



# Reaction Dynamics for Light Dripline Nuclei

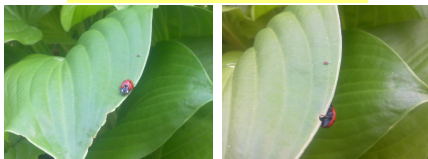
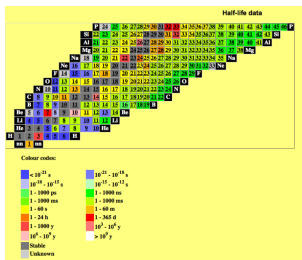
Phys. Scr. T152 (2013) 014019

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Tokyo, Japan, June 1th- 6th, 2014

# Entering the world of exotic nuclei: probing the unbound by walking at the drip line.

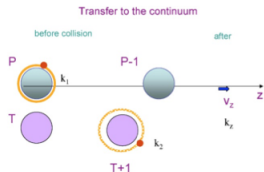


- Is there a **life** beyond the dripline?
- How can we discover it without getting lost?
- Extend our understanding of the *residual* nuclear force.
- Check the limits of validity of structure models such as the SHELL MODEL or "ab initio" models.
- Challenges in breakup reaction theory.

## Transfer to continuum states (inclusive reaction)

Kinematics and phase space ++

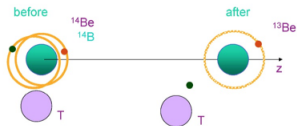
Single particle state properties (shell model)



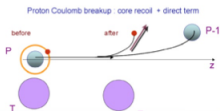
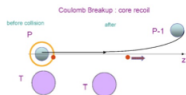
$$k_2 - k_1 = k_z$$

$$\epsilon_T \epsilon_{T+1} = m v^2 / 2$$

## Fragmentation reaction (coincidence)

Let us start with a two neutron halo nucleus like  $^{11}\text{Li}$  or  $^{14}\text{Be}$ 

## Coulomb breakup (inclusive or coincidence)



# A consistent formalism for all breakup reaction mechanisms

The core-target movement is treated in a semiclassical way, but neutron-target and/or neutron-core with a full QM method.

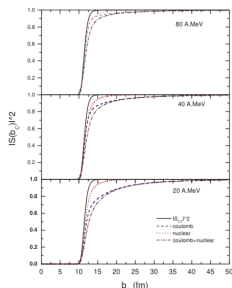
AB and DM Brink, PRC38, 1776 (1988), PRC43, 299 (1991), PRC44, 1559 (1991).

Early eikonal model: I. Tanihata, Prog. Part. Nucl. Phys. 35, 505 (1995), halo-core decoupling.

$$\frac{d\sigma}{d\xi} = c^2 S \int_0^\infty db_c \frac{dP_{-n}(b_c)}{d\xi} P_{ct}(b_c),$$

⊗

$$\xi \rightarrow \varepsilon_f, k_z, P_{//} \quad \text{also} \quad \text{ANC} = \sqrt{c^2 S C_i^2}$$



Use of the simple parametrization

$$P_{ct}(b_c) = |S_{ct}|^2 = e^{(-\ln 2 \exp[(R_s - b_c)/a])},$$

$$R_s \approx r_s (A_p^{1/3} + A_t^{1/3}) \quad r_s \approx 1.4 \text{ fm}$$

'strong absorption radius'

# Transfer to the continuum: from resonances to knockout reactions

First order time dependent perturbation theory amplitude: \*\*

$$A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle \phi_f(\mathbf{r}) | V(\mathbf{r}) | \phi_i(\mathbf{r} - \mathbf{R}(t)) \rangle e^{-i(\omega t - mvz/\hbar)} \quad (1)$$

$$\omega = \varepsilon_i - \varepsilon_f + \frac{1}{2}mv^2 \quad \mathbf{R}(t) = \mathbf{b}_c + vt$$

$$\begin{aligned} \frac{dP_{-n}(b_c)}{d\varepsilon_f} &= \frac{1}{8\pi^3} \frac{m}{\hbar^2 k_f} \frac{1}{2l_i + 1} \sum_{m_i} |A_{fi}|^2 \\ &\approx \frac{4\pi}{2k_f^2} \sum_{j_f} (2j_f + 1) (|1 - \bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2) \mathcal{F}, \end{aligned}$$

$\phi_f$  see (\*)

$$\mathcal{F} = (1 + F_{l_f, l_i, j_f, j_i}) B_{l_f, l_i}$$

$$B_{l_f, l_i} = \frac{1}{4\pi} \left[ \frac{k_f}{mv^2} \right] |C_i|^2 \frac{e^{-2\eta b_c}}{2\eta b_c} M_{l_f, l_i}$$

# Final continuum wave functions

(\*)

Final continuum state:

$$\phi_{l_f}(\mathbf{r}) = C_f k \frac{i}{2} (h_{l_f}^{(+)}(kr) - \bar{S}_{l_f} h_{l_f}^{(-)}(kr)) Y_{l_f, m_f}(\Omega_f),$$

$\bar{S}_{l_f}(\epsilon_f)$  is an optical model (**n-core in fragmentation reactions, n-target in knockout reactions**) S-matrix.

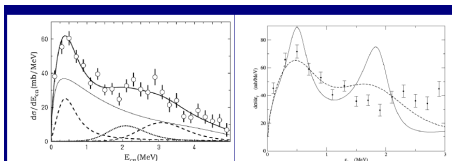
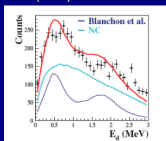
<sup>13</sup>Be puzzle or of the "elusive 1/2+ state in Be isotopes

Figure : (a) GSI, H. Simon et al. NPA791 (2007) 267.

Figure : (b) G.Blanchon et al. NPA784 (2007) 49.



(b) G. Randisi, N. Orr et al. Phys. Rev. C 89, 034320

Energies and widths of unbound  $p$ - and  $d$ -states in <sup>13</sup>Be and corresponding strength parameters for the  $\delta V$  potential

	$\epsilon_{res}$ (MeV)	$\Gamma_j$ (MeV)	$\alpha$ (MeV)
$1p_{1/2}$	0.67	0.28	8.34
$1d_{5/2}$	2.0	0.40	-2.36

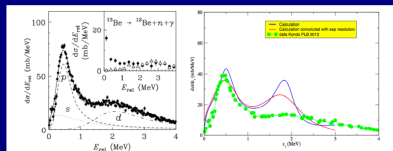


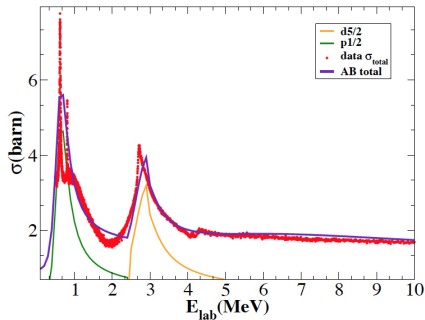
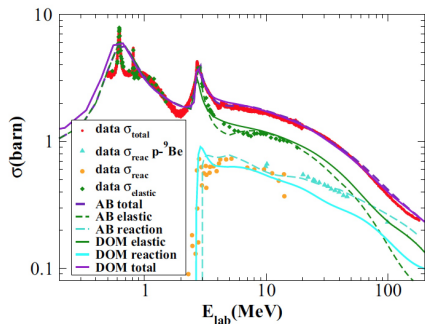
Figure : (b) RIKEN, Y. Kondo et al. PLB690 (2010) 245; G. Blanchon, private communication

$$(d\delta_l/d\epsilon)_{res} = 2/\Gamma_j$$

G. Blanchon et al.  
PRC82, 034313  
NPA A 784 (2007) 49

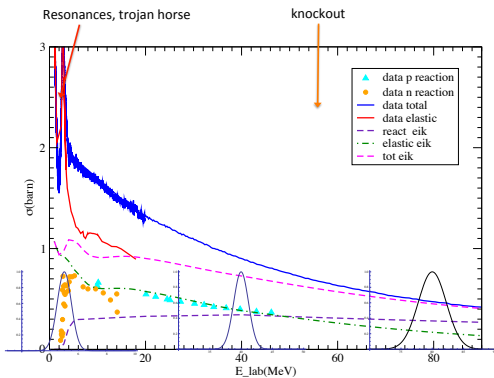
# n-<sup>9</sup>Be optical potential: A.B & R.J. Charity, PRC89, 024619 (2014),

data from <https://www-nds.iaea.org/exfor/exfor.htm>



(\*)

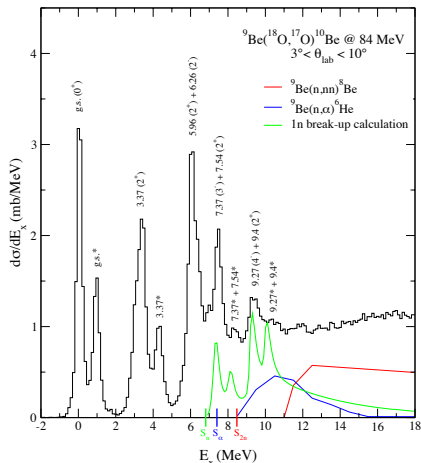




# Transfer to $^{10}\text{Be}$ resonances: missing mass experiment.

Paper in preparation

Diana Carbone, AB, Mariangela Bondi, F. Cappuzzello, M Cavallaro et al. MAGNEX Collaboration:  
1n and 2n transfer experimental campaign



$^{16}\text{O}$  has a degenerate  $g_s$  ( $1d_{5/2}, 2s_{1/2}$ )  
 $^{17}\text{O}$  has  $5/2^+$   $g_s$  and  $1/2^+$  first excited state at  $E^*=0.87\text{MeV}$

Position and widths of  $p_{1/2}$  and  $d_{5/2}$  resonances in  $^{10}\text{Be}$  perfectly reproduced

## Kinematics

From Eq.1 \*\* by the change of variables  $dt dx dy dz \rightarrow dx dy dz dz'$   
 $e^{-i(\omega t - mvz/\hbar)} \rightarrow e^{-ik_1 z'} e^{ik_2 z}$  neutron energies to neutron parallel momenta  
 with respect to core

$$k_1 = \frac{\varepsilon_f - \varepsilon_i - \frac{1}{2}mv^2}{\hbar v};$$

to target

$$k_2 = \frac{\varepsilon_f - \varepsilon_i + \frac{1}{2}mv^2}{\hbar v};$$

to core parallel momentum

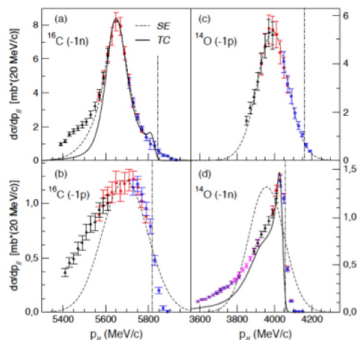
$$\begin{aligned} P_{//} &= \sqrt{E_r^2 - M_r^2} = \sqrt{(T_r + M_r)^2 - M_r^2} \\ &= \sqrt{(T_p + \varepsilon_i - \varepsilon_f)^2 + 2M_r(T_p + \varepsilon_i - \varepsilon_f)}, \end{aligned} \quad (2)$$

breakup threshold at  $\varepsilon_f = 0$

++\*\*

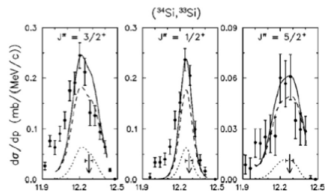
# Example "deformation" effects due to n-target interaction and kinematical cut-off.

F. Flavigny, A. Obertelli, AB et al., PRL 108, 252501 (2012). \*\*

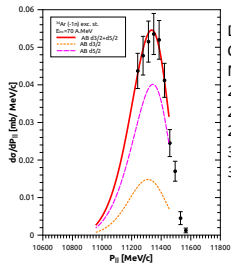
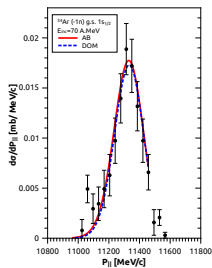
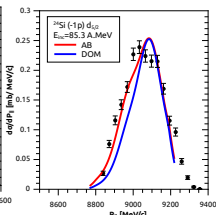
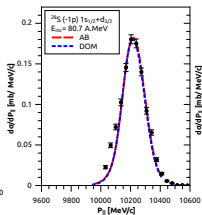
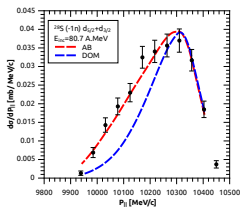


J. Enders et al.

PHYSICAL REVIEW C 65 034318



## Asymmetries at high incident energy



Data courtesy of A. Gade  
 Calculations G. Salvioni  
 MSc Thesis in preparation.  
 $^{28}\text{S} (-1n) 80.7 d5/2+d3/2$   
 $^{28}\text{S} (-1p) s1/2+d3/2 \text{ A.MeV}$   
 $^{24}\text{Si} (-1p) d5/2 85.3 \text{ A.MeV}$   
 $^{34}\text{Ar} (-1n) s1/2 \text{ gs } 70 \text{ A.MeV}$   
 $^{34}\text{Ar} (-1n) d3/2+d5/2$

# Origin of kinematical cut-off (phase space) and deformation effects

PRC60(1999) 054604, PRC44(1991) 1559, AB and GF Bertsch, PRC63(2001) 044604, F. Flavigny, A. Obertelli, AB et al., PRL 108, 252501 (2012). (+)

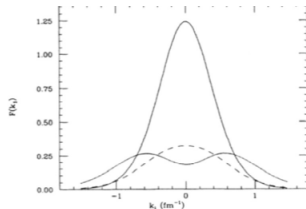
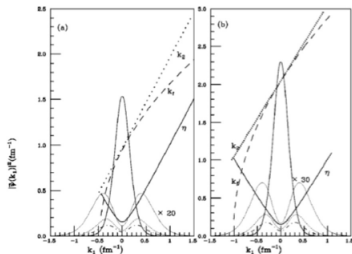
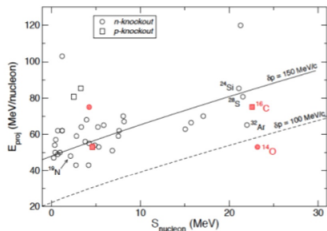
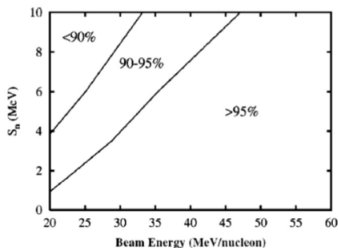
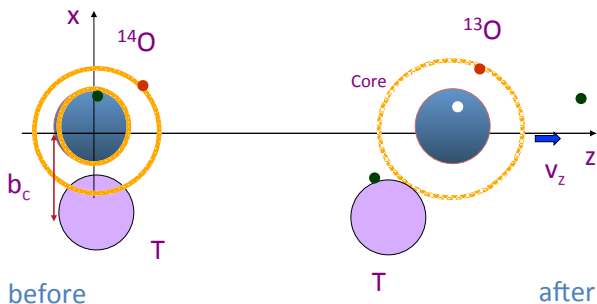


FIG. 11. Initial-state momentum distributions in  $^{20}\text{Ne}$  according to Eq. (2.3a). The solid curve is for the  $2s_{1/2}$  state, the dashed curve is for the  $1p_{1/2}$ , while the dotted curve is for the  $1p_{3/2}$ .



Removal of a deeply bound  $n/p$  while the weakly bound  $p/n$  is un-touched \*

### Knock out from a deeply bound state



reaction	$S_n$ [MeV]	$S_p$ [MeV]	$E_{inc}^{(m)}$ [A.MeV]	$\sigma^{exp}$ [mb]
${}^9\text{C} \rightarrow {}^8\text{C}$	14.25	1.3	63.8	3.
${}^{36}\text{Ca} \rightarrow {}^{35}\text{Ca}$	19.3	1.3	70	5
${}^{13}\text{O} \rightarrow {}^{12}\text{O}$	17.01	1.51	28.5	2.5
${}^{33}\text{Cl} \rightarrow {}^{32}\text{Cl}$	15.74	2.3	66.4	9
${}^{32}\text{Ar} \rightarrow {}^{31}\text{Ar}$	21.56	2.42	65.1	10.4
${}^{28}\text{S} \rightarrow {}^{27}\text{S}$	21.54	2.49	80.7	11.9
${}^{24}\text{Si} \rightarrow {}^{23}\text{Si}$	21.09	3.3	85.3	9.8
${}^{10}\text{C} \rightarrow {}^9\text{C}$	21.28	4.01	116.2	23.4
${}^6\text{Li} \rightarrow {}^5\text{Li}$	5.66	4.43	36.6	38.1
${}^{14}\text{O} \rightarrow {}^{13}\text{O}$	23.18 <del>17.06</del>	4.63	53	14 <del>4.7</del>
${}^{34}\text{Ar} \rightarrow {}^{33}\text{Ar}$	18.42 18.86	4.66	70	3.2 4.9
${}^7\text{Be} \rightarrow {}^6\text{Be}$	10.68 <del>10.98</del>	5.6	65.2	28.1 <del>7.7</del>
${}^{57}\text{Ni} \rightarrow {}^{56}\text{Ni}$	14.0 15.04	7.3	70.2	33.7
${}^{32}\text{S} \rightarrow {}^{31}\text{S}$	16.29 17.28	8.86	62.8	36
${}^{46}\text{Ar} \rightarrow {}^{45}\text{Ar}$	19.5 8.02 8.56	18.64	70	61.9 3.6
${}^{34}\text{Si} \rightarrow {}^{33}\text{Si}$	7.53 8.54 11.82	18.74	73.4	67 41 15
${}^{10}\text{Be} \rightarrow {}^9\text{Be}$	6.81	19.64 22.33	77.8	69.5
${}^{15}\text{C} \rightarrow {}^{14}\text{C}$	1.22 7.31 8.21 8.23	118.5 <del>21.08</del>	71.2	100.8 27.4 6.5 5.5
${}^{16}\text{C} \rightarrow {}^{15}\text{C}$	4.25 4.99	22.56	75	36.5 46

$$\sigma = C^2 S \int_0^\infty db_c P_{-n}(b_c) (1 - P_{-p}(b_c)) P_{ct}(b_c)$$

$$e^{-P_{-p}} \approx 1 - P_{-p}(b_c)$$

DPP from phase shift, AB, F. Carstoiu, NPA706, (2002) - typically  
 $\sim 10\%$  reduction in the cross sections.



Absolute cross sections

$$\sigma = \int d\xi \frac{d\sigma}{d\xi}$$

Ratios

$$\sigma_{exp} / \sigma_{Theo}$$

have been used to validate spectroscopic factors  $C^2S$  ( $=2j+1$ , in the IPM) for single particle orbitals from shell model or "ab initio" calculations, when available. This is similar to what has traditionally been done for transfer. However in transfer the core-target interaction is treated almost exactly thanks to optical potentials fitted to the elastic scattering.

C. Barbieri PRL103, 202502 (2009)

The reactions for transfer of a nucleon to or from the initial state  $|\Psi_0^A\rangle$  depend on the overlap wave function [8,9]

$$\psi_\alpha^{A\pm 1}(\mathbf{r}) = \langle \Psi_\alpha^{A\pm 1} | \psi^{(\dagger)}(\mathbf{r}) | \Psi_0^A \rangle, \quad (1)$$

where  $\alpha$  can label either particle or hole states. SFs are identified with the normalization integral of  $\psi_\alpha^{A\pm 1}(\mathbf{r})$  and give a "measure" of what fraction of the *final* wave function,  $|\Psi_\alpha^{A\pm 1}\rangle$ , can be factorized into a (correlated) core plus an independent particle or hole. Strong deviations from the independent particle model (IPM)—that is, a Slater determinant with fully occupied orbits—signal substantial correlations and imply the onset of nontrivial many-body dynamics. For stable nuclei, a large body of

neutron & proton  $\rightarrow$  transfer vs breakup F. Flavigny et al., PRL110 ,122503 (2013)

TABLE I. The normalization  $C^2S_{\text{exp}}$  for two OFs, phenomenological (WS) and microscopic (SCGF) [30]. For the WS OF, the  $r_0$  values were chosen to reproduce  $R_{\text{rms}}^{\text{HFB}}$ , except for  $^{16}\text{O}$  for which  $R_{\text{rms}}$  was taken from  $(e, e'p)$  data (see text). The SFs  $C^2S_{\text{th}}$  are obtained from shell-model calculations with the WBT interaction. In the second part, the analysis was performed with microscopic OFs and SFs. The two errors for  $C^2S_{\text{exp}}$  and  $R_s$  are the experimental and analysis errors.

Reaction	$E^*$ (MeV)	$J^\pi$	$R_{\text{rms}}^{\text{HFB}}$ (fm)	$r_0$ (fm)	$C^2S_{\text{exp}}$ (WS)	$C^2S_{\text{th}}$ $0p + 2h\omega$	$R_s$ (WS)	$C^2S_{\text{exp}}$ (SCGF)	$C^2S_{\text{th}}$ (SCGF)	$R_s$ (SCGF)
$^{14}\text{O} (d, t) ^{13}\text{O}$	0.00	$3/2^-$	2.69	1.40	1.69 (17)(20)	3.15	0.54(5)(6)	1.89(19)(22)	3.17	0.60(6)(7)
$^{14}\text{O} (d, ^3\text{He}) ^{13}\text{N}$	0.00	$1/2^-$	3.03	1.23	1.14(16)(15)	1.55	0.73(10)(10)	1.58(22)(2)	1.58	1.00(14)(1)
	3.50	$3/2^-$	2.77	1.12	0.94(19)(7)	1.90	0.49(10)(4)	1.00(20)(1)	1.90	0.53(10)(1)
$^{16}\text{O} (d, t) ^{15}\text{O}$	0.00	$1/2^-$	2.91	1.46	0.91(9)(8)	1.54	0.59(6)(5)	0.96(10)(7)	1.73	0.55(6)(4)
$^{16}\text{O} (d, ^3\text{He}) ^{15}\text{N}$ [19,20]	0.00	$1/2^-$	2.95	1.46	0.93(9)(9)	1.54	0.60(6)(6)	1.25(12)(5)	1.74	0.72(7)(3)
	6.32	$3/2^-$	2.80	1.31	1.83(18)(24)	3.07	0.60(6)(8)	2.24(22)(10)	3.45	0.65(6)(3)
$^{18}\text{O} (d, ^3\text{He}) ^{17}\text{N}$ [21]	0.00	$1/2^-$	2.91	1.46	0.92(9)(12)	1.58	0.58(6)(10)			

TABLE I: Summary of one-nucleon knockout results from  $^{14}\text{O}$  at 53 MeV/nucleon. The calculated inclusive cross sections  $\sigma_{TC}$  from the transfer-to-the-continuum approach are shown and compared to the measured ( $\sigma_{exp}$ ) cross sections. Theoretical spectroscopic factors  $C^2S$  are calculated with the WBT interaction [?]. Reduction factors are indicated and defined as  $R_f$  in order to distinguish from the strong absorption radius notation ( $R_s$ ).

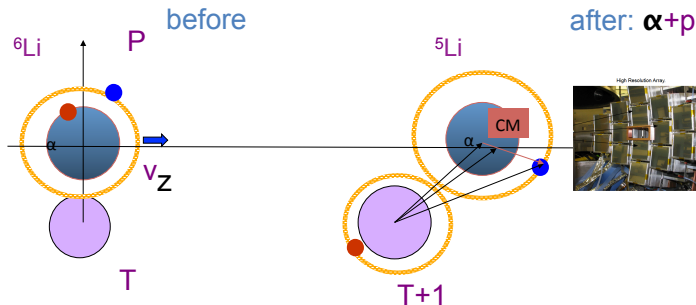
Res.	E (MeV)	$J^\pi$	$\sigma_{exp}$ (mb)	$C^2S$	$\sigma_{sp}$ (mb)	$\sigma_{sp}(no_p)$ (mb)	$\sigma_{TC}$ (mb)	$\sigma_{TC}(no_p)$ (mb)	$R_f$
$^{13}\text{N}$	0.0	$1/2^-$	58(4)	1.83	34.18		53		0.91
$^{13}\text{O}$	0.0	$3/2^-$	14(1)	3.15	10.94	8.6	34.47	27.1	0.52

# Proton unbound nuclei via invariant mass method

Interest: Two-proton radioactivity vs. 2n-halo by isospin symmetry  
 ${}^5\text{He}$ ,  ${}^6\text{He}$ ,  ${}^8\text{He}$ ,  ${}^{12}\text{Be}$  and IMME



## Proton unbound nuclei

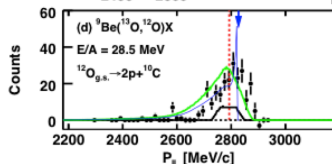
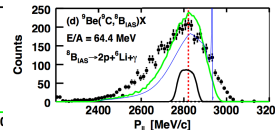
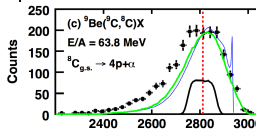
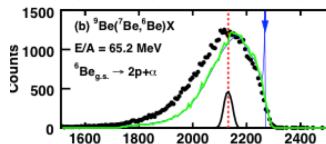
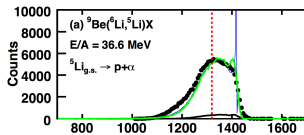
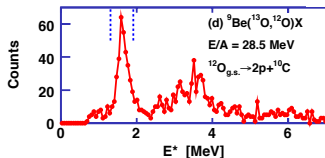
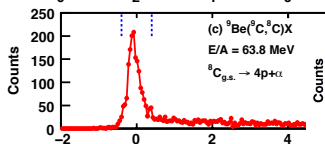
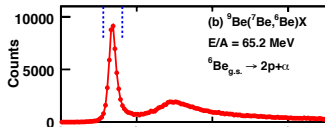
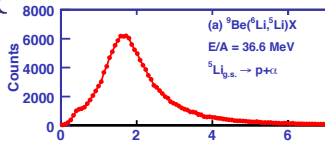


${}^6\text{Li}$ ,  ${}^7\text{Be}$ ,  ${}^9\text{C}$ ,  ${}^{13}\text{O}$  studied by knockout of a deeply bound **neutron**:

R. J. Charity & HiRA collaboration



Data: R. J. Charity and the HiRA collaboration, **preliminary**. Calculations: G. Salvioni & AB



# Structure inputs: Shell model and "ab initio" Variational MonteCarlo

$SF_{SM}$  by Mihai Horoi, private communication.

$SF_{VMC}$  from R. Wiringa website <http://www.phy.anl.gov/theory/research/overlap/>,

$ANC_{VMC}$  from Kenneth M. Nollett and R. B. Wiringa, PRC83, 041001(R) (2011).

	$S_n$ MeV	$ANC_{WS}$ $fm^{-1/2}$	$SF_{SM}$	$ANC_{VMC}$ $fm^{-1/2}$	$SF_{VMC}$	$S_p$ MeV	$ANC_{WS}$ $fm^{-1/2}$	$SF_{SM}$	$ANC_{VMC}$ $fm^{-1/2}$	$SF_{VMC}$
$\langle {}^6Li   {}^5Li \rangle$	5.66	2.85 2.89	$p_{1/2}$ 0.3301 $p_{3/2}$ 0.3384		$p_{1/2}$ 0.20463 $p_{3/2}$ 0.30566	4.59	2.66 2.36			$p_{1/2}$ 0.21363 $p_{3/2}$ 0.31905
$\langle {}^7Be   {}^6Be \rangle$	10.68	5.72	$p_{3/2}$ 0.5990	3.68(5.55)	$p_{3/2}$ 0.4389	5.61		$p_{1/2}$ 0.2523 $p_{3/2}$ 0.4888	1.652 1.89	$p_{1/2}$ 0.2423 $p_{3/2}$ 0.4727
$\langle {}^9C   {}^8C \rangle$	14.25	8.1	$p_{3/2}$ 0.8673	5.99 (7.9)	$p_{3/2}$ 0.5727	1.3	1.33	$p_{1/2}$ 0.0154 $p_{3/2}$ 0.9557	0.309 1.13	$p_{1/2}$ 0.1092 $p_{3/2}$ 0.9933
$\langle {}^9C   {}^8BIAS \rangle$	....					11.915				$p_{3/2}$ 0.16049
$\langle {}^9Li   {}^8Li_{IAS} \rangle$	14.89	6.3			$p_{3/2}$ 0.15754					
$\langle {}^{13}O   {}^{12}O \rangle$	17		$p_{3/2}$ 0.4990			1.5		$p_{1/2}$ 0.5844 $p_{3/2}$ 0.0670		

	$E_{inc}$ A.MeV	$\sigma_{exp}$ mb	$\sigma_{-n}$ mb	$\sigma_{-p}$ mb	$\sigma_{-n_{nop}}$ mb	$r_s$ fm
$\langle {}^6Li   {}^5Li \rangle$	36.6	38.1	44.53	47.41	38.5	1.53
$\langle {}^7Be   {}^6Be \rangle$	65.2	28.1	34.14	15.22	27.5	1.38
$\langle {}^9C   {}^8C \rangle$	63.8	3.		1.57(-1p <sub>CB</sub> )		1.4
			9.8	19.3 (22.3)	4.48	1.59
$\langle {}^9C   {}^8B_{IAS} \rangle$	64.4	1.24	...			1.4
$\langle {}^9Li   {}^8Li_{IAS} \rangle$			1.47			1.4
$\langle {}^{13}O   {}^{12}O \rangle$	28.5	2.5		2.32(-1p <sub>CB</sub> )		1.5
			3.9	1.9	3.6	

(-1p<sub>CB</sub>) direct proton Coulomb breakup

- Inclusive breakup reactions are dominated by final state interaction with the target at small incident energy: use as surrogate reaction
- At intermediate incident energy: strong interplay between projectile and target characteristics: "deformed" momentum distributions and cutoff effects. **PROTON TARGET?** see **A. Obertelli and T. Aumann talks**
- .... by the valence particle projectile momentum distribution at high incident energy: information on angular momentum of the initial state and possible dynamical core-target excitations
- Coincidence experiments of breakup particle experiments (using invariant mass method ) are more INdependent on incident energy but possible dependence on the initial state: necessity to link with methods discussed above and angular correlation of the decaying particles... : one the most interesting experiment to make...and interpret? Can enlighten different channels and reaction mechanisms.
- IN ADDITION
- Elastic scattering experiments and or total reaction cross section measurements: they can tell us about the typical **interaction** distances and help fixing the optical potentials.
- In the future more and more strongly bound nuclei will be studied at lower energies at ISOL-type facilities. In Europe: HI-Isolde, SPES, Spiral2, EURISOL (?).

Some of my co-authors and collaborators in historical order.

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G. Blanchon

F. Carstoiu

G. F. Bertsch

Ravinder Kumar

MAGNEX collaboration at INFN-LNS

F. Flavigny, A. Obertelli

R. J. Charity

G. Salvioni... see his talk at DREB2014 in Darmstadt



Preliminary information can be obtained from Dr. Angela Bonaccorso [bonac@df.unipi.it](mailto:bonac@df.unipi.it)  
 Local Organizing Committee : A. Bonaccorso (chair), G. Casini (co-chair), I. Bombaci, A. Kievsky, L. Marcucci  
 V. Rosso, M. Viviani.

## Re-writing Nuclear Physics textbooks: 30 years with RIBs and beyond

PISA 20-24 July 2015

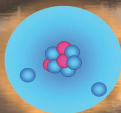
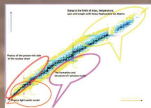


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Students from  
 all over the world  
 gather together to learn about  
 the wonders of  
 Physics with Exotic Nuclei