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Viscosity of hot nuclei

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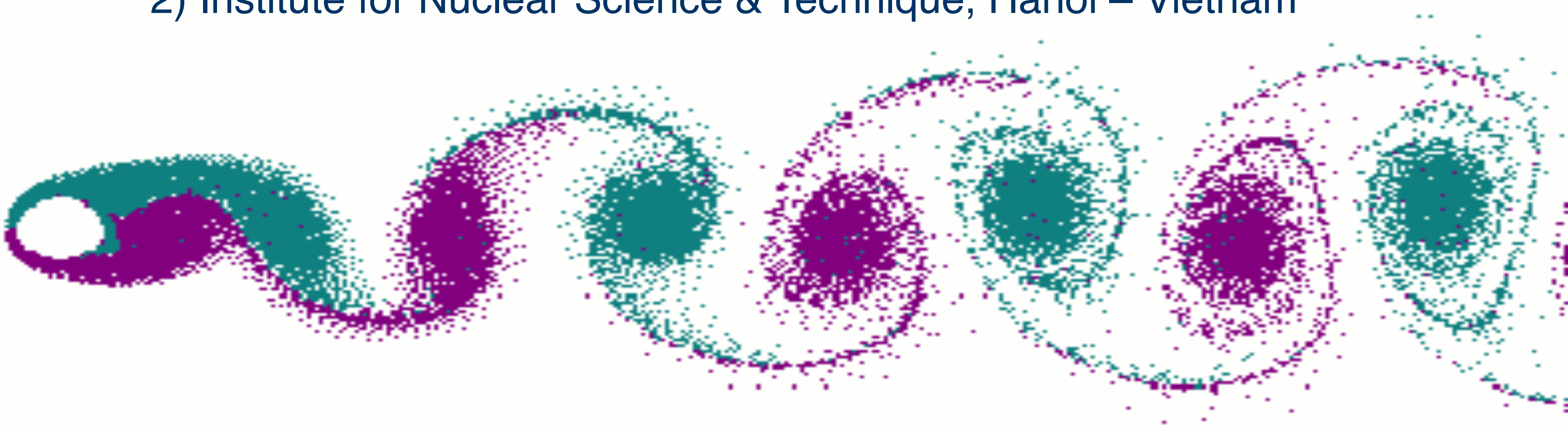


A brief history of viscosity

3 primary states of matter:

1) Solid: intermolecular attractions keep the molecules in fixed spatial relationships
2) Liquid: intermolecular attractions keep molecules in proximity, but not in fixed relationships.
3) Gas: molecules are separated and intermolecular attractions have little effect on their respective motions. **Plasma** is highly-ionized gas at high temperature.

Liquid and gas are **fluid: a** substance that continually deforms (flows) under an applied shear stress.

Viscosity is the resistance of a fluid which is being deformed by a stress.

Viscosity is the "thickness" or "internal friction" of a fluid.

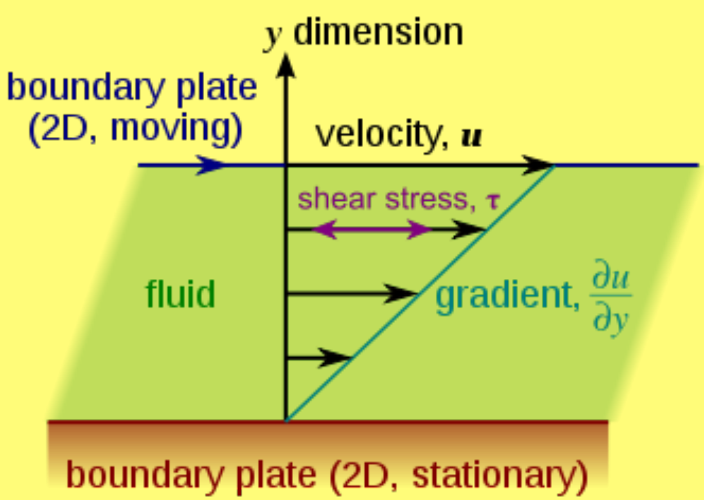
We say water is "thin". Honey is "thick"

Shear viscosity η

Unit: 1 *P* (poise) = 0.1 *Pa.s* = 1 *g / (cm.s)*

1*cP* = 1*mPa.s* = 0.001 *Pa.s*

named after Poiseuille (1797 – 1869)



$$\tau = \eta \frac{\partial u}{\partial y}$$

Substance	T (ºC)	η (cP)
Air	18	0.02
Water	20	1
Honey		2,000 – 10,000
Pitch	~ 20	230,000,000,000
Nucleus	-273	$(1-3)\times 10^{12}$
Lead glass	~ 500	$\sim 10^{14}$
QGP	4×10^{12}	$\sim 10^{14}$

$\phi = 1/\eta$ is called fluidity

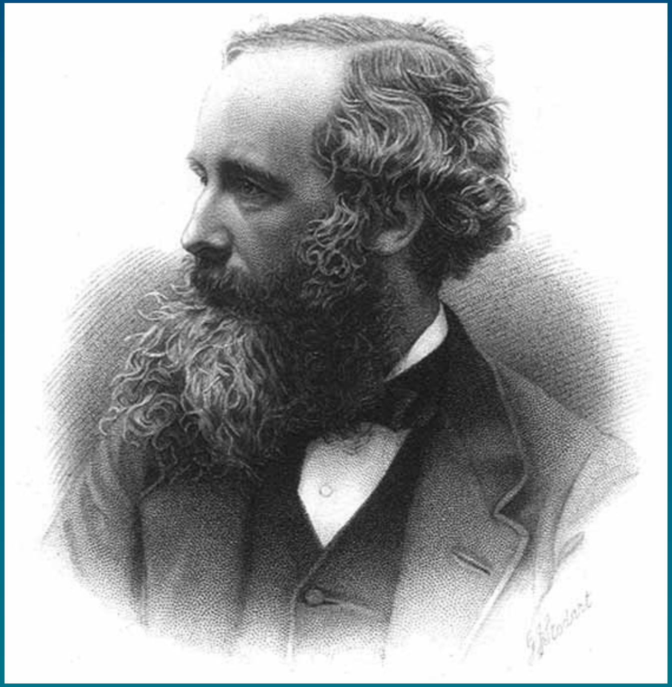
Viscosity of gases

In 1860 Maxwell showed that, if a gas is made of molecules, then its viscosity can be obtained by multiplying 3 quantities:

- *density of molecules* (e.g. 1.3 g/L for air)
- *average velocity of molecules* (250 m/s)
- *mean free path* (65 nanometers in air)

$$\eta = \frac{1}{3}\rho\bar{l}\bar{v}\,.$$

(In his 1860's paper "*Illustrations of Dynamical Theory of Gases*", Maxwell called it *internal friction*)



James Clark Maxwell
(1831-1879)

Conclusions by Maxwell:

1) Viscosity of a gas is independent of its pressure (density),
because the mean free path is inversely proportional to the density, i.e. a decrease of pressure by ½ reduces the density by ½ but increases the mean free path by 2.

This conclusion was known as Maxwell's law.

2) Viscosity of a gas increases with *T* (because *v* increases with *T* as T^n , $½ \le n \le 1$).

1) Maxwell himself could hardly believe that viscosity of a gas is independent of its pressure (density).

2) That viscosity increases with T also goes against the commonsense based on experience with liquids.

Maxwell wrote to Stokes asking for the experimental evidence. Stokes replied that Sabine did an experiment in 1829 showing that viscosity of a gas does vary with pressure*).

Maxwell was disappointed:

"Such a consequence of the mathematical theory is very startling and the only experiment I have met with on the subject does not seem to confirm it."

Maxwell decided to test this prediction himself. **The results (1865) by Maxwell the experimentalist confirmed the prediction by Maxwell the theorist: η= 0.0001878 (1+aT) for air (between 0.0167 ~ 1 *atm*).**

*) Stokes later admitted that the data analysis leading to this conclusion implicitly involved the assumption that viscosity vanishes at low pressures.

Viscosity of liquids

"Viscosity of a liquid is a very tough nut to crack".

E.M. Purcell (1912 – 1997)

Jean Louis Marie Poiseuille (1797 – 1869)

It cannot be related to the mean free path because of the strong interaction between its molecules.

Poiseuille was interested in the flow of human blood in narrow tubes.

Poiseuille's law (1844) for liquid flow through tubes:

$$\eta = \frac{\pi r^4 P t}{8VL},$$

t: elapsed time,
V: volume of liquid passing through the tube
P: hydrostatic pressure of the liquid
L: distance travel by the liquid in time *t*
r: radius of the tube

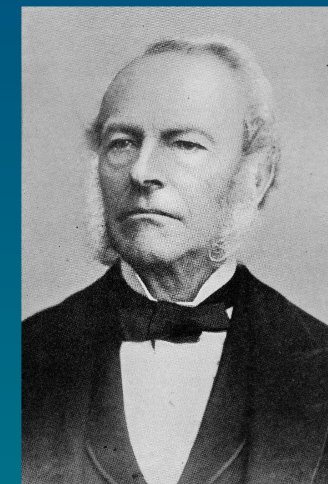
Fluid dynamics: C.L. Navier & G.G. Stokes

Navier-Stokes equations describe motion of a fluid in space:

Claude-Louis Navier
(1785 – 1836)

$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\text{Other body forces}}.$$

Derived first by Navier in 1831
and more rigorously by Stokes in 1845



Georges Gabriel Stokes
(1819 – 1903)

There is no proof that their solutions in three-dimensional case are smooth or do always exist with bound kinetic energy. **The Navier-Stokes existence and smoothness problem remains one of 7 most important problems in mathematics: 1,000,000 USD prize by Clay Mathematics Institute for a proof or counter-example**.

In the world of very small *Re* (viscosity is very large), e.g. microorganism, sperm, lava, paint, viscous polymer, etc. inertial forces are negligible, the flows obey the Stokes' equation (Stokes law).

Pitch Drop Experiment

World's longest continuously running laboratory experiment (Guinness record)

Pitch drop experiment at Univ. of Queensland (Australia) started dripping in 1930.

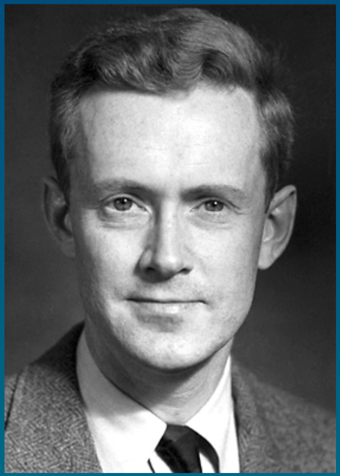
- 8 drops have fallen so far (28/11/2000), approximately 1 drop every 9 years.
- No one has ever seen a droplet fall live.
- There is enough pitch in the funnel to allow it to continue for at least another hundred years.

John Mainstone and the pitch drop experiment

Ig Nobel in 2005 to John Mainstone and Thomas Parnell (1881-1948). J. Mainstone attended the ceremony held at Harvard.

Viscosity of pitch is 230 billions times that of water.

Is fluid viscosity bound below?

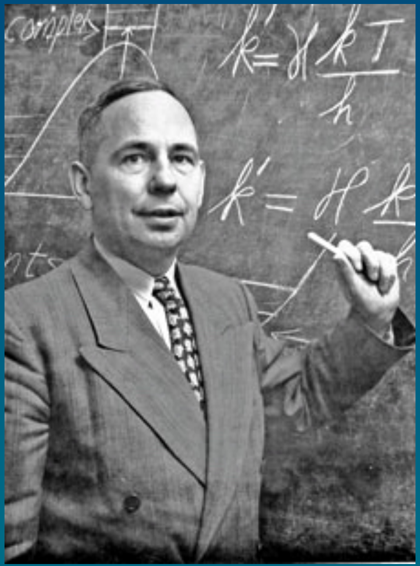


Edward Mills Purcell
(1912 – 1997)
1952 Nobel Prize in Physics
for the discovery of NMR

1) Maxwell's and Poiseuille's laws show that viscosity of a fluid can be infinite (ideal gas) but not zero.

2) "*If you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range but they stop at the same place. I don't understand that.*"

E.M. Purcell, *Life at low Reynolds number* (1976)



Henry Eyring
(1901- 1981)

3) Eyring (1936) assumed that the frequency of collisions of molecules in liquids is $k_B T/h$ and found

$$\eta \cong hne^{E/(k_B T)},$$

so at T$\rightarrow \infty$, viscosity $\rightarrow$ *hn* ($n = \rho/m$)

Viscosity of liquids can be very large (as that of pitch, e.g.) but cannot be too small.

Lower bound for η/s

For a relativistic fluid the particle number is not conserved. Instead of η/n it is better to study η/s. For many fluids $s/n \sim k_B$. From the Maxwell's law and the uncertainty relation one finds

$$m\bar{v}\bar{l} \geq \hbar \quad \rightarrow \quad \eta/s \geq \hbar/k_B .$$

Is it the lower bound? No, kinetic theory is not reliable in the regime $\eta/s \approx \hbar/k_B$.

Other methods are needed to determine the minimum value of η/s

Using string theory, **P. Kovtun**, **D.T. Son** and **A. Starinets (KSS),** PRL 94 (2005) 111601, **conjectured that the value**

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} , \qquad s = \frac{\rho}{A} S$$

is the universal lower bound for all fluids.

Why string theory?

When the temperature of the quark-gluon plasma is of the order of couple hundred MeV then the quark-gluon plasma is not a collection of weakly interacting quarks and gluons. It is not even a collection of strongly interacting quarks and gluons. Rather, it is a "hot soup" of strongly fluctuating quantum fields, in which particle-like "quarks" and "gluons" cease to exist. For a theorist, it is a mess.

(*Pavel Kovtun, QGP and string theory, RHIC News, Sep 11, 2007*)

Anti de Sitter/conformal field theory (AdS/CFT) correspondence is the conjectured equivalence between a string theory and gravity defined on one space, and a quantum field theory without gravity defined on the conformal boundary of this space, whose dimension is lower by one or more.

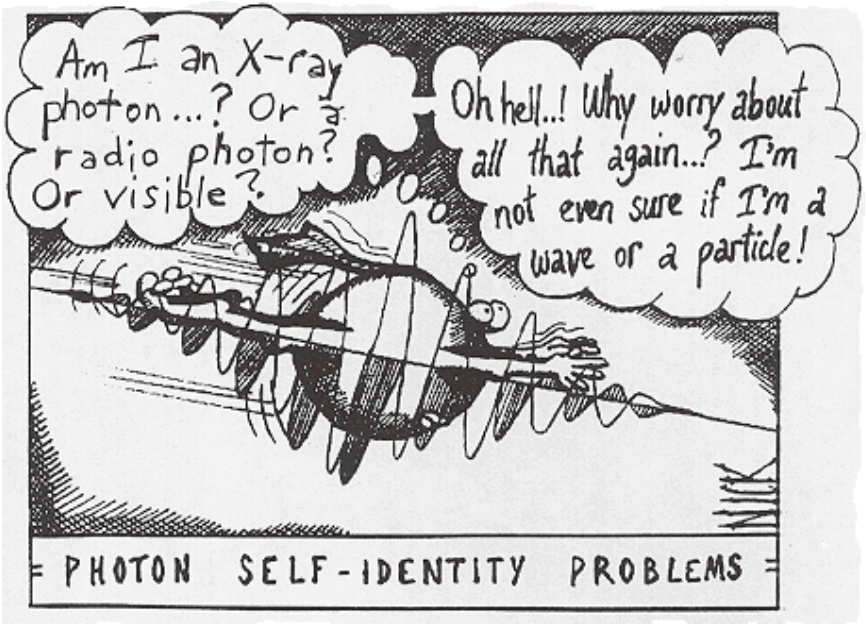
According to the AdS/CFT correspondence, *string theory and QCD-like field theory are simply two different representations of the same thing - just like position and momentum representations in quantum mechanics are simply two different views of exactly the same physics.* The string description of the QCD-like worlds is tractable when quantum fluctuations are strong, precisely when conventional QCD methods break down.

(*Pavel Kovtun, QGP and string theory, RHIC News, Sep 11, 2007*)

Physical meaning of the lower bound

Wave-particle duality: A particle is also a wave. De Broglie's wave length

$$\lambda = h / p \ .$$



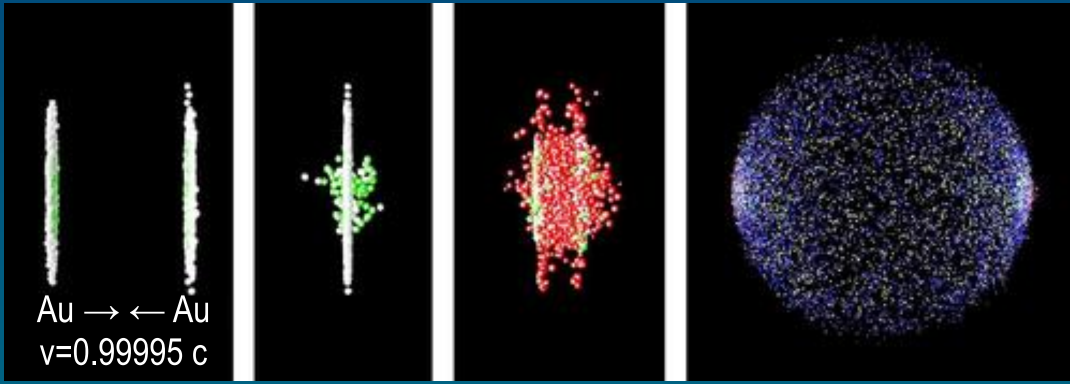
The mean free path cannot be shorter than the wave length, or else a particle does not have enough time to exist as "a particle".

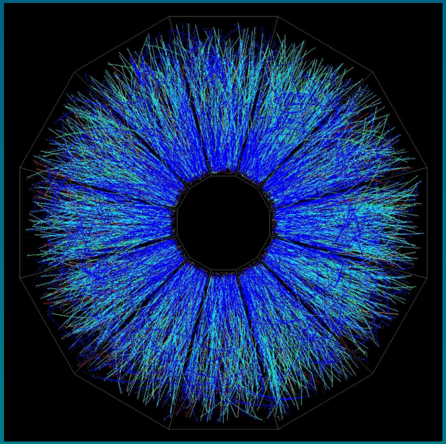
$$\frac{viscosity}{entropy\ density} = \frac{\hbar}{4\pi k_B}$$

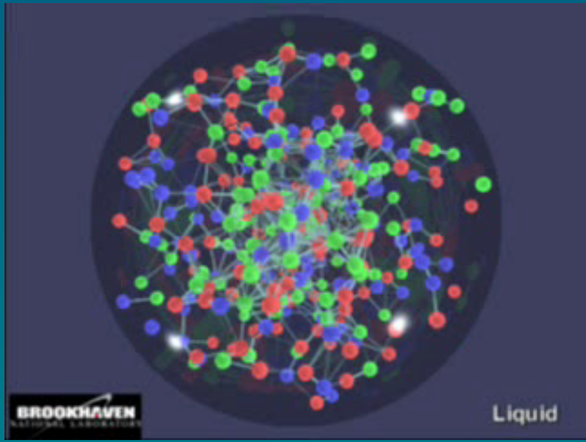
means the shortest mean free path of a particle is its wavelength, and the viscosity cannot be smaller than that. **The KSS conjecture relates (1) fluid dynamics, (2) thermodynamics with (3) quantum mechanics.**

Water (1 bar, 25 ^{o}C): **$\eta/s \approx$ 380 KSS** , liquid helium (including superfluid): **$(\eta/s)_{min} \approx$ 9 KSS**

Quark-Gluon Plasma







Experimental data from the RHIC (BNL) and LHC (CERN) have revealed that the matter formed in ultrarelativistic heavy-ion (Au-Au with $\sqrt{s_{NN}}$= 200 GeV, $T>T_c\sim175$ MeV, and Pb-Pb with $\sqrt{s_{NN}}$= 5.4 TeV) collisions is a nearly **perfect fluid** with extremely "low" viscosity: $\boldsymbol{\eta/s}$ **= (2 ~ 3) KSS**.

η / s in Hot Nuclei

One of fundamental explanations of giant resonance damping remains the friction term (viscosity) of neutron and proton fluids → **Viscosity can be extracted from the GR.**

I) Direct calculations using FLDM:

N. Auerbach & S. Shlomo

PRL 103 (2009) 172501

They found **η/s = (4-19) KSS** for heavy, **(2.5 – 12.5) KSS** for light nuclei.

Shortcomings:

1) the GDR width does not agree with experimental systematic at high T
2) The entropy $S = 2aT$ with constant level density parameter a;
3) Large uncertainties.

I propose to calculate η/s from the GDR width and energy at T≠0

NDD, PRC 84 (2011) 034309

By using the Green-Kubo relation, which expresses *h* in terms of correlation functions of shear stress tensors:

$$\eta(T) = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt d\mathbf{x} e^{i\omega t}\langle[T_{xy}(t,\mathbf{x}), T_{xy}(0,0)]\rangle \ ,$$

and the fluctuation-dissipation theorem, one obtains

$$\boxed{\eta(T) = \lim_{\omega\to 0}\frac{1}{2\omega i}[G_A(\omega) - G_R(\omega)] = -\lim_{\omega\to 0}\frac{\mathrm{Im}G_R(\omega)}{\omega} = \lim_{\omega\to 0}\frac{\sigma(\omega,T)}{C} \ ,}$$

with

$$G_R(\omega) = -i\int dt d\mathbf{x} e^{i\omega t}\theta(t)\langle[T_{xy}(t,\mathbf{x}), T_{xy}(0,0)]\rangle, \text{ and } G_A(\omega) = G_R(\omega)^*.$$

This exact formula relates the linear transport coefficient of any system (in a non-equilibrium or local thermodynamic equilibrium) to fluctuations in global thermodynamic equilibrium (such as thermal noise of electric and heat currents, collective vibrations, etc.)

$$C = \frac{\lim_{\omega\to 0}\left[\sigma_{GDR}(\omega, T=0)\right]}{\eta(0)}$$

GDR cross section

$$C = \frac{\lim_{\omega \to 0}[\sigma_{GDR}(\omega, T=0)]}{\eta(0)} ,$$

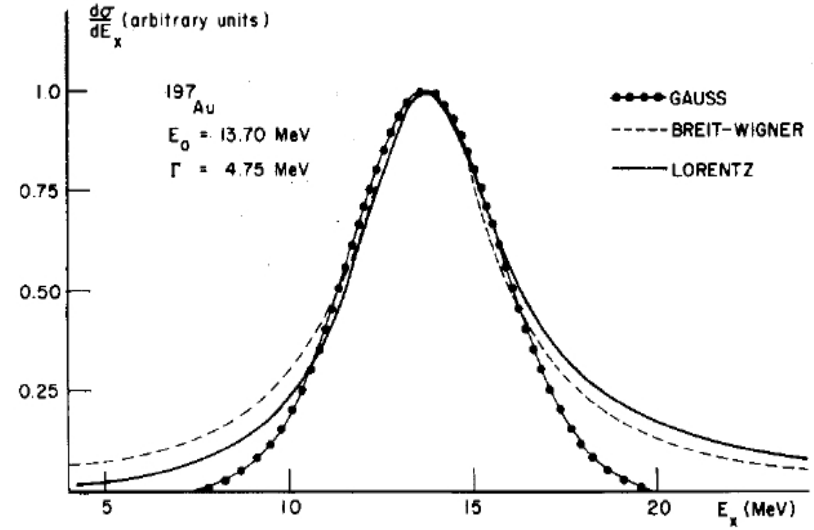


Fig. 1. Comparison of Gauss, Breit–Wigner and Lorentz line shape for the parameters of the GDR in ^{197}Au from refs. 1 and 3. The graph shows the typical rapid fall-off of the Gauss curve as compared to the others. While Breit–Wigner and Lorentz forms do not appear too different, the investigation presented in this paper shows that the Breit–Wigner curve better describes the dipole strength distribution in nuclei.

A) Breit-Wigner distribution:

$$\sigma_{GDR}(\omega) = \sigma_{GDR}^{int} f^{\mathrm{BW}}(\omega, E_{GDR}, \Gamma) \ ,$$

$$f^{\mathrm{BW}}(\omega, E_{GDR}, \Gamma) = \frac{1}{\pi} \frac{\Gamma/2}{[(\omega - E_{GDR})^2 + (\Gamma/2)^2]}$$

See: G. Breit & E. Wigner, Phys. Rev. 49 (1936) 519.

B) Lorentz distribution:

$$\sigma_{GDR}(\omega) = \sigma_0 E'_\nu \, f^L(\omega, E'_\nu, \Gamma), \qquad (E'_\nu)^2 = E_\nu^2 + (\Gamma/2)^2$$

$$f^{\mathrm{L}}(\omega, E'_\nu, \Gamma) = \frac{\omega}{E_\nu}\left[f^{\mathrm{BW}}(\omega, E_\nu, \Gamma) - f^{\mathrm{BW}}(\omega, -E_\nu, \Gamma)\right] = \frac{2}{\pi} \frac{\omega^2 \Gamma}{[\omega^2 - (E'_\nu)^2]^2 + \omega^2 \Gamma^2}$$

See: M. Danos & W. Greiner, Phys. Rev. 138 (1965) B876,
E.F. Gordon & R.Pitthan, Nucl. Instruments & Methods, 145 (1977) 569

Shear viscosity of hot nuclei

A) By using Breit-Wigner distribution:

$$\eta(T) = \eta(0)\frac{\Gamma(T)}{\Gamma(0)}\frac{E_{GDR}(0)^2 + [\Gamma(0)/2]^2}{E_{GDR}(T)^2 + [\Gamma(T)/2]^2}$$

B) By using Lorentz distribution:

$$\eta(T) = \eta(0)\frac{\Gamma(T)}{\Gamma(0)}\left\{\frac{E_{GDR}(0)^2}{E_{GDR}(0)^2 - [\Gamma(0)/2]^2 + [\Gamma(T)/2]^2}\right\}^2$$

Entropy density

$$s = \frac{S}{V} = \rho\frac{S}{A}$$

$$S = \int_0^T \frac{1}{\tau}\frac{\partial\mathcal{E}}{\partial\tau}d\tau \qquad\qquad S = S_F + S_B$$

$$S_\alpha^{\rm PDM} = -\sum_j N_j[p_j \ln p_j \pm (1\mp p_j)\ln(1\mp p_j)] \quad (\alpha = F, B)$$

F

$$n_j^{FD} = [\exp(E_j/T)+1]^{-1},$$

N_j = 2j+1 , (upper sign)

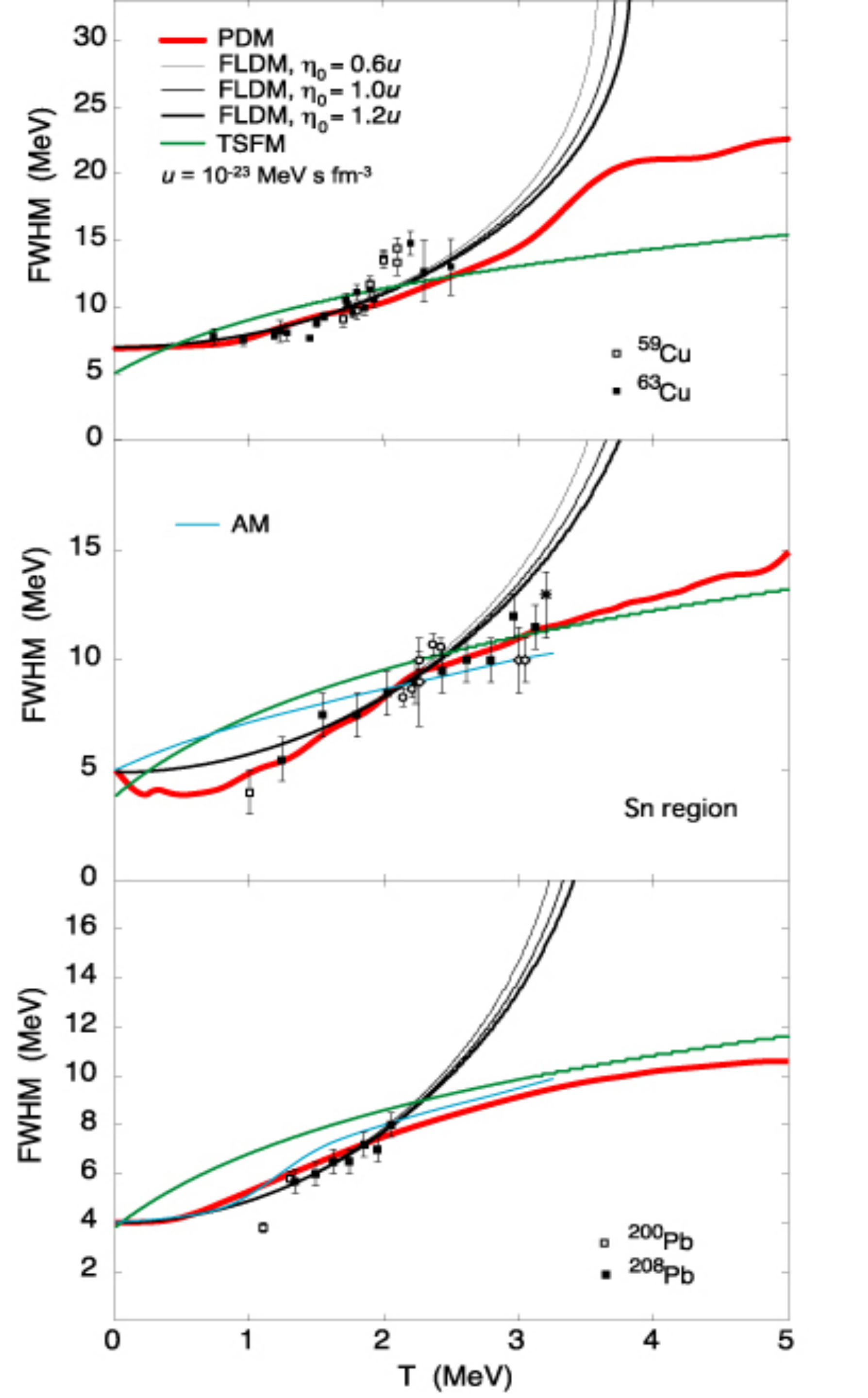
B

$$\nu_{GDR}^B = [\exp(E_{GDR}/T)-1]^{-1}$$

N_j = 1 , (lower sign)

GDR width at T ≠ 0

AM: Ormand, Bortignon & Broglia (1996)
PDM: Dang & Arima (1997)
TSFM: Kusnezov, Alhassid, Snover (1998)
FLDM: Kolomietz & Shlomo (2006)



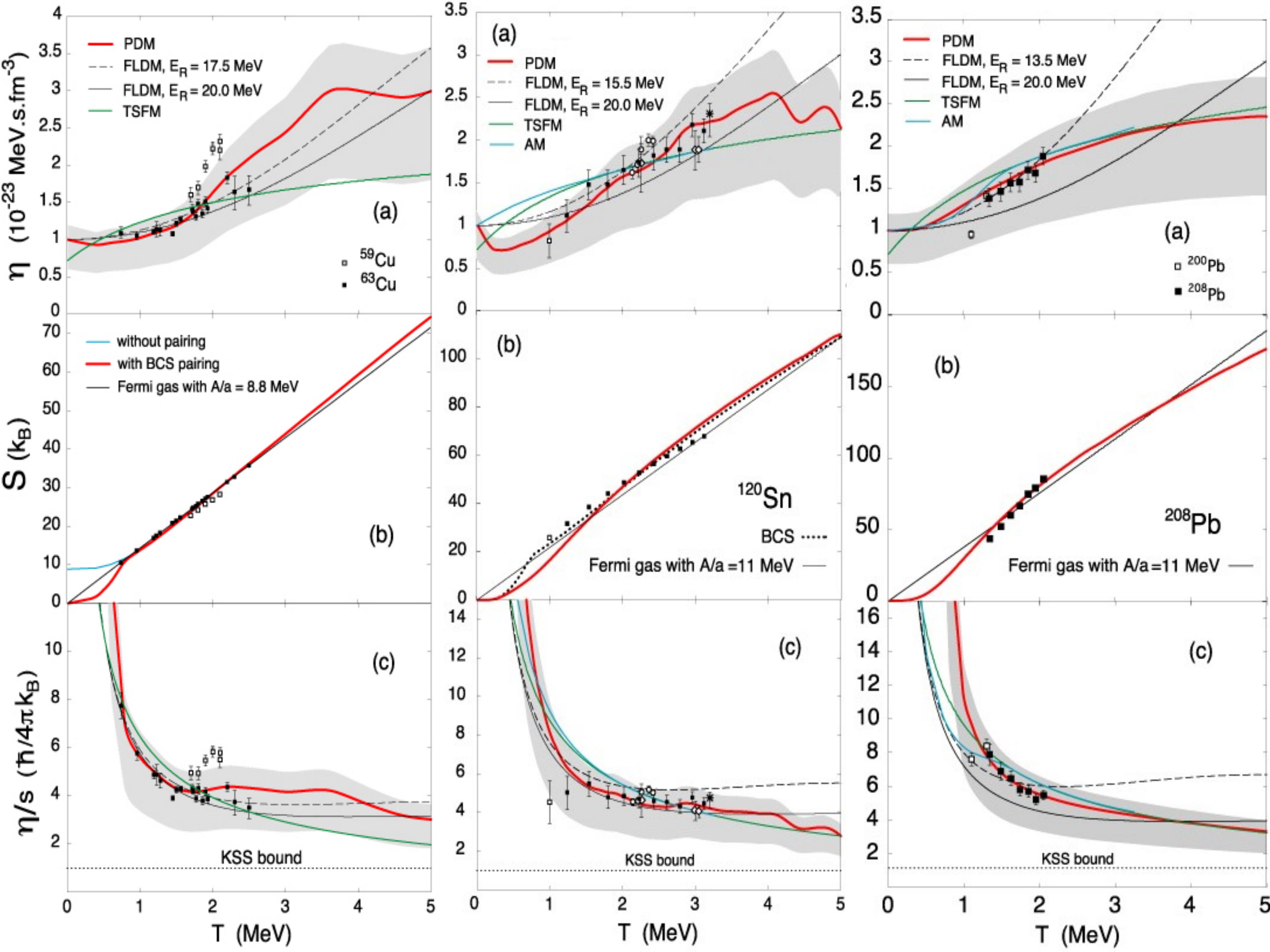
Model independent estimation:

$S \rightarrow -2\Omega / N \ln(n)$ with $n \rightarrow$ ½
at $T \rightarrow \infty$ implies $\Omega = N$ to conserve N,
$s \rightarrow 2\rho \ln 2 \approx 0.222$ (k_B).

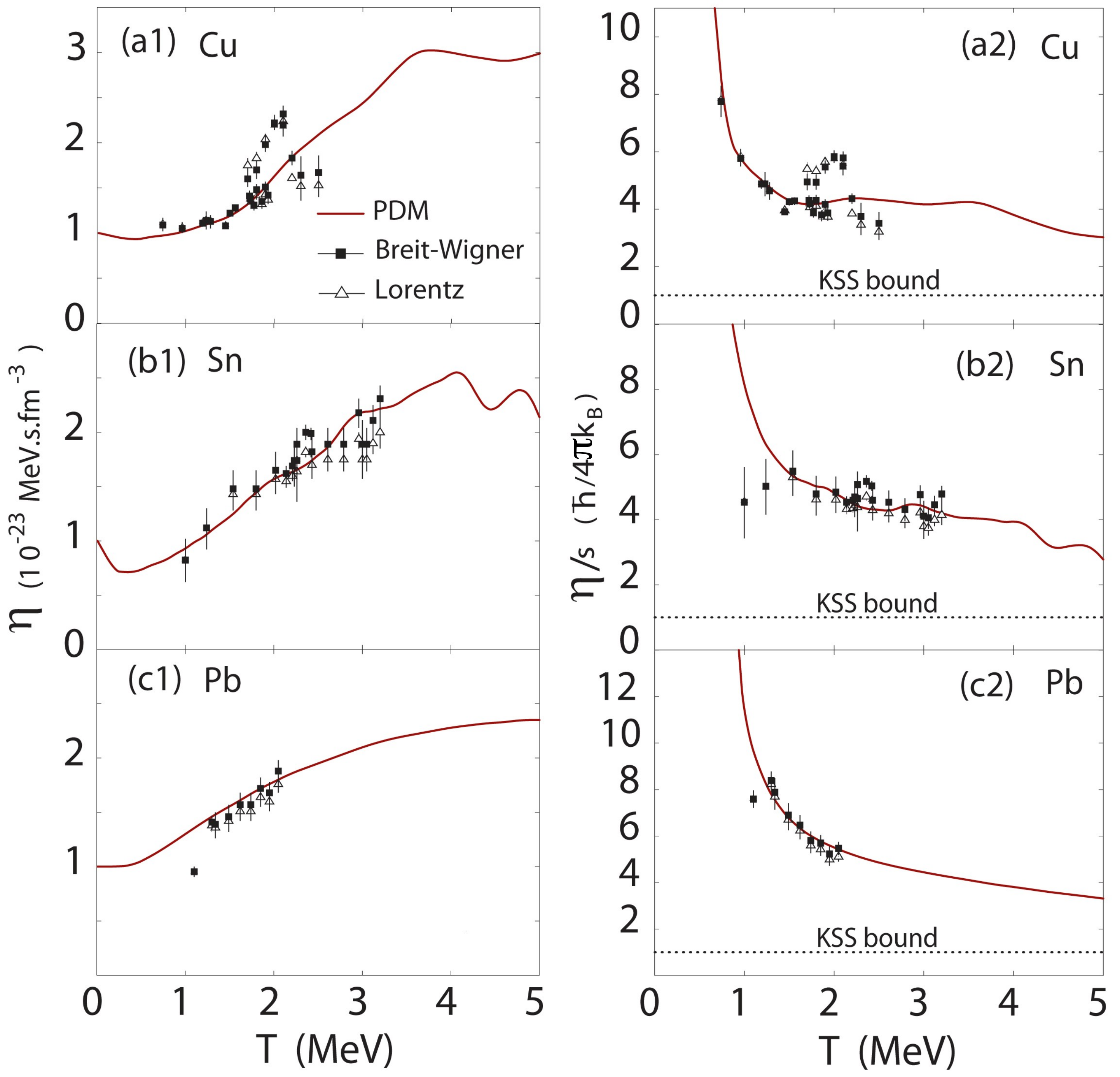
$$\frac{\eta}{s}(\infty) = 2.2^{+0.4}_{-0.9}\ KSS$$

with $\Gamma(\infty) = 3\ \Gamma(0) \approx 0.9 E_{\mathrm{GDR}}(0)$, and

$$\eta(0) = 1.0^{+0.2}_{-0.4} \times u\ (u = 10^{-23}\ \mathrm{Mev\ s\ fm^{-3}})$$



PDM
FLDM, E_R = 17.5 MeV
FLDM, E_R = 20.0 MeV
TSFM
(a)
59Cu
63Cu
η (10^-23 MeV.s.fm^-3)
without pairing
with BCS pairing
Fermi gas with A/a = 8.8 MeV
(b)
S (k_B)
(c)
η/s (ħ/4πk_B)
KSS bound
T (MeV)
(a)
PDM
FLDM, E_R = 15.5 MeV
FLDM, E_R = 20.0 MeV
TSFM
AM
(b)
120Sn
BCS
Fermi gas with A/a =11 MeV
(c)
KSS bound
T (MeV)
PDM
FLDM, E_R = 13.5 MeV
FLDM, E_R = 20.0 MeV
TSFM
AM
(a)
200Pb
208Pb
(b)
208Pb
Fermi gas with A/a =11 MeV
(c)
KSS bound
T (MeV)



(a1) Cu
(b1) Sn
(c1) Pb
(a2) Cu
(b2) Sn
(c2) Pb
PDM
Breit-Wigner
Lorentz
KSS bound
η (10^-23 MeV.s.fm^-3)
η/s (ħ/4πk_B)
T (MeV)

Conclusion

By adopting $\eta(0) = 1.0^{+0.2}_{-0.4} \times u \ (u = 10^{-23} \text{ Mev s fm}^{-3})$, it is shown that:

1) The shear viscosity η increases with T up to T ~ 3 - 3.5 MeV, and saturates at higher T; η *(T = 5 MeV)* ~ (1.3 – 3.5) *u;*

2) η/s decreases with increasing T, to reach (1.3 ~ 4.0) KSS at T= 5 MeV. These values are lower and of less uncertainty than the prediction by the FLDM (4 ~ 19 KSS).

Nucleons inside a hot nucleus at T ~ 5 MeV have nearly the same viscosity as that of QGP (2 – 3 KSS units) at T > 175 MeV.