Phonon damping model using random-phase-approximation operators

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A fully microscopic interpretation of the phonon damping model (PDM) is proposed. The phonon operator, which generates the collective excitation such as giant dipole resonance (GDR), is constructed from coherent particle-hole pairs. The phonon energy is determined as the solution of dispersion equation within the random-phase approximation (RPA). The phonon structure is found in terms of RPA and $Y$ amplitudes. The formalism is illustrated by numerical calculations within a schematic model. The results of calculations demonstrate the same feature of GDR as a function of temperature obtained previously within PDM-1, which employs structureless phonons.

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The phonon damping model (PDM) has been proposed recently in a version called PDM-1 [1], and developed further to include all the processes up to two-phonon ones in a version called PDM-2 [2]. This model has been shown to be efficient in the description of several important nuclear phenomena such as the observed increase and saturation of the width of giant dipole resonance (GDR) in highly excited nuclei [1–4], the electromagnetic cross sections of double GDR (DGDR) in $^{136}$Xe and $^{208}$Pb [5], and the appearance of the pygmy dipole resonance (PDR) in light neutron-rich nuclei [6]. The GDR has been generated so far within PDM using a structureless phonon. As such, the PDM has only three parameters, which are the unperturbed phonon energy $\omega_0$, the particle-hole ($ph$) interaction vertex $F_1$ at zero temperature ($T=0$), and the particle-particle ($pp$) and hole-hole ($hh$) interaction vertex $F_2$ at $T\neq 0$. The parameters $\omega_0$, $F_1$, and $F_2$ have been adjusted so that the calculated GDR energy and width at $T=0$ reproduce their experimentally extracted values. The value of $F_2$ is chosen at $T=0$ so that the GDR energy remains stable at $T\neq 0$. In order to construct a fully microscopic version of PDM, it is desirable to calculate the structure of phonons starting from a microscopic Hamiltonian with two-body residual interaction. This is the aim of the present work, in which we will construct the phonon operator from the coherent $ph$ pairs within the random-phase approximation (RPA) using a model Hamiltonian with a residual interaction in the form of separable two-body forces. The formalism is illustrated by numerical calculations within a schematic model. The results obtained will be discussed in comparison with the main feature of GDR as a function of $T$ obtained within PDM previously, and observed in experiments [7].

We consider the following model Hamiltonian with a two-body residual interaction in a separable form:

$$H = \sum_x E_x a_x^\dagger a_x - k \sum_{kl't'} f_{kl} f_{k'l'} B_{kl} B_{k'l'}^\dagger, \quad f_{kl} = f_{kl}. \quad (1)$$

where $B_{kl} = a_{kl}^\dagger$ with $(kl)$ running over all $(ph)$, $(pp)$, and $(hh)$ indices. The use of the separable force does not reduce the generality of the results, but it will allow an analytic solution of the schematic model, which will be considered later as an illustration of the formalism. For simplicity, we do not distinguish neutron and proton systems as well as the isoscalar and isovector parts of the parameter $k$ in Eq. (1) either. Using the standard RPA phonon operator $Q_v$,

$$Q_v = \sum_{ph} (X_{ph}^v B_{ph}^\dagger - Y_{ph}^v B_{ph}), \quad (2)$$

we express the $(ph)$ channel in the residual-interaction part of Eq. (1) in terms of $Q_v^p$ and $Q_v^s$ so that Eq. (1) can be rewritten as

$$H = \sum_x E_x a_x^\dagger a_x - k \sum_{klp} f_{kl} \sum_{pph} f_{ph} (X_{ph}^v Y_{ph}^v) (B_{kl}^\dagger + B_{kl}) \times (Q_v^p + Q_v^s)$$

$$= \sum_x E_x a_x^\dagger a_x - k \sum_{pph} \sum_{pph} f_{ph} (X_{ph}^v Y_{ph}^v)^2 (Q_v^p Q_v^s)$$

$$+ Q_v^p Q_v^s + 2 Q_v^p Q_v^s. \quad (3)$$

The scattering terms $k \Sigma_{ppp} f_{pp} f_{pp} B_{pp}^\dagger B_{pp}^\dagger$ and $k \Sigma_{hh} B_{hh} B_{hh}$ are neglected in the transition from Eqs. (1)–(3). As they contain no one-phonon terms, they have little influence on the damping of the RPA phonon excitations at $T=0$ in the lowest order, considered in the second term at the right-hand side (RHS) of Eq. (3). It has been shown in Ref. [8] that the effect caused by the scatter-
ing terms on the RPA phonon energies is negligible except for very low-lying states in transitional nuclei. At finite temperature, an approximate estimation of the effect due to the scattering terms can be made applying the standard method of lowering the order of Green’s functions within the same decoupling scheme, which was used to derive the equations of the PDM-1 [1,9]. We find that the inclusion of these terms leads only to the following renormalization of the single-particle energy $E_s'$:

$$E'_s = E_s - \delta E_s,$$

where $\lambda$ is the multipolarity of the phonon. The numerical estimation carried out for the hot GDR in $^{120}$Sn has shown that the change due to the effect of such a single-particle energy renormalization on the GDR width does not exceed that the change due to the effect of such a single-particle analytic solution, but has to be diagonalized numerically. In general, the RPA equation in this case does not have an analytic solution, but has to be diagonalized numerically. In the present work, as an illustration, we prefer to carry out the numerical analysis within a schematic microscopic model similar to what proposed in Ref. [11]. This model consists of $L$ single-particle levels with a degeneracy equal to 2, which have an equal spacing $E_\text{q}$ except for a shell gap $\Delta$ above the $L_{\text{gap}}$-th level. Thus, the single-particle energies measured from the Fermi energy $E_F$ are $E_I = (l-1)E_0 - E_F$ for $1 \leq l \leq L_{\text{gap}}$, and $E_I = (l-2)E_0 + \Delta - E_F$ for $L_{\text{gap}} < l \leq L$. The Fermi energy $E_F$ is defined at $T = 0$ from the condition $2\sum_{n=1}^{L_{\text{gap}}} N_F = N$. In order to mimic the GDR, we choose the following values for the parameters: $L = 200$, $L_{\text{gap}} = 100$, $E_0 = 0.15$ MeV, $\Delta = 6.62$ MeV, and $k = -0.18$ MeV$^{-1}$. The negative value of $k$ corresponds to the repulsive force, which takes place in the case of GDR. With this parameter’s selection, the Fermi level $E$ at $T = 0$ is located in the middle of the shell gap, i.e., between 100th and 101st levels. The schematic picture of energy levels is similar to Fig. 1(a) of Ref. [11]. The values of the matrix element $f_{kl}$ are chosen to be $f_{ph} = 0.018$ MeV for all $ph$ transitions, and $f_{pp'} = f_{hh'} = 0.4961$ MeV for all $pp$ and $hh$ ones. We assume that the giant resonance state, which we refer to hereafter as the GDR, is generated within RPA in the collective subspace $\{C\}$, which consists of $p$ and $h$ levels whose distance $E_h - E_p$ is equal to $E = 12$ MeV. The RPA phonon energy $\omega$ is then found analytically as

$$\omega_v = \omega_\nu - \delta \omega,$$

$$\delta \omega = -2k \sum_{ph} f_{ph}(X_{ph}^v + Y_{ph}^v)^2$$

is the “shifted” RPA energy, and

$$F_{kl}^{(v)} = -2k f_{kl} \sum_{ph} f_{ph}(X_{ph}^v + Y_{ph}^v).$$

Similar results can be obtained using a nonseparable force such as the Skyrme interaction or that of the shell-model after replacing $k_{kl'}^{\tau' \tau}$ with a two-body matrix element $V_{k_{kl'}^{\tau' \tau}}$. In this case $k \sum_{ph} f_{ph}(X_{ph}^v + Y_{ph}^v)^2$ in Eq. (6) obviously becomes $\sum_{pp'h'}X_{pp'h'}^v Y_{pp'h'}^v$. However, the RPA equation in this case does not have an analytic solution, but has to be diagonalized numerically. In Eq. (5) we retain the mean-field part $\sum_s E_s a_s^\dagger a_s$, in addition to $\sum_v \omega_v Q_v^\dagger Q_v$ as the former still and only contributes in the derivation that involves $a_s^\dagger a_s$ beyond RPA. Therefore, there is no double counting for this term either. The RPA equation produces many solutions for phonon excitations.
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\[ \omega = E \sqrt{1 - 8k \sum_{ph} f_{ph}^2 D_{ph}/E}, \quad (8) \]

where \( D_{kl} = n_l - n_k \) with the single-particle number \( n_k \) given by the Fermi-Dirac distribution \( n_k = \exp[(E_k - E_F)/T] + 1 \)^{-1}. The RPA amplitudes \( X_{ph} \) and \( Y_{ph} \), with \( n_h > n_p \) are given as

\[ X_{ph} = \frac{1}{N} \frac{f_{ph} \sqrt{D_{ph}}}{E - \omega}, \quad Y_{ph} = \frac{1}{N} \frac{f_{ph} \sqrt{D_{ph}}}{E + \omega}, \quad (9) \]

Using Eq. (9), we find that the RPA energy \( \omega \) is shifted down to \( \bar{\omega} \) by \( \delta \omega = -2k(E/\omega)\Sigma_{ph} f_{ph}^2 \sqrt{D_{ph}}/\Sigma_{ph} D_{ph} \) as given by Eq. (6). The Landau damping within this model is zero as there is only one phonon state, which generates the GDR. The resonance centroid energy \( \bar{\omega} \) is found within PDM as the solution of equation for the pole of the Green function for phonon propagation [1], which takes a simple form in this case

\[ \bar{\omega} - \bar{\omega} - \rho(\bar{\omega}) = 0, \quad \rho(\bar{\omega}) = -8k \sum_{kl} f_{kl}^2 D_{kl}(E_l - E_k)/(E_l - E_k)^2 - (\bar{\omega})^2. \quad (10) \]

The strength function \( S(\omega) \) and the phonon damping \( \gamma(\omega) \) are calculated according to Ref. [1] as

\[ S(E_\gamma) = \frac{1}{\pi} \frac{\gamma(E_\gamma)}{(E_\gamma - \bar{\omega})^2 + [\gamma(E_\gamma)]^2}, \quad \gamma(E_\gamma) = \pi k \sum_{kl} (f_{kl})^2 D_{kl} \delta(E_\gamma - E_l + E_k). \quad (11) \]

The calculations have been carried out at \( T = 0 \text{ to } 7 \text{ MeV}. \) The \( \delta \) function in the expression for the damping \( \gamma(E_\gamma) \) in Eq. (11) is smoothed using \( \delta(x) = (2\pi i)^{-1} [(x + i\epsilon)^{-1} + (x + i\epsilon)^{-1}] \) with \( \epsilon = 0.2 \text{ MeV} \) in the calculations. The Fermi energy \( E_F \) is found to be almost temperature independent. The RPA energy \( \omega \), the shifted RPA energy \( \bar{\omega} \), and the GDR energy \( \bar{\omega} \) are shown in Fig. 1 as a function of \( T \). The shift \( \delta \omega \) is \( 0.37 \text{ to } 0.38 \text{ MeV} \), which is only around \( 3\% \) of the GDR energy. The RPA energy \( \omega \) and the shifted RPA energy \( \bar{\omega} \) are found to be almost independent of \( T \). The GDR energy \( \bar{\omega} \) deviates from \( \bar{\omega} \) by at most \( \sim 0.7 \text{ MeV} \) at \( T = 1.5 \text{ to } 2 \text{ MeV} \). In general, this figure shows that the GDR energy is rather stable against varying \( T \). This also leads to a very weak increase of the matrix element (7) with increasing \( T \) so that they can be considered to be temperature independent, and equal to \( F_{pp} = F_1 = 0.67 \times 10^{-1} \text{ MeV} \), and \( F_{pp} = F_{hh} = F_2 = 0.17 \text{ MeV} \).

Shown in Fig. 2 are the calculated strength function \( S(E_\gamma) \) at several temperatures, and the full width at the half maximum (FWHM) \( \Gamma = 2\gamma(E_\gamma, \omega) \) as a function of \( T \). The width \( \Gamma \) is the total width caused by coupling of GDR phonon to all \( ph, pp \), and \( hh \) configurations [solid line in Fig. 2(b)]. The quantal width \( \Gamma_0 \) due to coupling of GDR phonon to only \( ph \) configurations, and the thermal width \( \Gamma_T \) due to coupling to only \( pp \) and \( hh \) configurations at \( T \neq 0 \) are also shown in Fig. 2(b) as the dashed and dotted lines, respectively. The total width \( \Gamma \) increases sharply with increasing \( T \) up to \( T = 3 \text{ to } 4 \text{ MeV} \), and saturates at \( T > 4 \text{ MeV} \) in agreement with the trend observed in experiments [7]. This feature

FIG. 1. The GDR energy as a function of temperature. The solid line is energy \( \bar{\omega} \) obtained from Eq. (10). The dashed line shows the RPA energy \( \omega \) (8), while the dotted line stands for the shifted RPA energy \( \bar{\omega} \) (6).

FIG. 2. Temperature dependence of the strength functions (a) and width (b) of GDR. In (a) solid, short-dashed, long-dashed, dotted, dash-dotted, and dash–double-dotted lines are results at \( T = 0, 1, 1.5, 2, 3, \) and 5 MeV, respectively. (b) The thick solid, dashed, and dotted lines are the total width \( \Gamma \), the quantal width \( \Gamma_0 \), and the thermal width \( \Gamma_T \), respectively, as described in the text.
is mostly dictated by the thermal width $\Gamma_T$. The quantal width $\Gamma_Q$ decreases with increasing $T$. The behavior of the quantal, thermal, and total widths as a function of $T$ is consistent with the results reported and discussed previously within PDM [1,2]. Figure 2(a) also shows that the low-energy part of the GDR shape ($E_g \lesssim 8–10$ MeV) becomes enhanced strongly, while the high-energy tail ($E_g > 17$ MeV) is depleted weakly with increasing $T$. As $T$ approaches the value of the shell gap $D$, the shape of the GDR peak becomes slightly skewed toward a higher energy, but the centroid remains nearly unchanged (see also Fig. 1). We also found that the maximal absolute value for the shift $\delta E_s$ of the single-particle energies in Eq. (4) is $2.124 \times 10^{-7}$ MeV at $T \approx 5.5$ MeV (with $\lambda = 1$ and the degeneracy $2j_s + 1 = 2$). At lower $T$ the shift is much smaller. Such a negligible shift practically causes no effect on the GDR width.

In conclusion, we have shown how the phonon operators, which PDM uses to generate GDR, can be constructed from RPA. This allows the PDM Hamiltonian to be derived from a microscopic Hamiltonian with a two-body residual interaction, confirming the microscopic foundation of PDM. The numerical analysis within a schematic model shows that the energy shift due to exclusion of double counting is not significant for GDR as it amounts to only few percents of the resonance energy obtained within RPA. This has been actually taken into account in Refs. [1,2] by adjusting the unperturbed energy $\omega_q$ of the structureless phonon so that the calculated GDR energy coincides with the value extracted in experiments. The numerical results within this simple schematic model also confirm the general features of GDR as a function of temperature. They include a position of the GDR, which is rather stable against $T$, and the behavior of the GDR width, which increases sharply with increasing $T$ up to $T \approx 3–4$ MeV, and saturates at higher $T$. It is also confirmed that the quantal width $\Gamma_Q$ due to coupling of GDR to only $\phi$ configurations does not stay constant, but decreases with increasing $T$.