Thermal shape and orientation fluctuation corrections for the hot giant dipole resonance within the static path approximation

Ahmad Ansari
Institute of Physics, Bhubaneswar, India

Nguyen Dinh Dang
RI-beam factory project office, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
and Institute for Nuclear Science and Technique, VAEC, Hanoi, Vietnam

Akito Arima
House of Councilors, 2-1-1 Nagata-cho, Chiyoda-ku, Tokyo 100-8962, Japan
and RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
(Received 31 July 2000; published 19 January 2001)

The γ-absorption cross section [i.e., giant dipole resonance (GDR)] of ^{106}Sn is calculated at temperature $T=2$ MeV and angular momenta $J=40$ and 55 following the linear response theory at finite temperature and incorporating the thermal shape as well as orientation fluctuation corrections within the static path approximation approach. The orientation fluctuation corrections have been included for the first time in such a calculation, and are found to be rather unimportant. Then without the inclusion of these effects the GDR is calculated at several more temperature and spin values. We find that at $T=4$ MeV the GDR width Γ shows an increase of only about 1.5 MeV compared to its value at $T=0.5$ MeV. However, at a fixed temperature $T=2$ MeV, the increase in the width at $J=69$ is by about 3 MeV compared to its value at $J=0$. Finally, we have also compared the increase of Γ at $T=2$ MeV as a function of J for ^{106}Sn and ^{120}Sn. The increase is steeper for the lighter isotope, as expected.

DOI: 10.1103/PhysRevC.63.024310 PACS number(s): 21.10.Pc, 24.10.Pa, 24.30.Cz, 24.60.−k

I. INTRODUCTION

The study of giant dipole resonance (GDR) at high temperatures and spins is still an active area of investigations experimentally [1–6] as well as theoretically [7–11]. There is the need for a fully microscopic approach that employs an effective many-body Hamiltonian and can reproduce the dependence of the GDR width Γ and shape as a function of temperature T and spin J. Recently a microscopic scheme was proposed in Ref. [8] in which the γ-absorption cross section $\sigma(E)$ as a function of the γ-ray energy E is computed in the linear response theory at finite temperature with averaging over the fluctuating shape parameters within the static path approximation (SPA) to the grand canonical partition function. The angular momentum dependence is included following the standard cranking approach. Very recently [12], we have applied this approach to the study of GDR parameters of ^{120}Sn. It was found that although the increase of Γ with T is very weak, the average over shape fluctuations within the SPA can provide a reasonable dependence on J. However, corrections for the fluctuation of the orientation angles of the rotational axis were not included.

As a continuation of the above work we have attempted here to investigate the effect of orientation fluctuation corrections with numerical calculations performed for ^{106}Sn. As this nucleus is lighter than ^{120}Sn, we expect a stronger dependence of Γ as well as an average of deformation parameters on the angular momentum [3].

In the next section we present a brief outline of the formalism. In Sec. III some numerical details will be presented along with a discussion of our results. Finally, in Sec. IV, we present some conclusions.

II. THEORETICAL FRAMEWORK

As we use the same formalism as that of Ref. [8], we give here only some essential equations to outline the steps involved in the calculations. The many-body Hamiltonian employed has a spherical single-particle part and a quadrupole-quadrupole interaction term:

$$\hat{H}=\hat{H}_0-\frac{1}{2}\chi_Q\sum_{\mu}(-1)^{\mu}\hat{Q}_{-\mu}\hat{Q}_{\mu}, \tag{1}$$

where $\hat{H}_0$ stands for the spherical part and the quadrupole operator $\hat{Q}_{\mu}$ is given as $\hat{Q}_{\mu}=(r^2/b^2)Y_{2\mu}$ with the usual harmonic-oscillator (HO) length parameter $b^2=\hbar/m\omega_0$, where $\hbar\omega_0=41A^{-1/3}$ MeV. The interaction strength parameter χ_Q (in MeV) is taken as [13]

$$\chi_Q=120A^{-5/3}f_c, \tag{2}$$

where f_c is a core polarization factor ($f_c\geqslant 1$) taken as unity if there is no assumption of an inert core. The angular momentum dependence is brought in following the standard cranking approach [8,14]. Taking the cranking axis to be the z axis in the laboratory frame of reference, the partition function within the SPA takes the form [15]

TABLE I. Core polarization factor f_c and the value of the deformation parameter in the ground state, β_0, in two model spaces for ^{106}Sn and ^{120}Sn.

	Space $S(A)$		Space $S(B)$	
Nucleus	f_c	β_0	f_c	β_0
^{106}Sn	1.60	0.116	1.75	0.118
^{120}Sn	1.75	0.104	2.0	0.109

$$Z_{SPA}=4\pi\left(\frac{\alpha}{2\pi T}\right)^{5/2}\int dD(\psi,\theta,\beta,\gamma)e^{-\alpha\beta^2/2T}\times z(\psi,\theta,\beta,\gamma,\omega,T), \tag{3}$$

where

$$z(\psi,\theta,\beta,\gamma,\omega,T)=\mathrm{Tr}\,e^{-(\hat{H}^{\omega}-\mu_p\hat{N}_p-\mu_n\hat{N}_n)/T} \tag{4}$$

and

$$\int dD(\psi,\theta,\beta,\gamma)=\int_0^{\pi}d\psi\int_0^{\pi}d\theta\sin\theta\int_0^{\infty}d\beta\beta^4\int_0^{\pi/3}d\gamma\sin(3\gamma). \tag{5}$$

In the above equations $\alpha=(\hbar\omega_0)^2/\chi_Q$ and $\hat{H}^{\omega}=\Sigma_i\hat{h}^{\omega}(i)$, with

$$\hat{h}^{\omega}=\hat{h}_0-\hbar\omega_0\beta\left(\frac{r^2}{b^2}\right)\left[\cos\gamma Y_{20}+\frac{1}{\sqrt{2}}(Y_{22}+Y_{2-2})\right]-\omega(\cos\theta\hat{j}_z-\sin\theta\cos\psi\hat{j}_x+\sin\theta\sin\psi\hat{j}_y), \tag{6}$$

where angular momentum operators are now in the body-fixed (principal axes) frame of reference. It is clear that the trace in Eq. (4) can be computed with the eigenvalues of the one-body Hamiltonian (6), which is complex Hermitian. If the orientation angles ψ and θ are set to zero value, it will amount to a simple z-axis cranking in the usual sense. The values of the chemical potentials $\mu_{p,n}$ and the cranking frequency ω can be adjusted to get the correct values of the number of particles and the expectation value of $\hat{J}_z$:

$$N_{p,n}=T\frac{\partial}{\partial\mu_{p,n}}\ln Z_{SPA}, \tag{7}$$

$$M=\langle\hat{J}_z\rangle=T\frac{\partial}{\partial\omega}\ln Z_{SPA}. \tag{8}$$

Using the eigenvalues and eigenfunctions of the above Hamiltonian (6) the γ-absorption cross section with the γ-ray energies E can be computed following the linear response theory as discussed in Refs. [8,14]:

$$\sigma(E,\beta,\omega,T)=-4\pi\alpha E\sum_{\mu=-1,0,1}\mathrm{Im}\,R_{D_{1\mu}D_{1-\mu}}\times(E-\omega\mu,\beta,T), \tag{9}$$

where $\alpha=e^2/\hbar c$, and R is the response function matrix with $D_{1\mu}$ denoting the dipole operator expressed in tensor representation. In the above, β is used to stand for all the shape parameters including the orientation angles. In Cartesian coordinates the $3\times3R$ matrix is

$$R(E)=\frac{R^0(E)}{1-\chi_D R^0(E)} \tag{10}$$

for a separable dipole interaction with the interaction strength [14]

$$(\chi_D)_i=3m\omega_i^2\frac{A}{NZ} \tag{11}$$

with the deformed harmonic oscillator frequency

$$\omega_i\simeq\omega_0\left[1-\sqrt{5/4\pi}\beta\cos\left(\gamma-\frac{2\pi}{3}i\right)\right], \tag{12}$$

where $i=1$, 2, and 3 means x, y, and z, respectively. The matrix R^0 is given by

$$R^0=R_p^0+R_n^0, \tag{13}$$

where

$$(R^0_{i,j})_{p,n}=\sum_{k,k'}\frac{\langle k|(\hat{D}_i^{\dagger})^{p,n}|k'\rangle\langle k'|(\hat{D}_j)^{p,n}|k\rangle}{(E+i\eta)-E_{kk'}}f_{kk'}. \tag{14}$$

The dipole operators are defined as

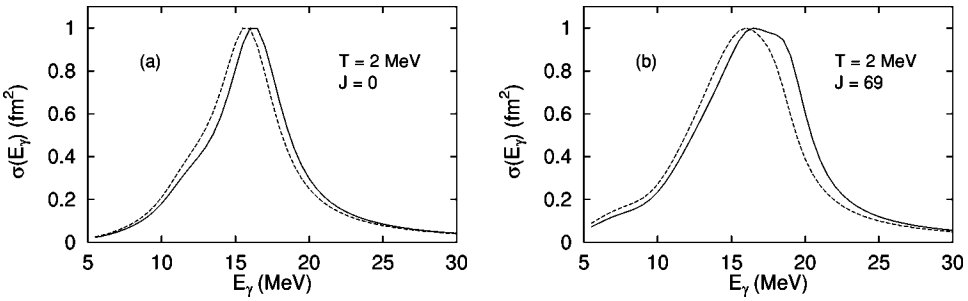

FIG. 1. Normalized γ-absorption cross section $\sigma(E)$ of the GDR in ^{106}Sn. The solid curve corresponds to the calculation in the large model space $S(A)$ as discussed in the text. The dashed curve similarly corresponds to $S(B)$, the smaller model space. (a) Results obtained at $T=2$ MeV and $J=0$; (b) those at $T=2$ MeV and $J=69$.

$$\hat{D}_i^p = \frac{N}{A} x_i^p, \quad \hat{D}_i^n = -\frac{Z}{A} x_i^n, \tag{15}$$

where $x_i = x,y,z$ for $i = 1,2,3$, respectively. The ket $|k\rangle$ is the eigenstate of the mean-field Hamiltonian (6) with eigenvalues ϵ_k and $E_{kk'} = \epsilon_k - \epsilon_{k'}$, $f_{kk'} = f_{k'} - f_k$ with f_k being the standard Fermi distribution functions. An imaginary energy $i\eta$ is added to E in order to have a finite GDR width.

Thus, main numerical calculation involves the computation of the matrix R^0 as a function of various variables such as the γ-ray energy, temperature, cranking frequency, and the shape parameters. The thermal-averaged value of the cross section is finally calculated as

$$\sigma(E,\omega,T) = \frac{\int dD(\psi,\theta,\beta,\gamma) e^{-\alpha\beta^2/2T} z(\psi,\theta,\beta,\gamma,\omega,T) \sigma(\psi,\theta,\beta,\gamma,E,\omega,T)}{\int dD(\psi,\theta,\beta,\gamma) e^{-\alpha\beta^2/2T} z(\psi,\theta,\beta,\gamma,\omega,T)}. \tag{16}$$

It may be noted that, if the orientation angles are set to zero, the limit of γ integration should be such that it allows for full shape evolution with the possibility of rotation about and perpendicular to the prolate as well as the oblate symmetry axes. In our notation (see Fig. 1.4 in Ref. [16]) we have $0 \leqslant \gamma \leqslant \pi$ for the z-axis cranking and $-\pi/3 \leqslant \gamma \leqslant 2\pi/3$ for the x-axis cranking.

III. SOME NUMERICAL DETAILS AND RESULTS

As in our recent paper [12], the dipole interaction strength χ_D is reduced by 25% as compared to the one given by Eq. (11) above. This is required to get the GDR energy corresponding to the peak of the cross section, E_0, close to the experimental value of about 16 MeV. For the quadrupole interaction strength χ_Q [Eq. (2)], the value of the core polarization factor f_c depends upon the dimension of the model space such that the equilibrium value of the quadrupole deformation parameter, β_0, comes around 0.1 for ^{106}Sn as well as for ^{120}Sn [17]. Since our main emphasis in the present work is to estimate the effect of orientation fluctuation corrections at high spins, we have explored the possibility of using a smaller basis space compared to the one employed in [12], because now we have a four-dimensional integral instead of a two-dimensional one. Thus, with $N = Z = 28$ as the inert core, we consider here two model spaces with the spherical single-particle energies calculated using the spherical Nilsson potential with A-dependent parameters [18].

(i) Space $S(A)$ consists of 54 negative parity and 86 positive parity orbitals (total 140) extending the basis space up to $N = 6$ major shells, where N is the principal quantum number.

(ii) Space $S(B)$ has a truncated positive-parity basis space compared to $S(A)$ keeping only the $0i_{13/2}$ orbitals from the $N = 6$ shell, i.e., only 44 positive parity orbitals (with total 98).

The adopted values of the parameter f_c depending upon the basis space and the nucleus are listed in Table I along with the corresponding β_0 values for the ground state. The value of the smearing parameter η appearing in the matrix R is taken to be 2 MeV as in [12]. Also the radial matrix elements are similarly reduced by the appropriate factors $(N_0 + 3/2)/(N + 3/2)$, where $N_0 = 3$ and $N > 3$ are the principal quantum number (see Ref. [19] for a discussion of the use of such a reduction factor for the $\hat{Q} \cdot \hat{Q}$ interaction). For ^{106}Sn we have 22 proton and 28 neutron active particles. Therefore, for this nucleus the model space $S(B)$ with 98 orbitals including the high-j orbitals $0i_{13/2}$ should be quite adequate. Use of different model spaces amounts to use of different effective Hamiltonians, and with only one controlling parameter f_c, these cannot be made fully equivalent. However, we would like that the qualitative results should be similar in the two model spaces $S(A)$ and $S(B)$, at least for ^{106}Sn. For this purpose we have compared in Fig. 1(a) the GDR plot for ^{106}Sn at $T = 2$ MeV and $\omega = 0$ obtained in the two model spaces, where the solid curve corresponds to the large space $S(A)$ and the dashed one corresponds to $S(B)$. The peak positions are somewhat shifted but the overall shapes are similar. For space $S(A)$, the value of the width parameter Γ is 5.15 MeV which becomes 5.35 MeV in space $S(B)$. In Ref. [12] the value of Γ was determined by a fit to a single Lorentzian shape, whereas here it is directly estimated at the half-maximum of the cross section. Following the x-axis cranking approach without the inclusion of orientation fluctuation corrections, Fig. 1(b) shows a similar comparison at $T = 2$ MeV and $J \approx 69$. In this case $\Gamma = 8.07$ MeV for space $S(A)$ (solid curve) and 7.58 MeV for space $S(B)$ (dashed curve). Now the two values differ by 0.5 MeV, implying that the bigger space should be more suitable.

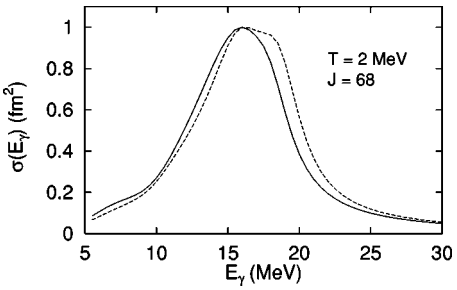

FIG. 2. The same as in Fig. 1 at $T = 2$ MeV and $J = 68$ calculated in the small model space $S(B)$. The solid curve is obtained by cranking about the x axis whereas the dashed curve corresponds to cranking about the z axis.

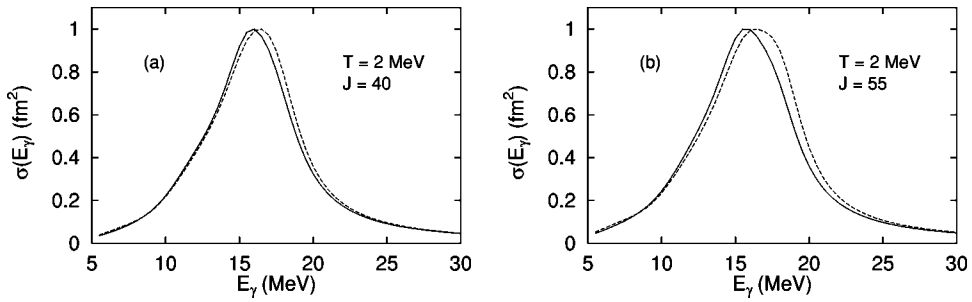

FIG. 3. Same as in Fig. 1 with cranking about the z axis in the model space $S(B)$. The solid curve is with the inclusion of the orientation fluctuation corrections, whereas the dashed one is without inclusion of such corrections. (a) Results at $T=2$ MeV and $J=0$; (b) those at $T=2$ MeV and $J=55$.

In Fig. 2 we show a comparison between results obtained with the x-axis cranking (solid curve) and those obtained with the z-axis cranking (dashed curve) at $T=2$ MeV and $J=68$ within the model space $S(B)$. The orientation fluctuation corrections are still not included. Figure 2 shows that these results are more or less similar. Hence, to study the effect of orientation fluctuations we choose the z-axis cranking so that we have additional integrations only over two angular variables [see Eq. (6)].

In Fig. 3(a) we show the cross section $\sigma(E)$ at $T=2$ MeV and $J\simeq 40$ for ^{106}Sn in the model space $S(B)$. The solid curve is obtained including the orientation fluctuation corrections and the dashed one is without such corrections. For the solid curve we obtain the values of $E_0=15.9$ MeV and $\Gamma=6.22$ MeV, compared to 16.39 MeV and 6.42 MeV, respectively, for the dashed curve. Figure 3(b) shows a similar comparison at $T=2$ MeV and $J\simeq 55$. Now $E_0=15.71$ MeV and $\Gamma=6.90$ MeV for the solid curve, which become 16.39 MeV and 7.19 MeV, respectively, for the dashed curve. Comparing Figs. 3(a) and 3(b), it is clear that the effect of the orientation fluctuation correction is not very significant. Here it is worth pointing out that in the presence of ψ and θ integrations the integration limit of γ is only from 0 to $\pi/3$, whereas in the absence of angular integrations the integration limit for γ is from 0 to π for the z-axis cranking. Therefore, it seems that, out of principal-axis planes, the orientation fluctuation is quite small, leading to almost the same results with and without orientation fluctuation corrections. In fact, at $T=2$ MeV and $J=55$ we find the average $\langle\psi\rangle=\langle\theta\rangle=90°$, which corresponds to a rotation about an oblate symmetry axis, J_y, in the present convention [see Eq. (6)]. But the thermal average of γ is about 28°, not 60°. It is similar to a negative value of average γ in case of x-axis cranking [12], as will be seen later on. Since inclusion of the orientation fluctuation correction consumes much more CPU time, it is not worth the effort. For instance, using 13-point trapezoidal method for ψ and 10-point Gaussian quadrature for the remaining three variables on a 64-bit DEC Alpha AXP workstation of RIKEN takes about 20 h in the smaller model space $S(B)$ for one value of T and ω and a fixed value of the chemical potentials.

In view of the above, we have further performed x-axis cranking calculation for ^{106}Sn exactly like the one for ^{120}Sn in our recent study [12] in the large basis space $S(A)$. Of course, we use $f_c=1.6$ for ^{106}Sn instead of 1.75 for ^{120}Sn (see Table I).

Figure 4(a) shows the plot of $\sigma(E)$ at $\omega=0$. The solid, dashed, and solid curves with diamond, dash-dotted, and dotted curves are the results at $T=0.5$, 1, 2, 3, and 4 MeV, respectively. The diamond points indicate the value of E at which actual numerical calculations have been performed in steps of $\delta E=0.5$ MeV. The average values of the deformation parameters at these respective temperatures are $\langle\beta\rangle=0.120$, 0.166, 0.226, 0.243, and 0.249 and $\langle\gamma\rangle=24.2°$, 24.3°, 24.3°, 25.6°, and 26.7°. As the figure shows, the value of E_0 is about 16 MeV, compared to its values of 16.3 MeV at $T=0.5$ MeV and 15.9 MeV at $T=4$ MeV. We obtain $\Gamma=4.47$, 4.67, 5.15, 5.54, and 5.93 MeV at $T=0.5$ to 4 MeV, corresponding to the curves in Fig. 4(a). The increase

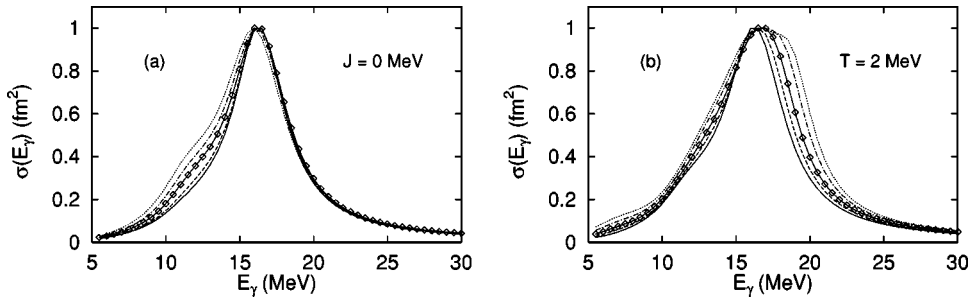

FIG. 4. (a) Same as in Fig. 1 at various T and $J=0$ with the calculation performed in the model space $S(A)$. The solid, dashed, and solid curves with diamonds, dash-dotted, and dotted curves correspond to $T=0.5$, 1, 2, 3, and 4 MeV, respectively. (b) $T=2$ MeV and various J values. The solid, dashed, and solid curves with diamond, dash-dotted, and dotted curves are for $J=0$, 26, 40, 57, and 69, respectively.

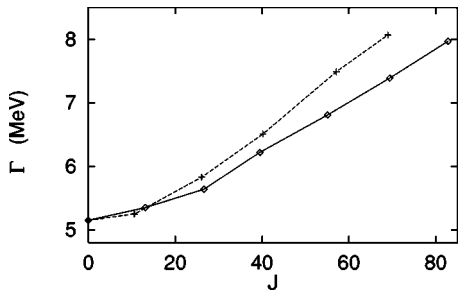

FIG. 5. GDR width Γ as a function of J at $T=2$ MeV for ^{106}Sn (dashed curve with crosses) and ^{120}Sn (solid curve with diamonds) following x-axis cranking in the model space $S(A)$. The crosses and diamonds correspond to the actual values of J for which the calculation is done.

of the width is only by about 1.5 MeV at $T=4$ MeV compared to that at $T=0.5$ MeV. This increase is by about 1.2 MeV in the case of ^{120}Sn.

Figure 4(b) depicts the variation of the GDR shape of ^{106}Sn as a function of J at $T=2$ MeV. The value of the GDR energy is $E_0=16.2$ MeV at $J=0$ and 16.5 MeV at $J=69$. The five curves—solid, dashed, solid with diamonds, dash-dotted, and dotted ones—correspond to $J=0$, 26, 40, 57, and 69, respectively. The corresponding values of the width Γ are 5.15, 5.83, 6.51, 7.48, and 8.07 MeV. The width increases by about 3 MeV at the highest spin compared to its value at $J=0$. The corresponding value for ^{120}Sn is found to be about 2 MeV. The average values of the shape parameters are $\langle\beta\rangle=0.226$, 0.225, 0.224, 0.225, and 0.228 and $\langle\gamma\rangle=24.2°$, 9.1°, $-0.5°$, $-11.8°$, and $-18.8°$ at $T=2$ MeV and $J=0$, 26, 40, 57, and 69, respectively. Thus, the average value of β remains around 0.23 whereas γ shows a change from positive triaxial to negative triaxial shape for an increase of J from 0 to 69. In the mean-field approximation at very high spins this nucleus is expected to exhibit a noncollective rotation about the oblate symmetry axis which corresponds to $\gamma=-60°$ in our convention. In this sense, the thermal average of about $-19°$ at $J=69$ and $T=2$ MeV should be quite reasonable. It may be emphasized that besides the constancy of $\langle\beta\rangle$, the width shows quite a reasonable increase with the increase of J [3].

Finally, in Fig. 5 we show the relative increase of Γ as a function of J at $T=2$ MeV for ^{106}Sn and ^{120}Sn within the x-axis cranking approach in the large basis $S(A)$. As expected [3,9], the increase of Γ is more rapid for the lighter isotope. The main problem in the present approach is that the increase of the width with temperature is not enough.

IV. CONCLUSIONS

We have reported on the calculations of some further investigations in the line of our recent work [12]. The following conclusions can be drawn from these studies.

(1) We find that, at high spins for a given temperature, the x-axis cranking and z-axis cranking results on the GDR properties are qualitatively similar.

(2) In the z-axis cranking approach the orientation fluctuation corrections (besides those in the deformation parameters) have been incorporated for the first time in this approach. It is found that the effect is not quite significant. This important result justifies the calculations in which this effect is neglected.

(3) Then without inclusion of orientation fluctuation corrections and following the x-axis cranking approach, the GDR properties of ^{106}Sn are studied in detail as for ^{120}Sn in Ref. [12]. The increase of the width as a function of temperature is not sufficient, but at a fixed temperature $T=2$ MeV the increase of Γ as a function of J is quite satisfactory. This increase is steeper for ^{106}Sn than for ^{120}Sn.

ACKNOWLEDGMENTS

A.A. gratefully acknowledges the hospitality of the RI-beam factory of RIKEN, where a major part of the work was done during his visit.

[1] J. J. Gaardhøje, Annu. Rev. Nucl. Part. Sci. **42**, 483 (1992).
[2] A. Bracco *et al.*, Phys. Rev. Lett. **74**, 3748 (1995).
[3] M. Mattiuzzi *et al.*, Nucl. Phys. **A612**, 262 (1997).
[4] T. Baumann *et al.*, Nucl. Phys. **A635**, 428 (1998).
[5] M. P. Kelly, K. A. Snover, J. P. S. van Schagen, M. Kicinska-Habir, and Z. Trznadel, Phys. Rev. Lett. **82**, 3404 (1999).
[6] M. Kmiecik *et al.*, Nucl. Phys. **A674**, 29 (2000).
[7] W. E. Ormand, P. F. Bortignon, R. A. Broglia, and A. Bracco, Nucl. Phys. **A614**, 217 (1997).
[8] B. K. Agrawal, A. Ansari, and P. Ring, Nucl. Phys. **A615**, 183 (1997).
[9] D. Kuznesov, Y. Alhassid, and K. Snover, Phys. Rev. Lett. **81**, 542 (1998).
[10] N. Dinh Dang and A. Arima, Phys. Rev. Lett. **80**, 4145 (1998); Nucl. Phys. **A636**, 427 (1998).
[11] N. Dinh Dang, K. Tanabe, and A. Arima, Phys. Rev. C **58**, 3374 (1998); Nucl. Phys. **A645**, 536 (1999).
[12] A. Ansari, N. Dinh Dang, and A. Arima, Phys. Rev. C **62**, 011302(R) (2000).
[13] S. Åberg, Phys. Lett. **157B**, 9 (1985).
[14] P. Ring *et al.*, Nucl. Phys. **A419**, 261 (1984).
[15] B. Lauritzen, P. Arve, and G. F. Bertsch, Phys. Rev. Lett. **61**, 2835 (1989).
[16] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
[17] A. Leprêtre, H. Beil, R. Berger, P. Carlos, A. de Miniac, A. Veyssiere, and K. Kernbach, Nucl. Phys. **A219**, 39 (1974).
[18] S. G. Nilsson *et al.*, Nucl. Phys. **A131**, 1 (1969).
[19] M. Baranger and K. Kumar, Nucl. Phys. **A110**, 490 (1968).