Extending the Eikonal Approximation to Low Energy

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Halo nuclei

Exotic nuclear structures are found far from stability
In particular halo nuclei with
peculiar quantal structure :

- Light, n-rich nuclei
- Low S_n or S_{2n}

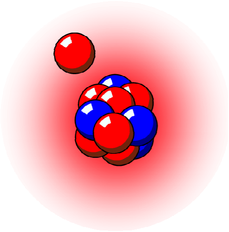


Exhibit large matter radius
due to strongly clusterised structure :
neutrons tunnel far from the core and form a halo

One-neutron halo

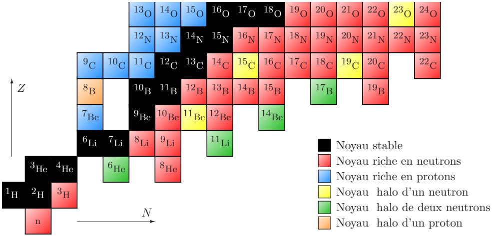
^{11}Be ≡ ^{10}Be + n

^{15}C ≡ ^{14}C + n

Two-neutron halo

^{6}He ≡ ^{4}He + n + n

^{11}Li ≡ ^{9}Li + n + n



Proton halœs are also possible, but less probable

Reactions with halo nuclei

Halo nuclei are fascinating objects
but difficult to study [$\tau_{1/2}$(^{6}He)= 0.8 s]
⇒ require indirect techniques, like reactions

Elastic scattering [see A. Di Pietro on Wednesday]

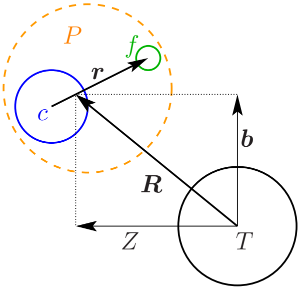
Breakup ≡ dissociation of halo from core
by interaction with target [see A. Bonaccorso on Thursday]

Need an good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction
coupled to a realistic model of projectile

Framework

Projectile (P) modelled as a two-body system :
core (c)+loosely bound nucleon (f) described by

$H_0 = T_r + V_{cf}(\boldsymbol{r})$



V_{cf} adjusted to reproduce bound state Φ_0 and resonances

Target T seen as structureless particle

P-T interaction simulated by optical potentials
$\Rightarrow$ breakup reduces to three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0(\boldsymbol{r})$

Continuum Discretised Coupled Channel (CDCC)

Solve the three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

by expanding Ψ on eigenstates of H_0

$$\Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \chi_i(\boldsymbol{R})\Phi_i(\boldsymbol{r}) \qquad \text{with } H_0\Phi_i = \epsilon_i\Phi_i$$

Leads to set of coupled-channel equations (hence CC) :

$$[T_R + \epsilon_i + V_{ii}]\chi_i + \sum_{j\neq i} V_{ij}\chi_j = E_T\chi_i,$$

with $V_{ij} = \langle\Phi_i|V_{cT} + V_{fT}|\Phi_j\rangle$

The continuum has to be discretised (hence CD)

[Austern *et al.* , Phys. Rep. 154, 125 (1987)]

[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]

Fully quantal approximation

No approximation on P-T motion, nor restriction on energy

But expensive computationally (at high energies)

Eikonal approximation

Three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0$

Eikonal approximation : factorise $\Psi = e^{iKZ}\widehat{\Psi}$

$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\widehat{\Psi}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2 + \epsilon_0$

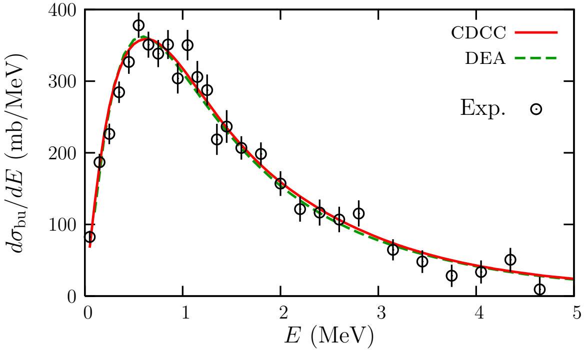
$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

^{15}C+Pb @ 68AMeV : energy distribution



- Excellent agreement between CDCC and DEA

  [P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]

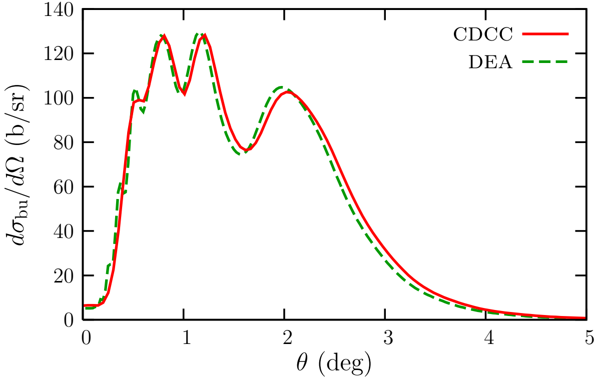
- Excellent agreement with experiment

  [Nakamura *et al.* PRC 79, 035805 (2009)]

⇒ Confirms the validity of the approximations

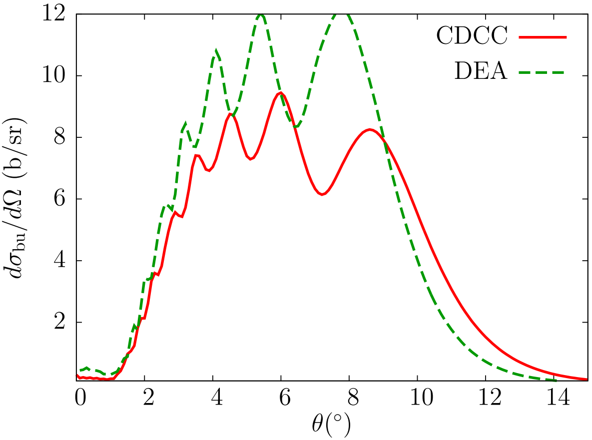
. . . and the two-body structure of ^{15}C

^{15}C+Pb @ 68AMeV : angular distribution



- DEA agrees well with CDCC
- Though a slight shift compared to CDCC…

^{15}C+Pb @ 20AMeV



DEA too high and too forward due to lack of Coulomb deflection

[P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]

Can E-CDCC solve the problem ?

Eikonal-CDCC (E-CDCC)

Solving the eikonal problem expanding Ψ upon H_0 eigenstates $\Phi_i(\boldsymbol{r})$ assuming discretised continuum [Ogata *et al.* PRC **68**, 064609 (2003)]

$$\Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \xi_i(\boldsymbol{b},Z)\Phi_i(\boldsymbol{r})e^{i\{K_iZ+\eta_i\ln[K_iR-K_iZ]\}}$$

⇒ set of coupled equations

$$\frac{\partial}{\partial Z}\xi_i(\boldsymbol{b},Z) = \frac{1}{i\hbar v_i(R)}\sum_{i'}\mathcal{F}_{ii'}(\boldsymbol{b},Z)\xi_{i'}(\boldsymbol{b},Z)e^{i(K_{i'}-K_i)Z}\mathcal{R}_{ii'}(b,Z),$$

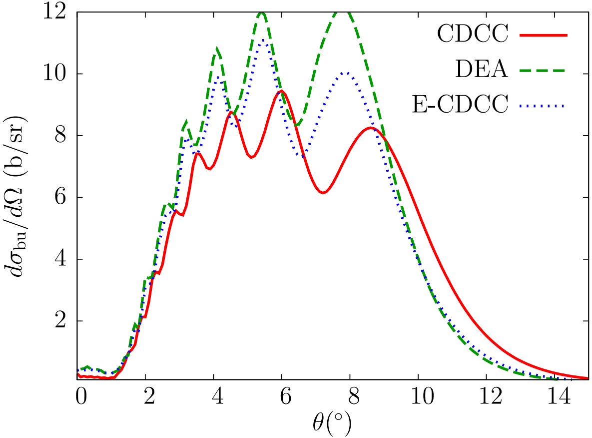
with coupling potential

$$\mathcal{F}_{ii'}(\boldsymbol{b},Z) = \left\langle\Phi_i\left|V_{cT}+V_{fT}-V_{\mathrm{C}}\right|\Phi_{i'}\right\rangle_{\boldsymbol{r}}.$$

E-CDCC takes proper account of energy conservation : $v_i(R)$

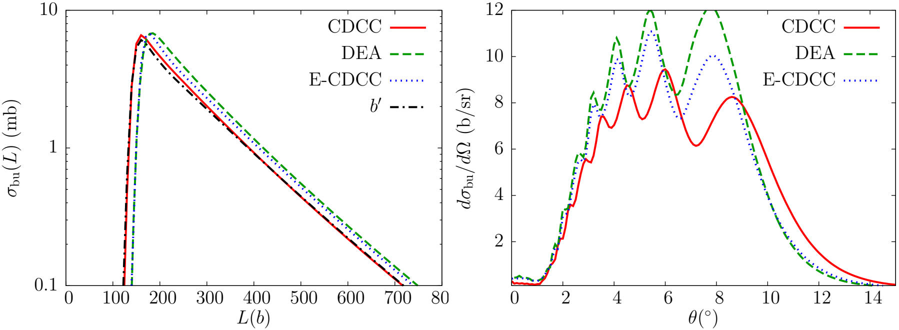
$\mathcal{R}_{ii'}(b,Z) = \frac{(K_{i'}R-K_{i'}Z)^{i\eta_{i'}}}{(K_iR-K_iZ)^{i\eta_i}}$ accounts for part of the Coulomb distortion

^{15}C+Pb @ 20AMeV



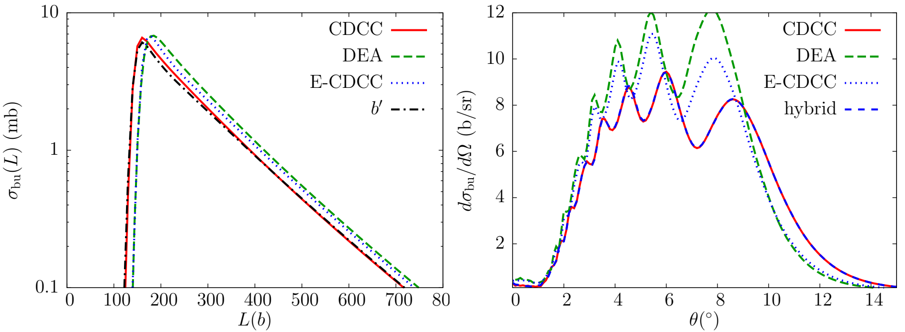
- E-CDCC also too high and too forward

^{15}C+Pb @ 20AMeV



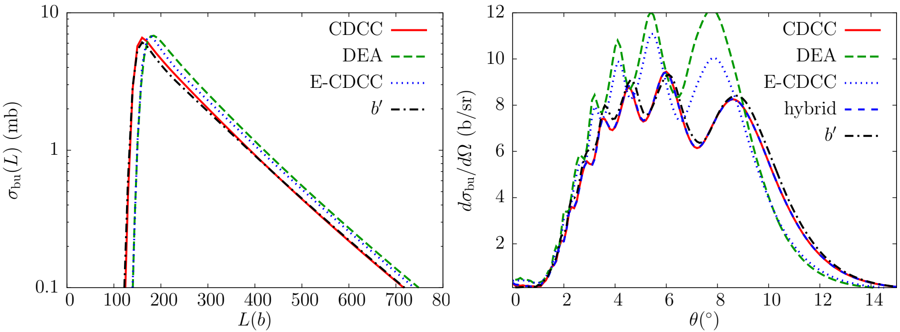
- E-CDCC also too high and too forward
  Shift in $L \Rightarrow$ correction $b \rightarrow b'$ (classical closest approach)

^{15}C+Pb @ 20AMeV



- E-CDCC also too high and too forward
  Shift in $L \Rightarrow$ correction $b \to b'$ (classical closest approach)
- hybrid solution : CDCC at low L (b) and eikonal at large L (b)
  $\Rightarrow$ excellent agreement with full CDCC

^{15}C+Pb @ 20AMeV



- E-CDCC also too high and too forward
  Shift in $L \Rightarrow$ correction $b \to b'$ (classical closest approach)
- hybrid solution : CDCC at low L (b) and eikonal at large L (b)
  $\Rightarrow$ excellent agreement with full CDCC
- Improve eikonal using Coulomb correction : $b \to b'$

Summary and prospect

- Halo nuclei exhibit very exotic structure : core + halo
- Studied mostly through reactions
  - elastic scattering
  - breakup
- Mechanism of reactions with halo nuclei understood
  CDCC & eikonal agree at $70A$MeV
- At $20A$MeV, eikonal fails, due to Coulomb deflection
  But :
  - E-CDCC can be extended to hybrid version
    ⇒ agreement with full CDCC
  - Simple Coulomb correction works fine
    Can it be improved ?
    How far down can we go in energy ? HIE-ISOLDE ?