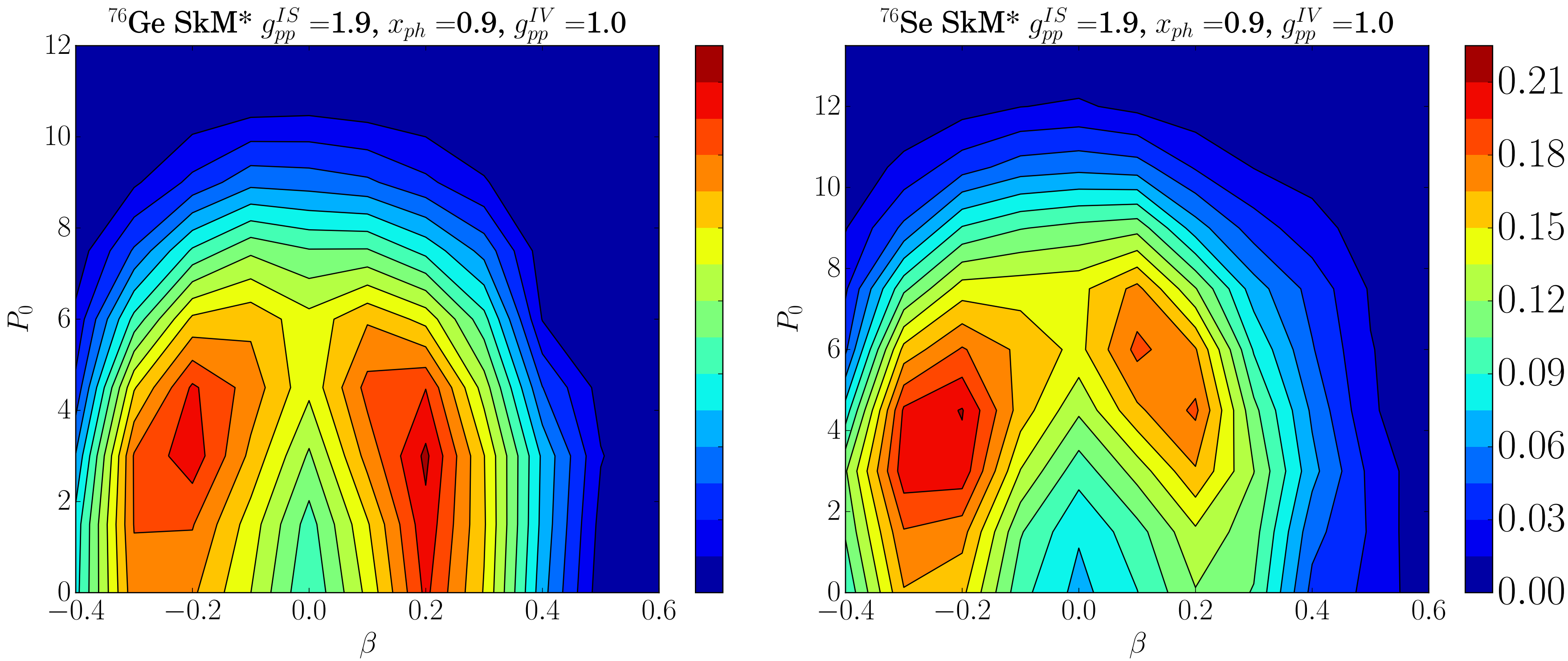
^{76}Ge SkM* g_{pp}^{IS} =1.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
^{76}Se SkM* g_{pp}^{IS} =1.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24



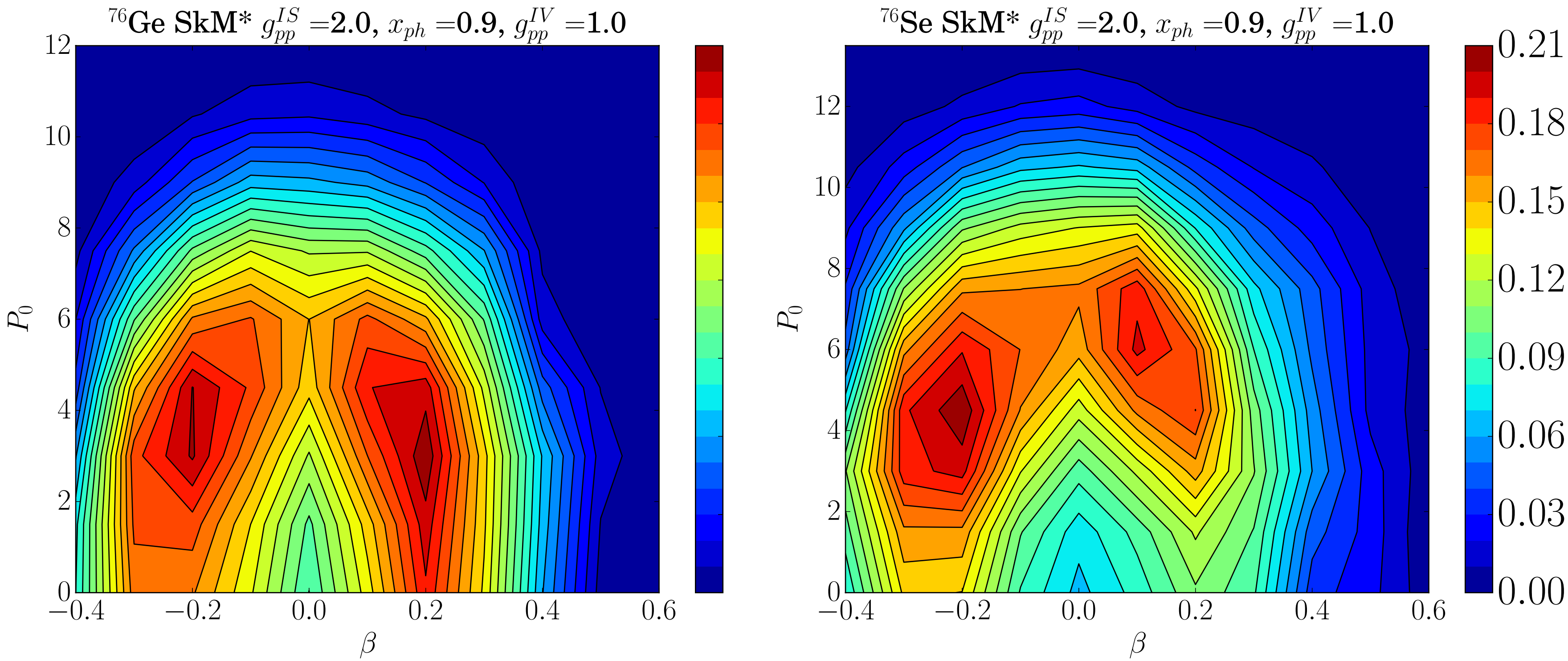
^{76}Ge SkM* g_{pp}^{IS} =1.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24



^{76}Ge SkM* g_{pp}^{IS} =1.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

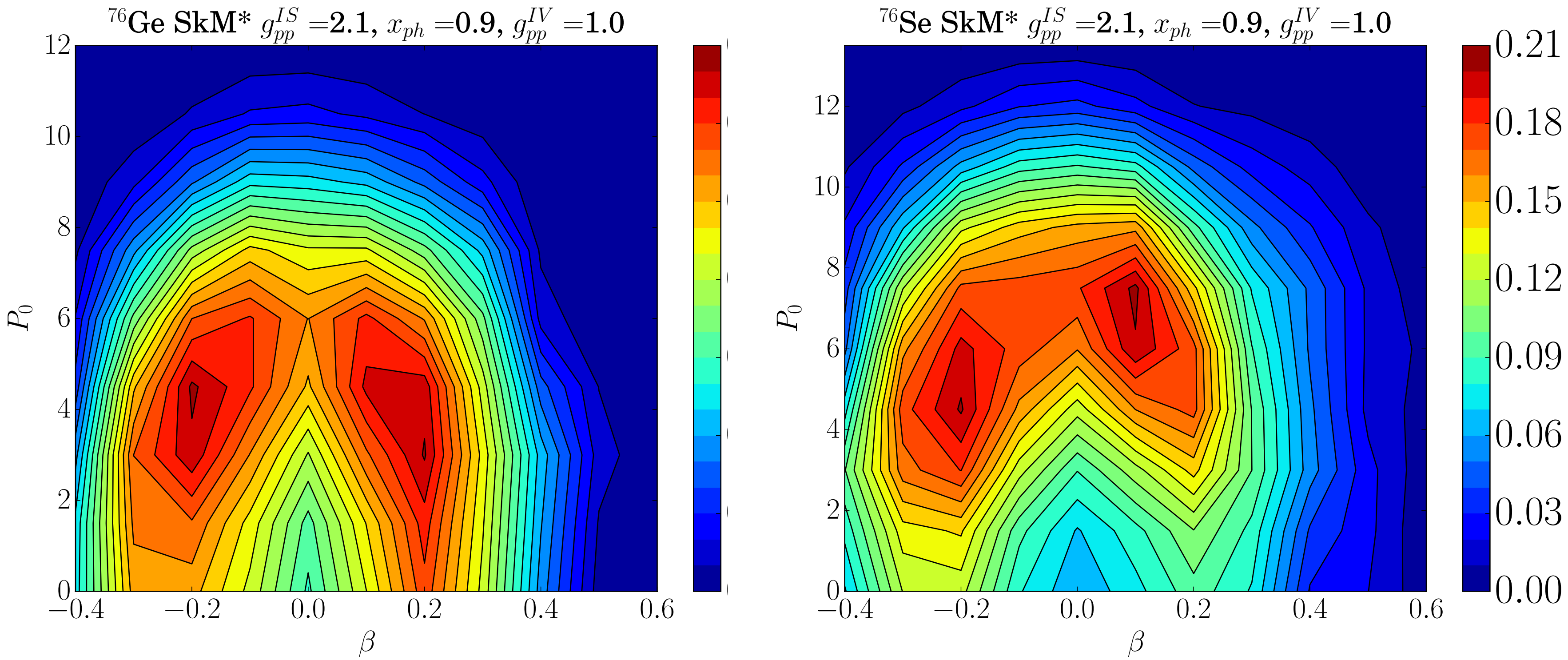


^{76}Ge SkM* g_{pp}^{IS} =1.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

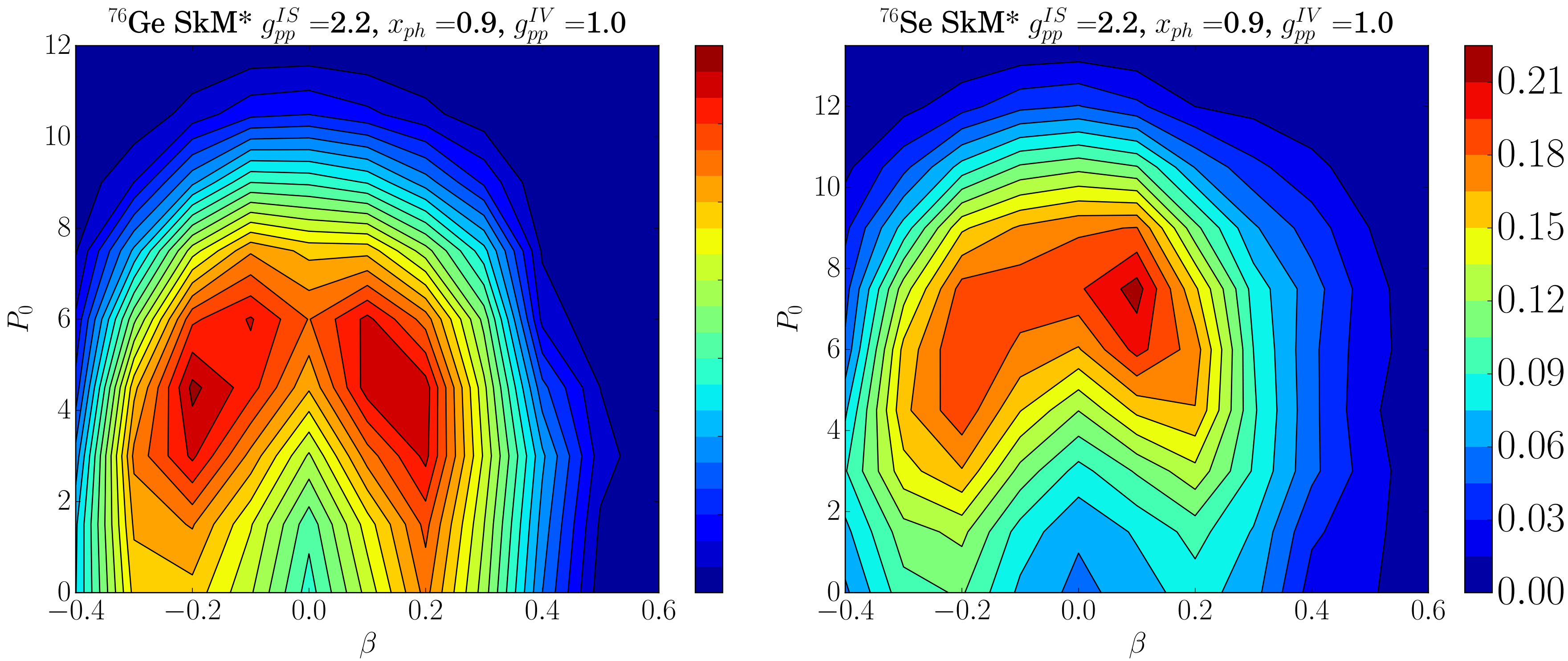


^{76}Ge SkM* g_{pp}^{IS} =2.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

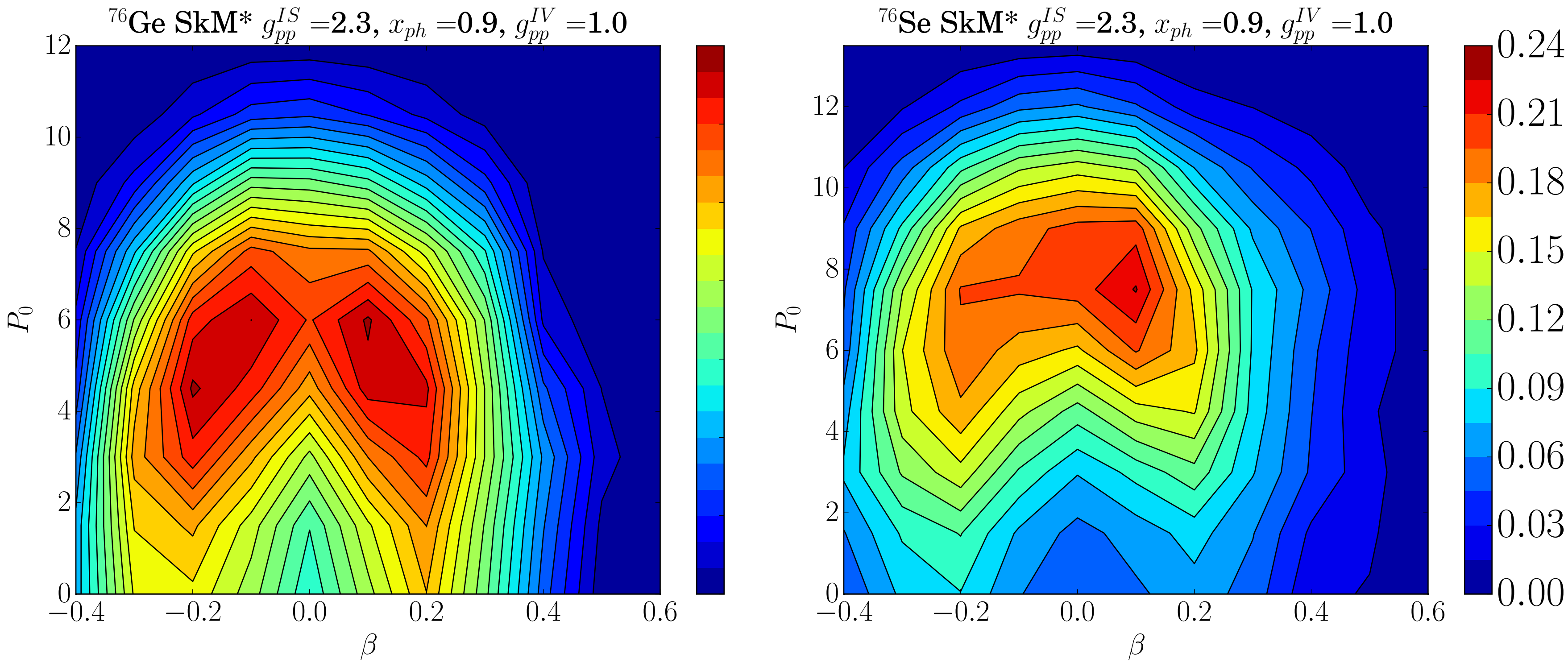
$g^{T=0}/g^{T=1} = 2.0$



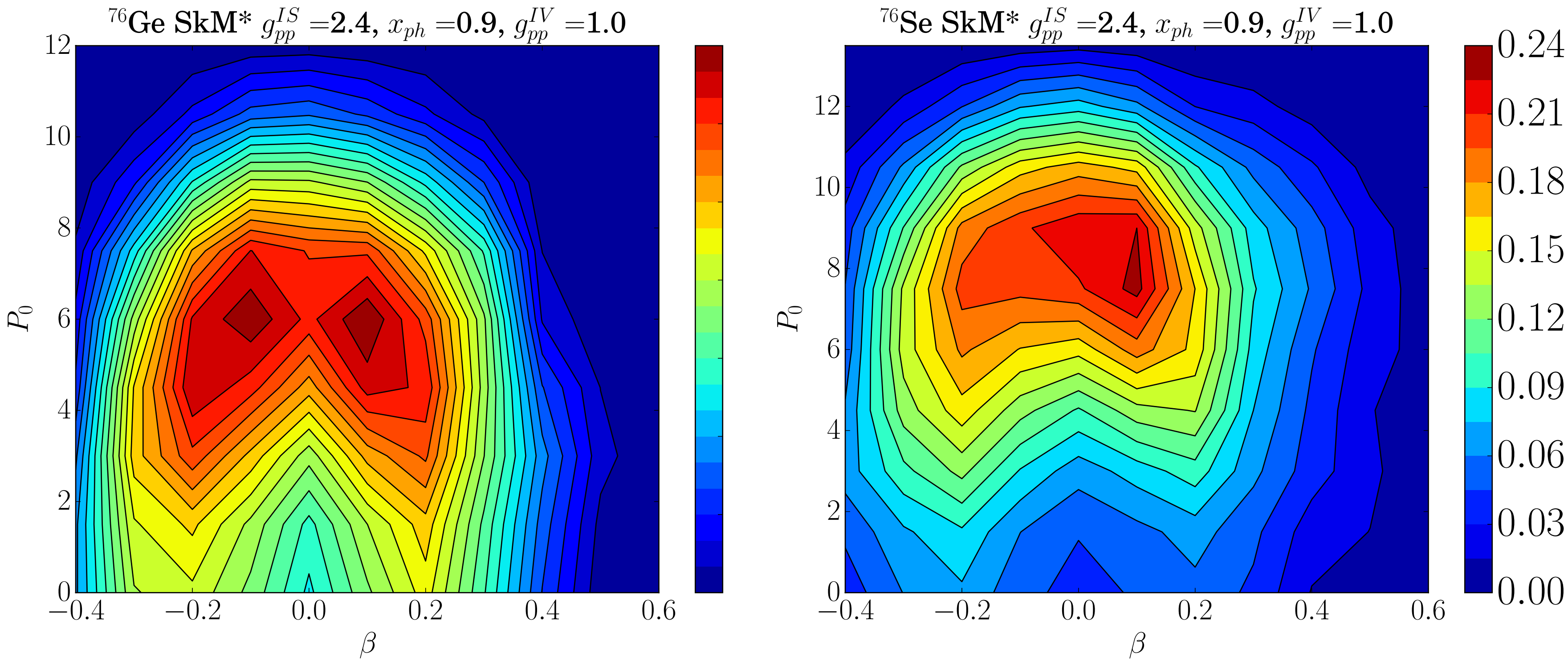
^{76}Ge SkM* g_{pp}^{IS} =2.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
β
^{76}Se SkM* g_{pp}^{IS} =2.1, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
β
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00



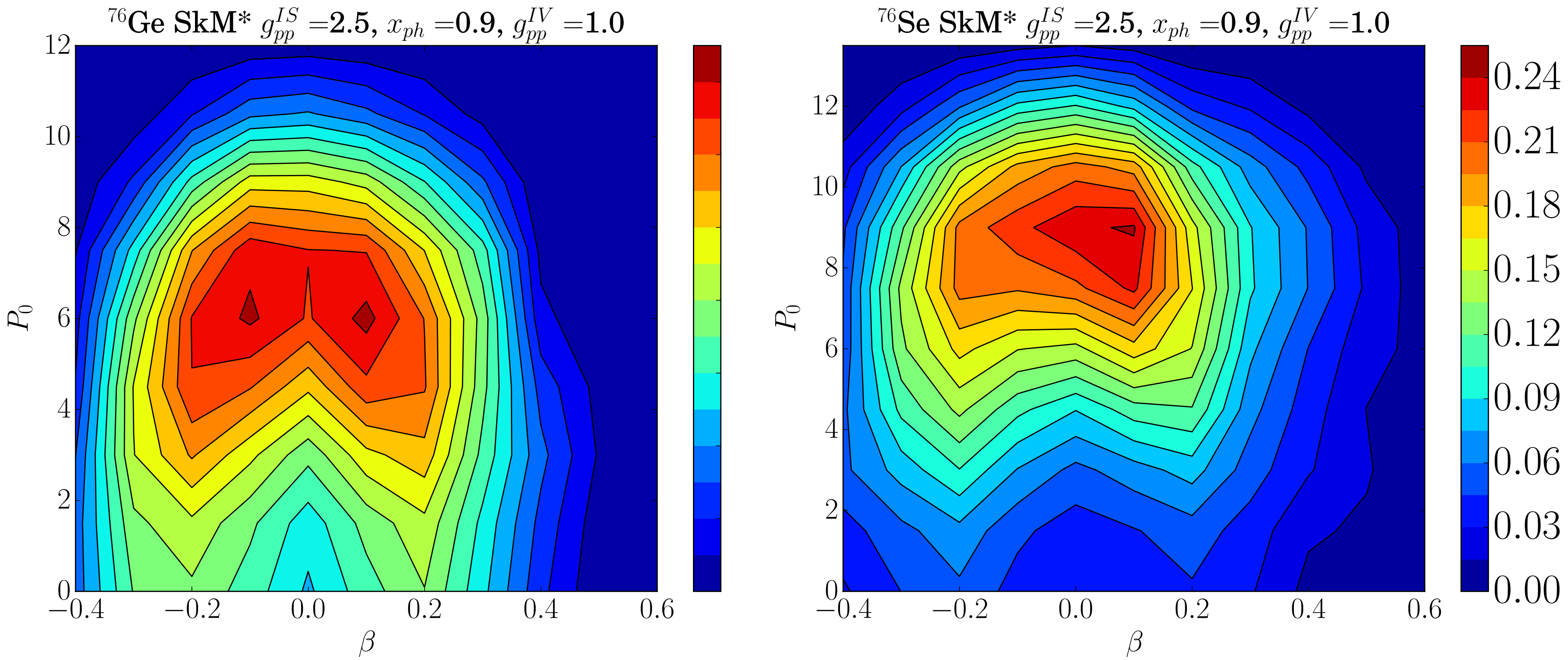
^{76}Ge SkM* g_{pp}^{IS} =2.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
^{76}Se SkM* g_{pp}^{IS} =2.2, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

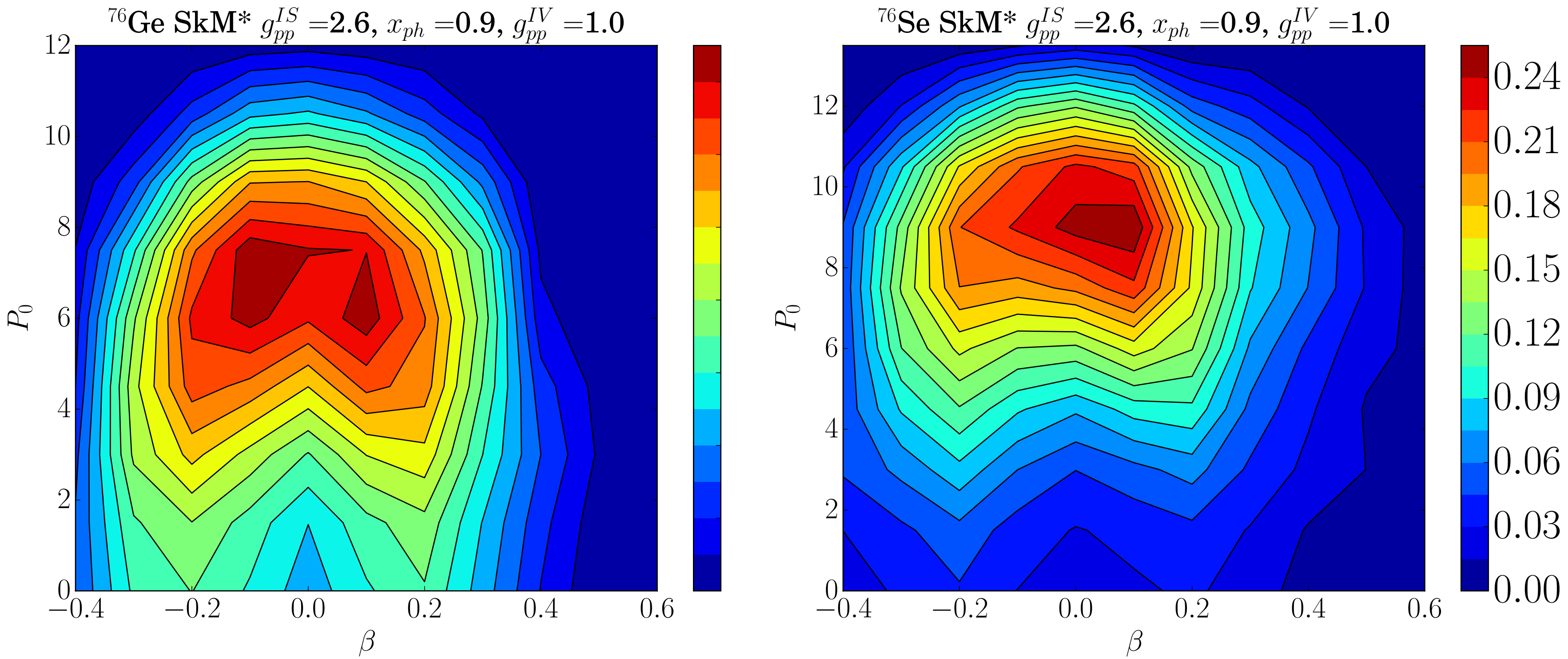


^{76}Ge SkM* g_{pp}^{IS} =2.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.3, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24

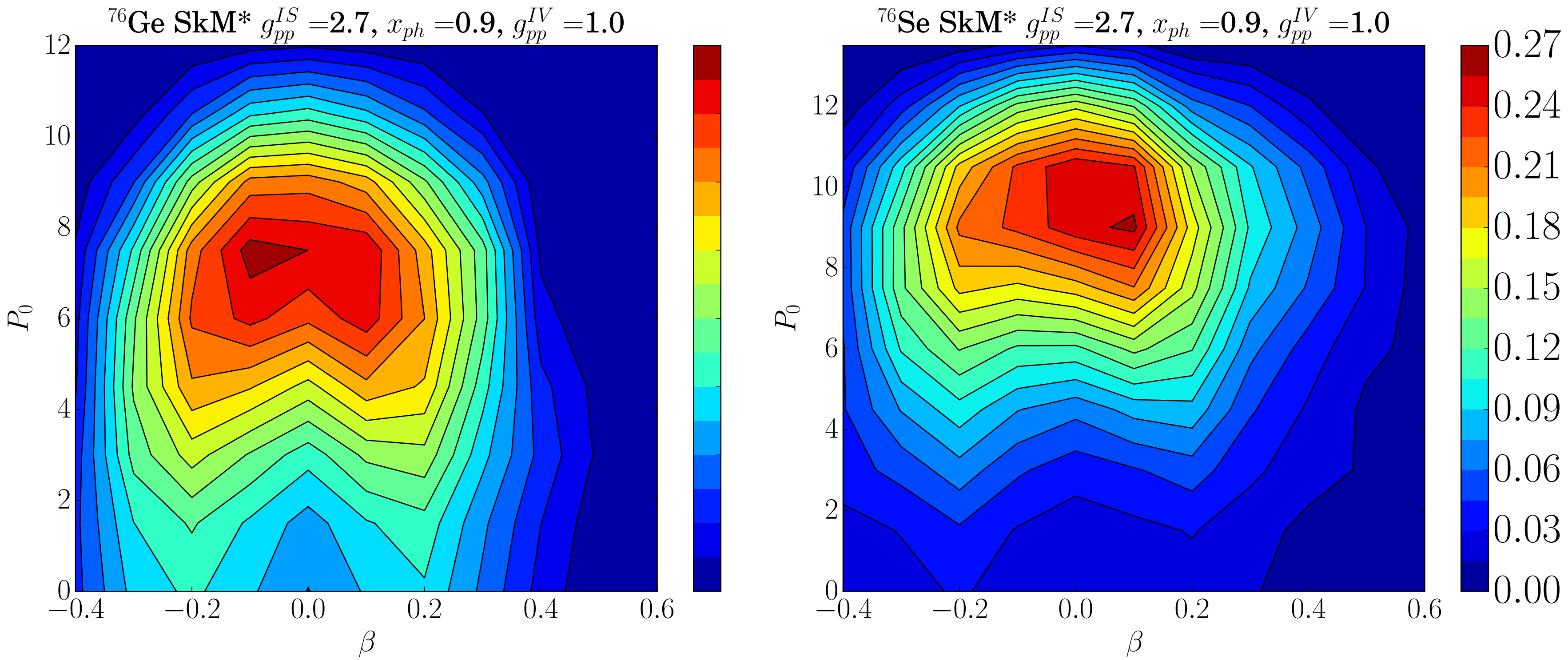


^{76}Ge SkM* g_{pp}^{IS} =2.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.4, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

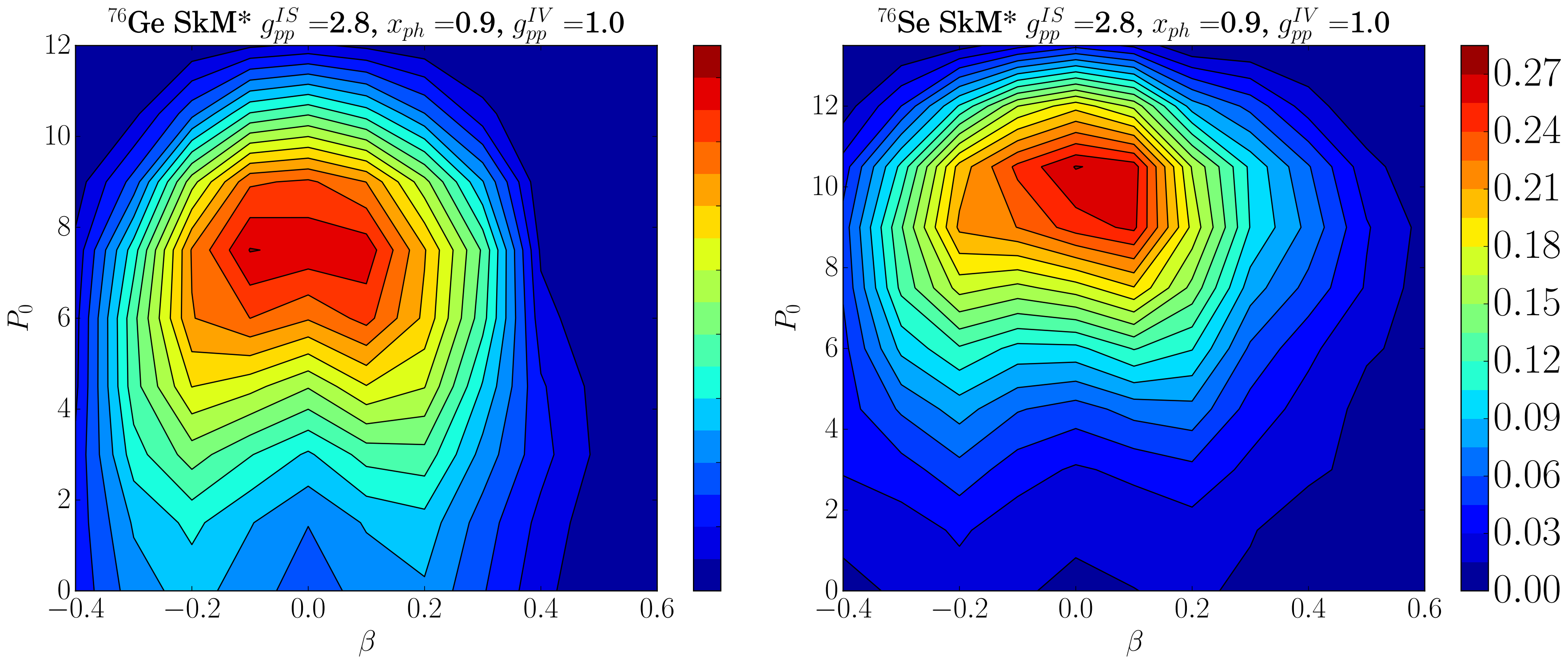

^{76}Ge SkM* g_{pp}^{IS} =2.5, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
^{76}Se SkM* g_{pp}^{IS} =2.5, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00



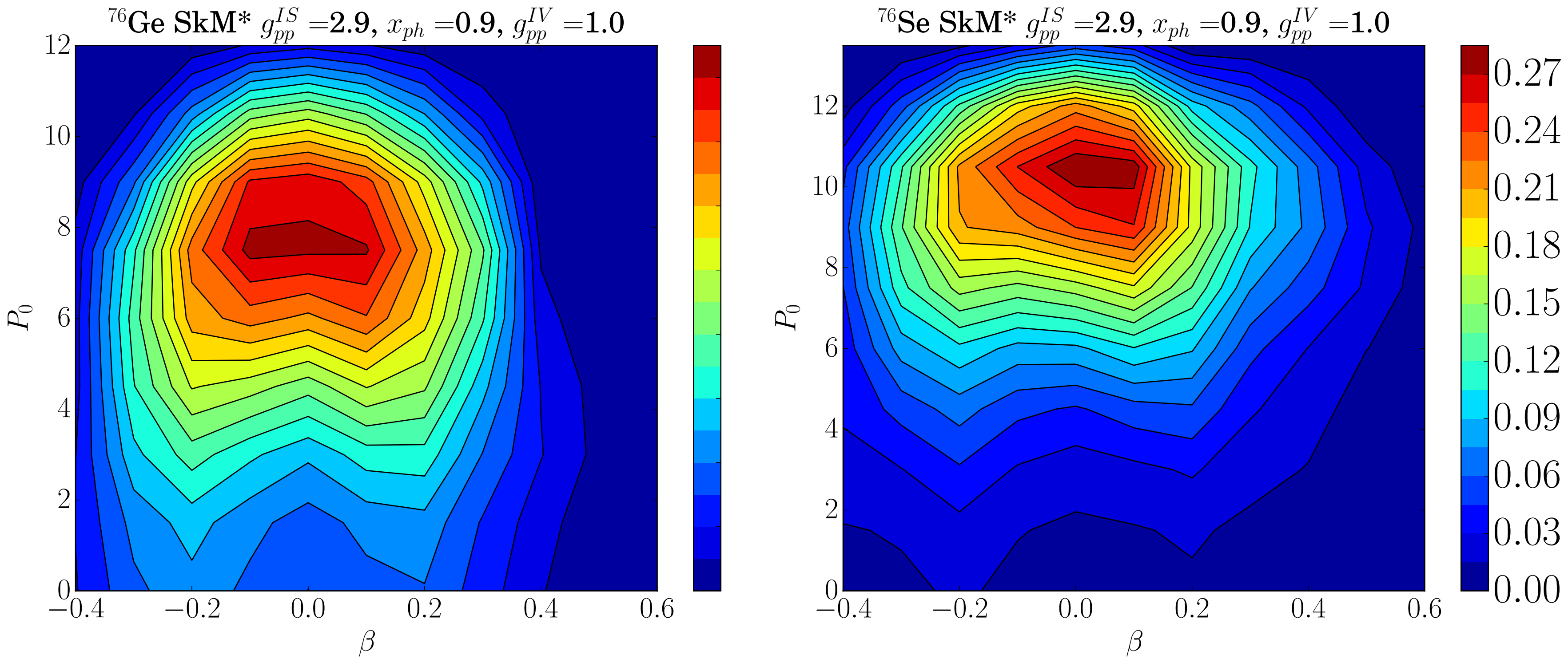
^{76}Ge SkM* g_{pp}^{IS} =2.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.6, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
−0.4
−0.2
0.0
0.2
0.4
0.6
0
2
4
6
8
10
12
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24



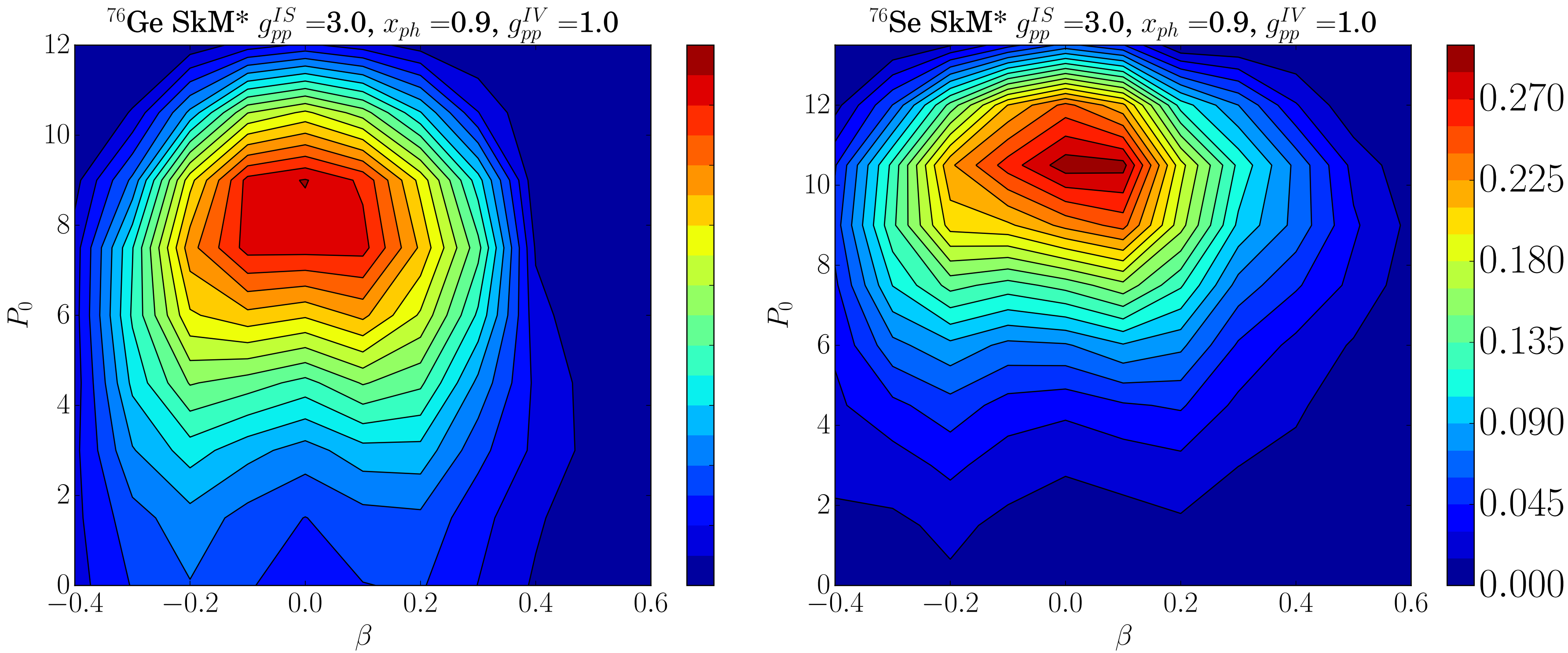
^{76}Ge SkM* g_{pp}^{IS} =2.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.7, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0 2 4 6 8 10 12
−0.4 −0.2 0.0 0.2 0.4 0.6
0.00 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24 0.27



^{76}Ge SkM* g_{pp}^{IS} =2.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.8, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.27
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00

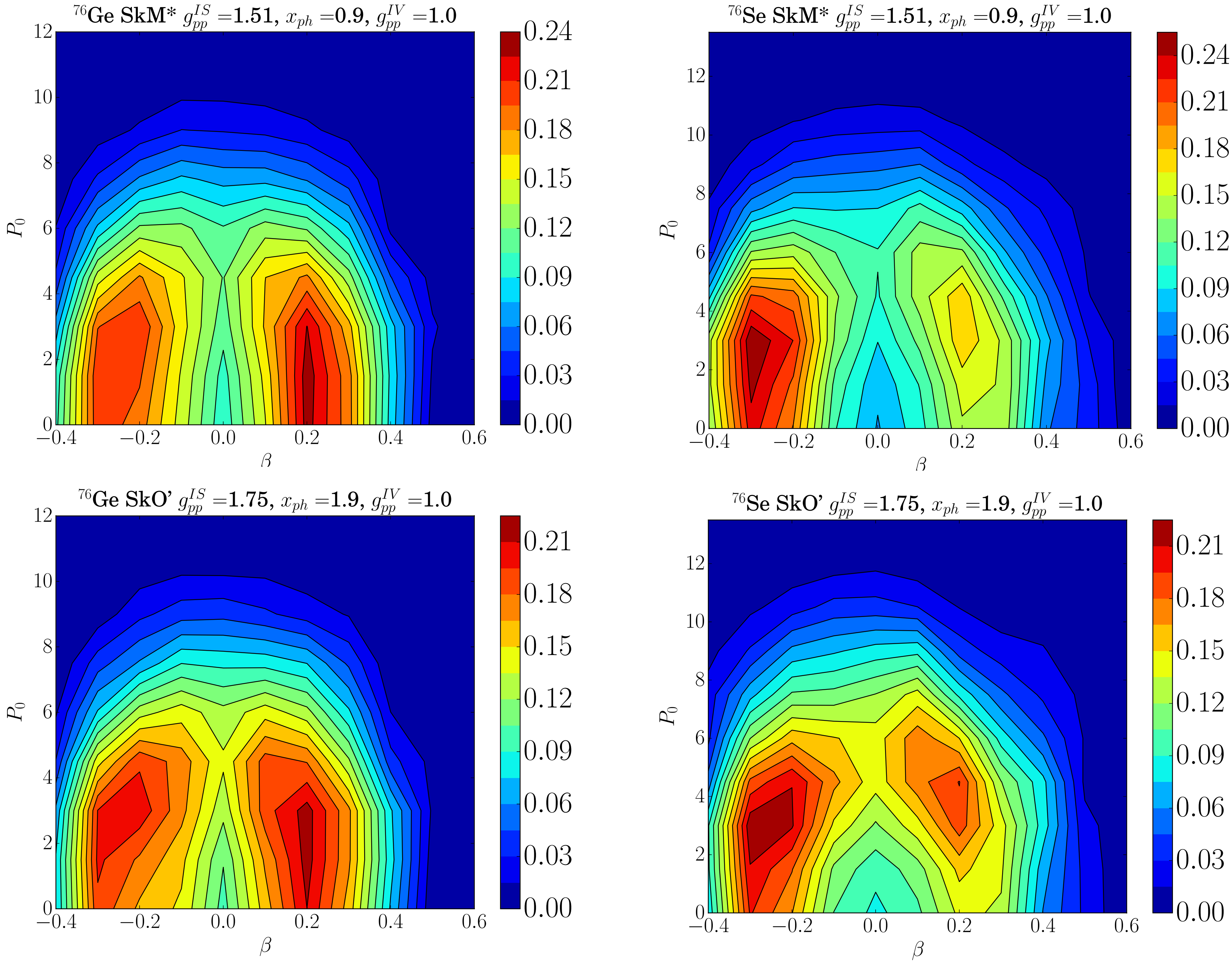


^{76}Ge SkM* g_{pp}^{IS} =2.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =2.9, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0
2
4
6
8
10
12
−0.4
−0.2
0.0
0.2
0.4
0.6
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
0.27



^{76}Ge SkM* g_{pp}^{IS} =3.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =3.0, x_{ph} =0.9, g_{pp}^{IV} =1.0
P_0
β
0.000
0.045
0.090
0.135
0.180
0.225
0.270

$g^{T=0}/g^{T=1} = 3.0$


^{76}Ge SkM* g_{pp}^{IS} =1.51, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Se SkM* g_{pp}^{IS} =1.51, x_{ph} =0.9, g_{pp}^{IV} =1.0
^{76}Ge SkO' g_{pp}^{IS} =1.75, x_{ph} =1.9, g_{pp}^{IV} =1.0
^{76}Se SkO' g_{pp}^{IS} =1.75, x_{ph} =1.9, g_{pp}^{IV} =1.0
P_0
β
−0.4 −0.2 0.0 0.2 0.4 0.6
0 2 4 6 8 10 12
0.00 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24

Summary

- 0nu nuclear matrix elements are calculated using generator coordinate method including both axial quadrupole deformation and isoscalar/ isovector proton-neutron pairing degrees of freedom.

- The approach explores the physics of beyond QRPA and shell model
  - accurate description of pn correlation
  - large single-particle model space

Future extensions

- Improve effective interaction (from shell model)
- Inclusion of triaxiality
- Formulation based on DFT (theoretical problems in projections)