

# Shell evolution, spectroscopy, theoretical uncertainties in neutron-rich calcium isotopes studied with chiral three-body forces

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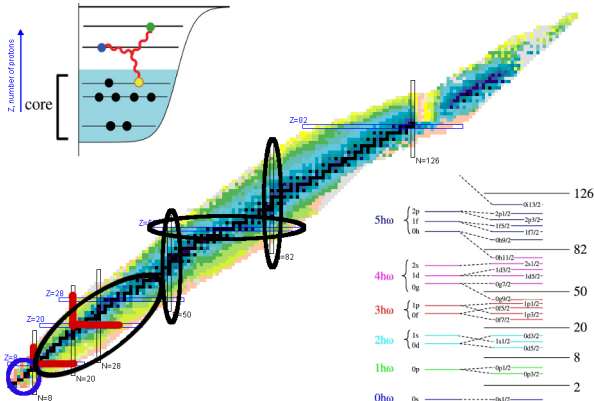
Institut für Kernphysik (TU Darmstadt) and ExtreMe Matter Institute (EMMI)

Advances in Radioactive Isotope Science, ARIS 2014

Tokyo, 6 June 2014



# Nuclear landscape



Big variety of nuclei in the nuclear chart,  $A \sim 2 \dots 300$

Systematic *ab initio* calculations only possible in the lightest nuclei

Hard many-body problem: approximate methods suited for different regions

## Shell Model:

Solve the problem choosing the relevant degrees of freedom





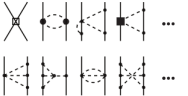
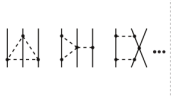

Use realistic nucleon-nucleon (NN) and three-nucleon (3N) interactions

# Nuclear forces in chiral EFT

Chiral EFT: low energy approach to QCD for nuclear structure energies

Short-range couplings are fitted to experiment once

Systematic expansion of nuclear forces

	2N force	3N force	4N force
LO		—	—
NLO		—	—
N <sup>2</sup> LO			—
N <sup>3</sup> LO			

pion exchanges  
contact terms

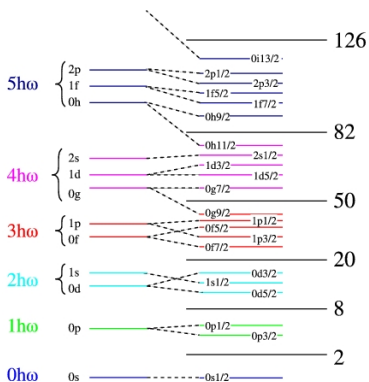
NN fitted to:

- NN scattering
- $\pi$ -N scattering

3N fitted to:

- $^3\text{H}$  Binding Energy
- $^4\text{He}$  radius

# Medium-mass nuclei: shell model



To keep the problem feasible, the configuration space is separated into

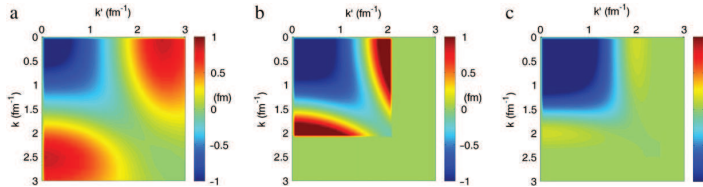
- Outer orbits: orbits that are always empty
- Valence space: the space in which we explicitly solve the problem
- Inner core: orbits that are always filled

Solve in valence space:  $H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$

$H_{eff}$  is obtained in many-body perturbation theory (MBPT)  
includes the effect of inner core and outer orbits

# Renormalization group (RG) and MBPT

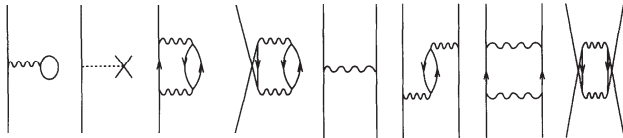
Better convergence of chiral forces after RG transformation



Single Particle Energies

Two-Body Matrix Elements

Many-body perturbation theory to third order: obtain effective shell model interaction in the valence space



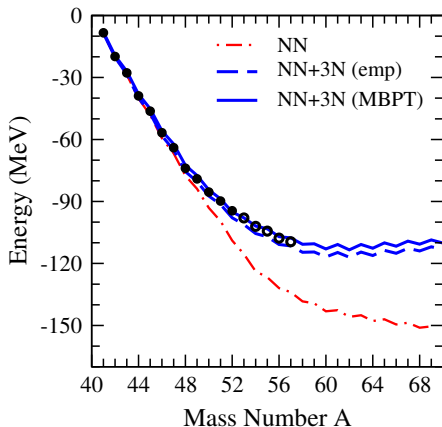
Solve many-body problem with shell model code ANTOINE

Diagonalize up to  $10^{10}$  Slater determinants *Caurier et al. RMP 77 (2005)*

$$|\phi_\alpha\rangle = a_{i_1}^+ a_{i_2}^+ \dots a_{i_A}^+ |0\rangle \quad |\Psi\rangle_{eff} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle$$

# Ca isotopes: masses

Ca isotopes: explore nuclear shell evolution  $N = 20, 28, 32?, 34?$



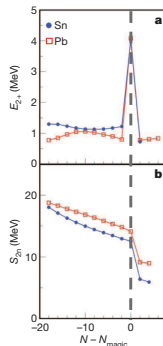
Ca measured from  $^{40}\text{Ca}$  core

3N forces repulsive contribution,  
chiral NN-only forces too attractive

Probe shell  
evolution:

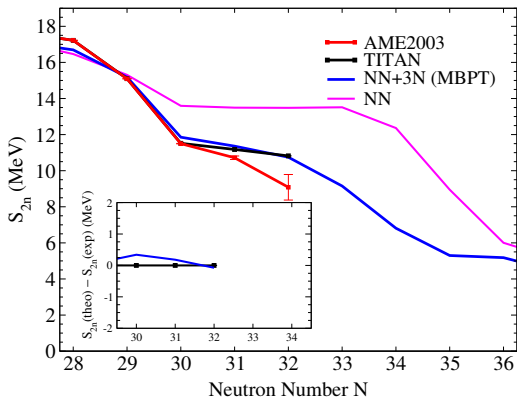
Mass-differences  
 $2_1^+$  energies

Jones et al.  
Nature 465 454 (2010)



# Two-neutron separation energies

Compare  $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$  with experiment



$S_{2n}$  in  $^{52}\text{Ca}$  predicted in disagreement with old measurements

Precision measurements with TITAN changed AME 2003  $\sim 1.74$  MeV in  $^{52}\text{Ca}$

More flat behavior in  $^{50}\text{Ca}$ – $^{52}\text{Ca}$

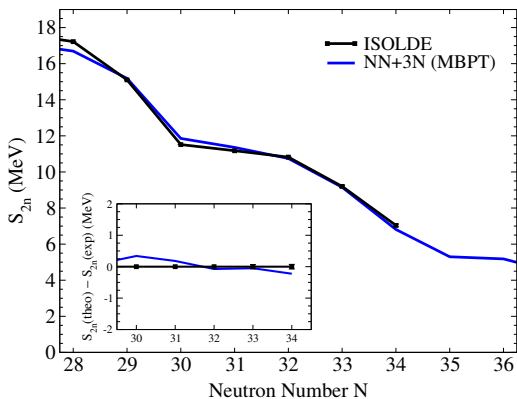
3N forces needed

Gallant et al.

PRL 109 032506 (2012)

# $^{54}\text{Ca}$ mass and $N = 32$ shell closure

## Recent measurement of $^{53,54}\text{Ca}$ at ISOLDE



Excellent agreement with theoretical prediction

$S_{2n}$  evolution:  
 $^{52}\text{Ca}$ – $^{54}\text{Ca}$  decrease  
similar to  $^{48}\text{Ca}$ – $^{50}\text{Ca}$   
unambiguously establishes  
 $N = 32$  shell closure

LETTER

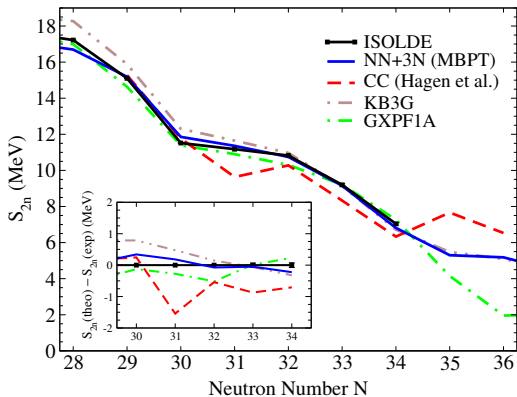
Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz<sup>1</sup>, D. Beck<sup>2</sup>, K. Blaum<sup>3</sup>, Ch. Borgmann<sup>3</sup>, M. Breitenfeldt<sup>4</sup>, R. B. Cakiri<sup>5,7</sup>, S. George<sup>1</sup>, F. Herfurth<sup>1</sup>, M. Kowalska<sup>3</sup>, S. Kreim<sup>3,8</sup>, D. Lunney<sup>9</sup>, V. Manea<sup>9</sup>, J. Menéndez<sup>6,7</sup>, D. Neidherr<sup>2</sup>, M. Rosenbusch<sup>1</sup>, L. Schwab<sup>1</sup>, A. Schwenk<sup>2,6</sup>, J. Simonis<sup>6,7</sup>, J. Stanja<sup>10</sup>, R. N. Wolf<sup>1</sup> & K. Zuber<sup>10</sup>



# Two-neutron separation energies

Compare to other theoretical calculations



## Phenomenology

good agreement  
masses/gaps as input

## Coupled-Cluster calculations

good agreement  
phenomenological 3N forces  
Hagen et al. PRL109 032502 (2012)

## LETTER

**Masses of exotic calcium isotopes pin down nuclear forces**

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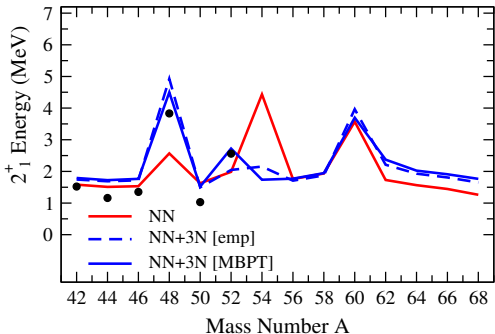
# Shell closures and $2_1^+$ energies

$2_1^+$  energies characterize shell closures

Correct closure at  $N = 28$  when 3N forces are included

Holt et al. JPG39 085111(2012)

Holt, JM, Schwenk,  
JPG40 075105 (2013)

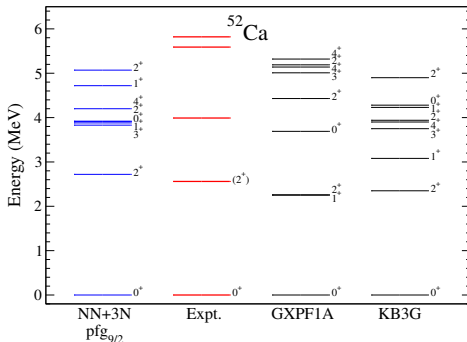
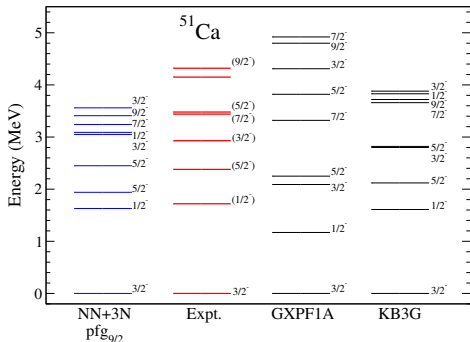


- 3N forces enhance closure at  $N = 32$
- 3N forces reduce strong closure at  $N = 34$   
Expt: suggest  $N = 34$  shell closure  
 $E(2_1^+) = 2.04$  MeV Steppenbeck et al. Nature 502 207(2013)



# Excitation spectra

## Spectra for neutron-rich calcium isotopes



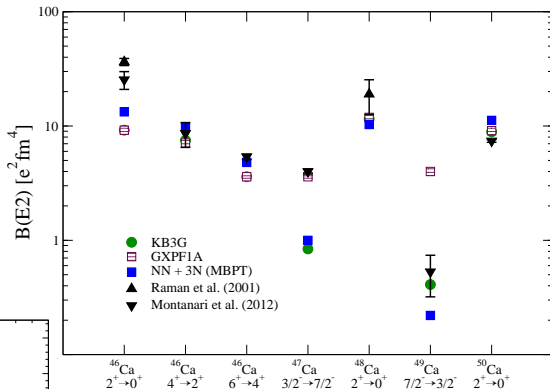
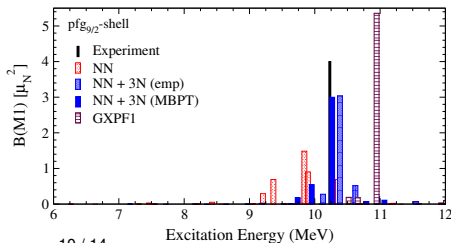
Good agreement with experiment when available,  
comparable to phenomenological interactions

Predictions in very neutron-rich nuclei, test in upcoming experiments

# Electromagnetic transitions

B(E2)s in reasonable agreement with experiment span three orders of magnitude

Similar quality as phenomenological interactions

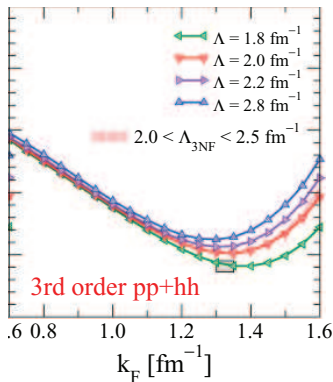


B(M1) strength in  $^{48}\text{Ca}$   
NN+3N good agreement  
experiment strength and energy

# Towards theoretical uncertainties

Estimate theoretical uncertainties  
allows meaningful comparison to experiment  
and better predictions of properties of non-accessible isotopes

- Theoretical uncertainties associated to nuclear force:  
Explore sensitivity of results with respect to cutoff of RG evolution of unevolved chiral Hamiltonian  
Impose correct nuclear matter saturation
- Consider different unevolved chiral Hamiltonians (outlook)
- Theoretical uncertainties associated to the many-body approach



Hebeler et al. PRC 83 031301 (2011)

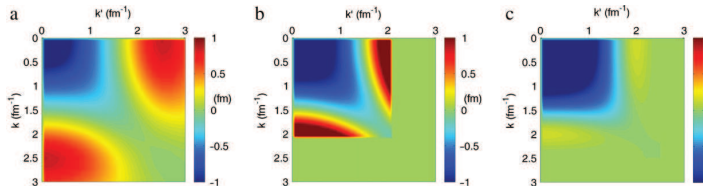
# Chiral EFT and RG

Original Hamiltonian  
includes:

Chiral NN force  
up to  $N^3\text{LO}$   
Chiral 3N force  
up to  $N^2\text{LO}$

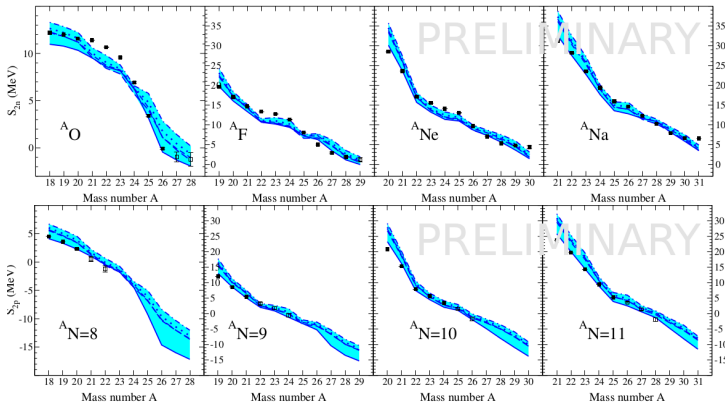
	2N force	3N force	4N force
LO		—	—
NLO		—	—
$N^2\text{LO}$			—
$N^3\text{LO}$			

Evolve through  
RG transformation  
to improve the  
convergence  
in many-body  
calculation



# Theoretical uncertainties in $sd$ nuclei

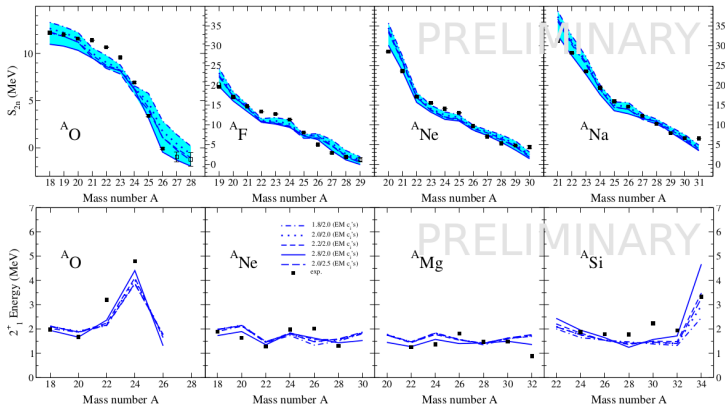
## Sensitivity to resolution-scale dependence of RG-evolved nuclear forces



Experimental trends very well reproduced in  $S_{2n}$ 's and  $S_{2p}$ 's  
Uncertainties in  $S_{2n}$ 's  $\sim 1 - 3$  MeV, in spectra much smaller  $\sim 500$  keV

# Theoretical uncertainties in $sd$ nuclei

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# Summary

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Shell Model calculation based on chiral effective field theory including NN+3N forces and many-body perturbation theory

- Predicted neutron rich Ca  $S_{2n}$ 's with NN+3N forces agree with recent measurements of  $^{51,52}\text{Ca}$  (TRIUMF) and  $^{53,54}\text{Ca}$  (ISOLTRAP)
- Shell structure: prominent closure established at  $N = 32$
- Predicted  $^{54}\text{Ca } 2_1^+$  in good agreement with measurement at RIBF
- Shell structure: suggested shell closure at  $N = 34$  to be complemented with mass measurements at  $^{55}\text{Ca}$  and  $^{56}\text{Ca}$
- Excitation spectra, B(E2) and B(M1) transitions
- Towards theoretical uncertainty quantification

# Collaborators

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K. Hebler, J. D. Holt,  
A. Schwenk, J. Simonis



ISOLTRAP Collaboration



TITAN Collaboration