Phase-space representation for nuclear potentials

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Outline

Nucleon-nucleon interaction

- Realistic NN potentials
- Unitary transformations and effective interactions

Phase-space representation

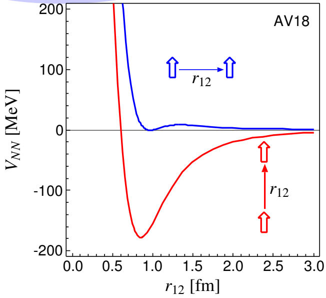
- Kirkwood representation
- Momentum dependence in phase-space representaion
- Results for realistic NN potentials

NN interaction and correlations

Realistic NN potentials

- describing two-nucleon properties (scattering, deuteron) with high accuracy
- different potentials available, e.g.
  Argonne V18 Wiringa, Stoks, Schiavilla, PRC **51**, 38 (1995)
  N^3LO from Chiral effectiv field theory Entem, Machleidt, PRC **68**, 041001 (2003)

$S = 1, T = 0$



- repulsive core:
  nucleons can not get closer than ≈ 0.5 fm→ **central correlations**
- strong dependence on the orientation of the spins due to the tensor force → **tensor correlations**
- the nuclear force will induce **strong short-range correlations** in the nuclear wave function

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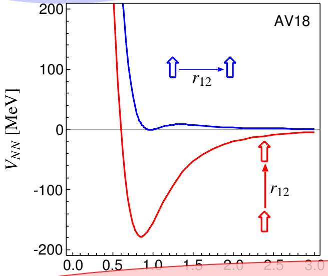

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Use unitary transformations to obtain "soft" effective realistic interaction

Unitary Correlation Operator Method

- NN interaction induces strong **central** and **tensor correlations**
  → Many-body methods working with (superpositions of) Slater determinants require huge model space sizes

- **Unitary Correlation Operator Method (UCOM)**:
  Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 51 (2010)
  Unitary transformation $\mathbf{C}$ imprints correlation on simple model state $|\Psi\rangle$

$$|\hat{\Psi}\rangle = \mathbf{C}\,|\Psi\rangle = \mathbf{C}_\Omega \mathbf{C}_r\,|\Psi\rangle$$

- Work with transformed operators and simple model states, e.g.

$$\langle\hat{\Psi}|\,\mathbf{H}\,|\hat{\Psi}'\rangle=\langle\Psi|\,\mathbf{C}^\dagger\mathbf{H}\mathbf{C}\,|\Psi'\rangle =: \langle\Psi|\,\mathbf{H}_{\text{eff}}\,|\Psi'\rangle$$

- $\mathbf{H}_{\text{eff}}$ for local NN potential (e.g. Argonne V18) contains **quadratic momentum dependence** replacing short-range repulsion and short-range tensor:

$$\mathbf{H}_{\text{eff}} = \mathbf{C}_r^\dagger\left(\frac{\vec{\mathbf{p}}^2}{2\mu} + V(\mathbf{r})\right)\mathbf{C}_r = \frac{\vec{\mathbf{p}}^2}{2\mu} + V_{\text{eff}}(\mathbf{r},\mathbf{p})$$

Weber, Feldmeier, Hergert, Neff, Phys. Rev. C **89**, 034002 (2014)

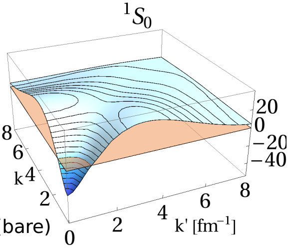
Similarity Renormalization Group

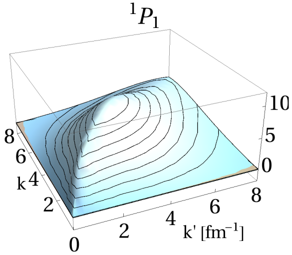
- **Similarity Renormalization Group (SRG)**:
  Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)
  Evolve Hamiltonian and unitary transformation matrix

$$\frac{d\mathbf{H}_\alpha}{d\alpha} = (2\mu)^2 \left[\left[\mathbf{T}_{\text{int}}, \mathbf{H}_\alpha \right], \mathbf{H}_\alpha \right] \quad \rightarrow \quad \mathbf{H}_\alpha = \mathbf{U}_\alpha^\dagger \mathbf{H} \mathbf{U}_\alpha$$

- Unitary transformation $\mathbf{U}_\alpha$ to obtain “soft” effective realistic interaction
- SRG drives the Hamiltonian towards a band-diagonal structure
- Performed in matrix element representation → **momentum dependence ?**

AV18: $\langle k(L0)J;T|\,\mathbf{V}_\alpha\,|k'(L0)J;T\rangle$ in MeVfm^3





$\alpha = 0.00\text{fm}^4$ (bare)

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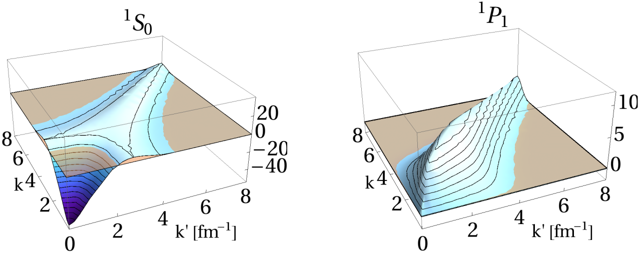

$\alpha = 0.01\text{fm}^4$

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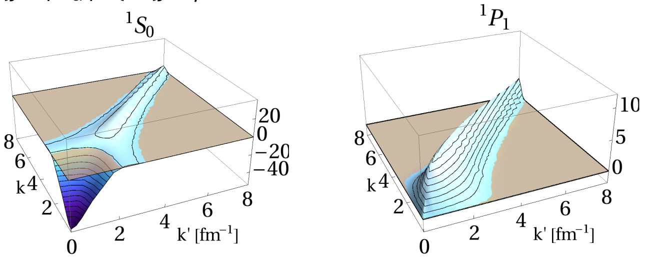


$\alpha = 0.04\text{fm}^4$

Representation of NN interactions

- effective realistic NN potentials given usually as momentum matrix elements
- can be used in many-body calculations with shell model or plane wave basis
- but not for Fermionic Molecular Dynamics and cluster models
- matrix element representation not intuitive, not transparent

- Other representation to study and visualize NN potentials
  - How does the potential look in position space?
  - What happens to the repulsive core?
  - What is the range of the interaction?
  - What is the momentum dependence?

⇒ Phase-space representation

Phase-space representation

- Kirkwood representation Kirkwood, Phys. Rev., **44**, 31 (1933)
  - **phase-space distribution** for a given state $|\phi\rangle$:

$$f_{\text{ps}}(\vec{r},\vec{p}) = (2\pi)^{3/2} \langle\vec{r}|\phi\rangle\langle\phi|\vec{p}\rangle\langle\vec{p}|\vec{r}\rangle$$

  - **phase-space representation** of an operator **O**:

$$O_{\text{ps}}(\vec{r},\vec{p}) = (2\pi)^{3/2} \langle\vec{r}|\,\mathbf{O}\,|\vec{p}\rangle\langle\vec{p}|\vec{r}\rangle$$

  - **quantum expectation value** (analogue to classical expression)

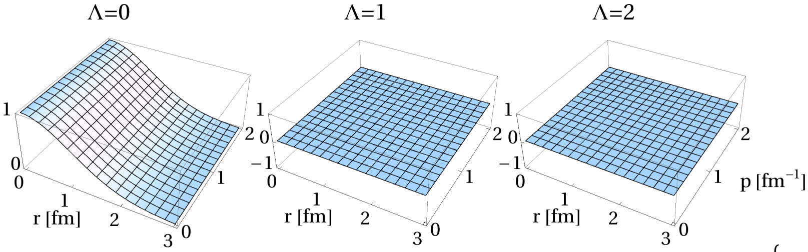
$$\langle O \rangle = \langle\phi|\,\mathbf{O}\,|\phi\rangle = \int d^3r \int d^3p\, f^*_{\text{ps}}(\vec{r},\vec{p})\, O_{\text{ps}}(\vec{r},\vec{p})$$

- Study phase-space representation of effective NN interactions
- Multipole expansion with Legendre polynomials P_Λ:

$$V_{\text{ps}}(\vec{r},\vec{p}) = \sum_\Lambda i^\Lambda V_\Lambda(r,p)\, P_\Lambda(\widehat{\vec{r}}\cdot\widehat{\vec{p}}).$$

Phase-space representation of local $\mathbf{V} = V(\mathbf{r})$

$V_\Lambda(r,p)$ (arb. units)



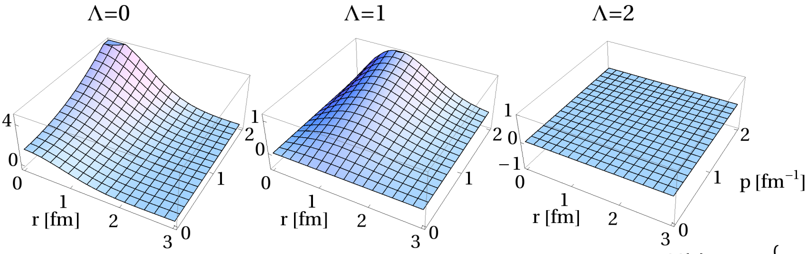
$V(r) = \exp\left\{-\frac{r^2}{2\mathrm{fm}^2}\right\}$

local: $\mathbf{V} = V(\mathbf{r})$

$$\rightarrow \quad V_{\mathrm{ps}}(\vec{r},\vec{p}) = V(r) = V(r) P_0(\widehat{\vec{r}} \cdot \widehat{\vec{p}})$$

Phase-space representation of $\mathbf{V} = \frac{1}{2}\left(\vec{\mathbf{p}}^{\,2} V(\mathbf{r}) + V(\mathbf{r})\vec{\mathbf{p}}^{\,2}\right)$

$V_\Lambda(r,p)$ (arb. units)



$V(r) = \exp\left\{-\frac{r^2}{2\text{fm}^2}\right\}$

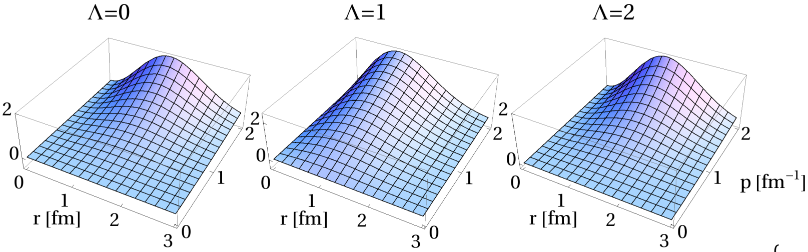
quadratic momentum dependence: $\mathbf{V} = \frac{1}{2}\left(\vec{\mathbf{p}}^{\,2} V(\mathbf{r}) + V(\mathbf{r})\vec{\mathbf{p}}^{\,2}\right)$

$$\rightarrow V_{\text{ps}}(\vec{r},\vec{p}) = \left(V(r)\,p^2 - \frac{1}{2}V''(r) - \frac{V'(r)}{r}\right) - i\frac{V'(r)}{r}\,\vec{r}\cdot\vec{p}$$

$$= \left(V(r)\,p^2 - \frac{1}{2}V''(r) - \frac{V'(r)}{r}\right) P_0(\hat{\vec{r}}\cdot\hat{\vec{p}}) - i\frac{V'(r)}{r}\,rp\,P_1(\hat{\vec{r}}\cdot\hat{\vec{p}})$$

Phase-space representation of $\mathbf{V} = V(\mathbf{r})\, \vec{\mathbf{L}}^2$

$V_\Lambda(r,p)$ (arb. units)



$V(r) = \exp\left\{-\frac{r^2}{2\text{fm}^2}\right\}$

quadratic angular momentum dependence: $\mathbf{V} = V(\mathbf{r})\, \vec{\mathbf{L}}^2$

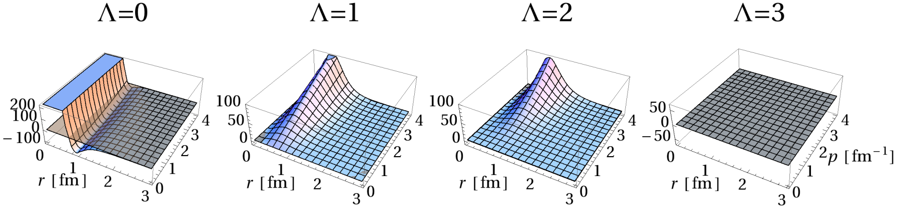
$$\rightarrow V_{\text{ps}}(\vec{r},\vec{p}) = V(r)\,(\vec{r}\times\vec{p})^2 + 2iV(r)\,\vec{r}\cdot\vec{p}$$

$$= \frac{2}{3}V(r)\,(rp)^2\; P_0(\hat{\vec{r}}\cdot\hat{\vec{p}}) + 2iV(r)rp\,P_1(\hat{\vec{r}}\cdot\hat{\vec{p}}) - \frac{2}{3}V(r)\,(rp)^2\; P_2(\hat{\vec{r}}\cdot\hat{\vec{p}})$$

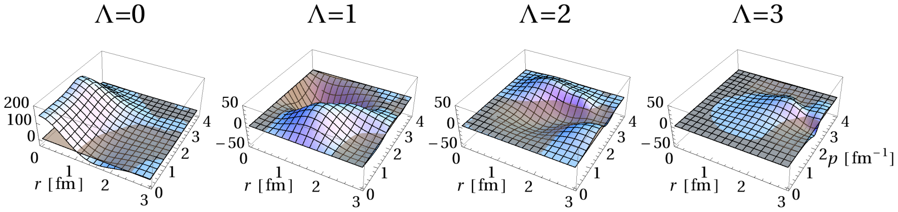
Bare potentials in phase-space representation

S=0, T=1: $V_\Lambda(r,p)$ in MeV

AV18



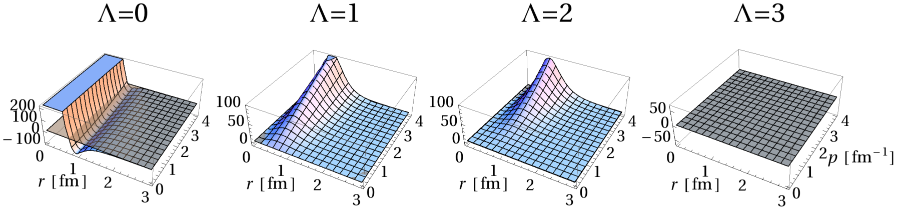
N^{3}LO



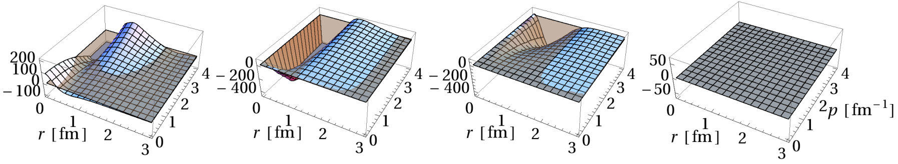
UCOM Argonne potential in phase-space representation

$S = 0$, $T = 1$: $V_\Lambda(r, p)$ in MeV

AV18



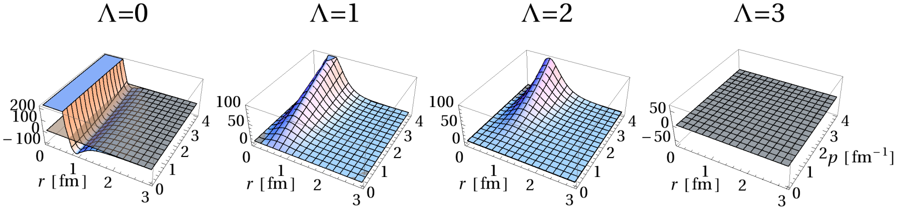
AV18 UCOM



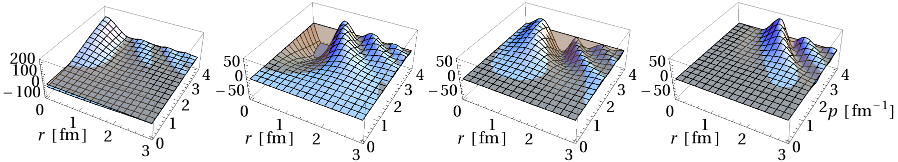
SRG Argonne potential in phase-space representation

$S = 0$, $T = 1$: $V_\Lambda(r,p)$ in MeV

AV18



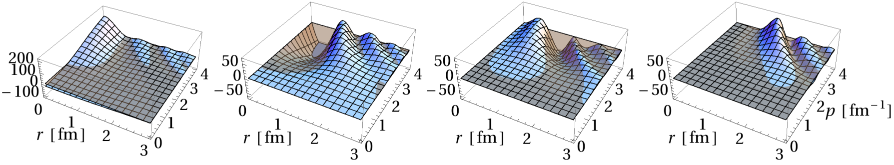
AV18 SRG



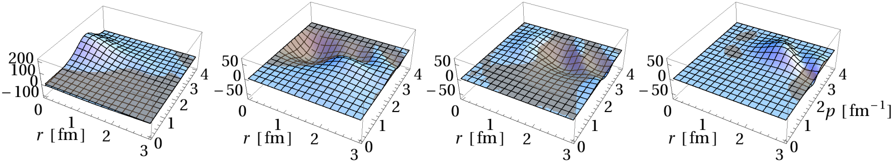
SRG potentials in phase-space representation

$S = 0$, $T = 1$: $V_\Lambda(r, p)$ in MeV

AV18 SRG



N^{3}LO SRG



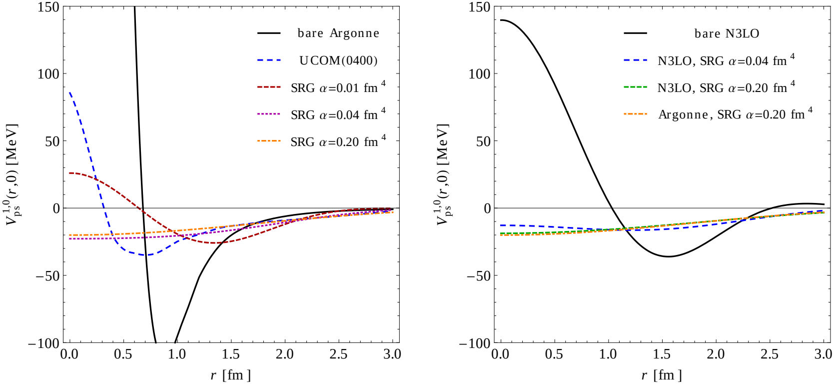
Local projection

Wendt et al. introduced the **local projection** of a potential

Wendt, Furnstahl, Ramanan, Phys. Rev. C **86**, 014003 (2012)

This is the same as the phase space representation for $\Lambda = 0$ and $p = 0$:

$$V_{loc}(r) = V_{\Lambda=0}(r, p = 0)$$



Summary

Realistic effective interactions

- UCOM and SRG soften the interaction by unitary transformation
- transformed interactions are non-local
- no “intuitive” picture in partial wave matrix element representation

Phase-space Representation

- investigate momentum-dependence
- non-locality reflected in p- and Λ-dependence
- form of regulators reflected in N^{3}LO phase-space representation
- AV18 UCOM has quadratic momentum-dependence
- AV18 and N^{3}LO SRG have more complicated momentum-dependence

Outlook

- extend phase-space representation to $S = 1$ channels
- what can we learn for the operator representation for effective interactions