Phonon damping model using random-phase-approximation operators

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A fully microscopic interpretation of the phonon damping model (PDM) is proposed. The phonon operator, which generates the collective excitation such as giant dipole resonance (GDR), is constructed from coherent particle-hole pairs. The phonon energy is determined as the solution of dispersion equation within the random-phase approximation (RPA). The phonon structure is found in term of RPA X and Y amplitudes. The formalism is illustrated by numerical calculations within a schematic model. The results of calculations demonstrate the same feature of GDR as a function of temperature obtained previously within PDM-1, which employs structureless phonons.

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The phonon damping model (PDM) has been proposed recently in a version called PDM-1 [1], and developed further to include all the processes up to two-phonon ones in a version called PDM-2 [2]. This model has been shown to be efficient in the description of several important nuclear phenomena such as the observed increase and saturation of the width of giant dipole resonance (GDR) in highly excited nuclei [1–4], the electromagnetic cross sections of double GDR (DGDR) in ^{136}Xe and ^{208}Pb [5], and the appearance of the pygmy dipole resonance (PDR) in light neutron-rich nuclei [6]. The GDR has been generated so far within PDM using a structureless phonon. As such, the PDM has only three parameters, which are the unperturbed phonon energy ω_q, the particle-hole (ph) interaction vertex F_1 at zero temperature ($T=0$), and the particle-particle (pp) and hole-hole (hh) interaction vertex F_2 at $T\neq 0$. The parameters ω_q and F_1 have been adjusted so that the calculated GDR energy and width at $T=0$ reproduce their experimentally extracted values. The value of F_2 is chosen at $T=0$ so that the GDR energy remains stable at $T\neq 0$. In order to construct a fully microscopic version of PDM, it is desirable to calculate the structure of phonons starting from a microscopic Hamiltonian with two-body residual interaction. This is the aim of the present work, in which we will construct the phonon operator from the coherent ph pairs within the random-phase approximation (RPA) using a model Hamiltonian with a residual interaction in the form of separable two-body forces. The formalism is illustrated by numerical calculations within a schematic model. The results obtained will be discussed in comparison with the main feature of GDR as a function of T obtained within PDM previously, and observed in experiments [7].

We consider the following model Hamiltonian with a two-body residual interaction in a separable form:

$$H=\sum_s E_s a_s^\dagger a_s - k\sum_{klk'l'} f_{kl}f_{k'l'}B_{kl}^\dagger B_{k'l'}, \quad f_{kl}=f_{lk}, \tag{1}$$

where $B_{kl}^\dagger = a_k^\dagger a_l$ with (kl) running over all (ph), (pp), and (hh) indices. The use of the separable force does not reduce the generality of the results, but it will allow an analytic solution of the schematic model, which will be considered later as an illustration of the formalism. For simplicity, we do not distinguish neutron and proton systems as well as the isoscalar and isovector parts of the parameter k in Eq. (1) either. Using the standard RPA phonon operator $Q_\nu^\dagger$

$$Q_\nu^\dagger = \sum_{ph}(X_{ph}^{(\nu)}B_{ph}^\dagger - Y_{ph}^{(\nu)}B_{ph}), \tag{2}$$

we express the (ph) channel in the residual-interaction part of Eq. (1) in terms of $Q_\nu^\dagger$ and Q_ν so that Eq. (1) can be rewritten as

$$\begin{aligned} H=&\sum_s E_s a_s^\dagger a_s - k\sum_{kl\nu} f_{kl}\left[\sum_{ph} f_{ph}(X_{ph}^{(\nu)}+Y_{ph}^{(\nu)})\right](B_{kl}^\dagger + B_{kl})\\ &\times(Q_\nu^\dagger + Q_\nu)\\ =&\sum_s E_s a_s^\dagger a_s - k\sum_{\nu\nu'}\left[\sum_{ph} f_{ph}(X_{ph}^{(\nu)}+Y_{ph}^{(\nu)})\right]^2 (Q_\nu^\dagger Q_{\nu'}^\dagger\\ &+Q_\nu Q_{\nu'} + 2Q_\nu^\dagger Q_{\nu'}). \end{aligned} \tag{3}$$

The scattering terms $k\Sigma_{pp'p_1p_1'}f_{pp'}f_{p_1p_1'}B_{pp'}^\dagger B_{p_1p_1'}$ and $k\Sigma_{hh'h_1h_1'}f_{hh'}f_{h_1h_1'}B_{hh'}^\dagger B_{h_1h_1'}$ are neglected in the transition from Eqs. (1)–(3). As they contain no one-phonon terms, they have little influence on the damping of the RPA phonon excitations at $T=0$ in the lowest order, considered in the second term at the right-hand side (RHS) of Eq. (3). It has been shown in Ref. [8] that the effect caused by the scatter-

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ing terms on the RPA phonon energies is negligible except for very low-lying states in transitional nuclei. At finite temperature, an approximate estimation of the effect due to the scattering terms can be made applying the standard method of lowering the order of Green's functions within the same decoupling scheme, which was used to derive the equations of the PDM-1 [1,9]. We find that the inclusion of these terms leads only to the following renormalization of the single-particle energy E_s:

$$E'_s = E_s - \delta E_s,$$

$$\delta E_s = \frac{2\lambda+1}{2j_s+1} k \sum_{s'} [f_{ss'}]^2 (1-2n_{s'}) (ss'=pp' \text{ or } hh'), \tag{4}$$

where λ is the multipolarity of the phonon. The numerical estimation carried out for the hot GDR in ^{120}Sn has shown that the change due to the effect of such a single-particle energy renormalization on the GDR width does not exceed 6% within the temperature interval $0 \leqslant T \leqslant 6$ MeV (see Figs. 8 and 9 of Ref. [10]). Furthermore, only the term, which contains $2Q_\nu^\dagger Q_{\nu'}$ in the RHS of Eq. (3), contributes, together with the single-particle mean field [the first term at the RHS of Eq. (3)], in the procedure of linearizing the equation of motion to obtain the RPA equation. This gives $\Sigma_\nu \omega_\nu Q_\nu^\dagger Q_\nu$ as the result, where ω_ν is the phonon energy found as the solution of RPA equation. Therefore, in order to avoid double counting with retaining this term in the form as of the first line of Eq. (3), it must be subtracted from the part of Hamiltonian linearized after RPA. As the result, we can rewrite the Hamiltonian (3) in the following form:

$$H = \sum_s E_s a_s^\dagger a_s + \sum_\nu \tilde{\omega}_\nu Q_\nu^\dagger Q_\nu + \sum_{kl\nu} F_{kl}^{(\nu)} a_k^\dagger a_l (Q_\nu^\dagger + Q_\nu), \tag{5}$$

where

$$\tilde{\omega}_\nu = \omega_\nu - \delta\omega, \quad \delta\omega = -2k \left[\sum_{ph} f_{ph} (X_{ph}^{(\nu)} + Y_{ph}^{(\nu)}) \right]^2 \tag{6}$$

is the "shifted" RPA energy, and

$$F_{kl}^{(\nu)} = -2k f_{kl} \sum_{ph} f_{ph} (X_{ph}^{(\nu)} + Y_{ph}^{(\nu)}). \tag{7}$$

Similar results can be obtained using a nonseparable force such as the Skyrme interaction or that of the shell-model after replacing $kf_{kl}f_{k'l'}$ with a two-body matrix element $V_{klk'l'}$. In this case $k[\Sigma_{ph} f_{ph}(X_{ph}^{(\nu)} + Y_{ph}^{(\nu)})]^2$ in Eq. (6) obviously becomes $\Sigma_{php'h'} V_{php'h'} (X_{ph}^{(\nu)} + Y_{ph}^{(\nu)})(X_{p'h'}^{(\nu)} + Y_{p'h'}^{(\nu)})$. However, the RPA equation in this case does not have an analytic solution, but has to be diagonalized numerically. In Eq. (5) we retain the mean-field part $\Sigma_s E_s a_s^\dagger a_s$ in addition to $\Sigma_\nu \tilde{\omega}_\nu Q_\nu^\dagger Q_\nu$ as the former still and only contributes in the derivation that involves $a_k^\dagger a_l$ beyond RPA. Therefore, there is no double counting for this term either. In general, the RPA equation produces many solutions for phonon excitations. Only few of them are collective, which can generate the giant resonance such as GDR. They are formed by coherent ph components in the subspace $\{C\}$ of the total ph space, which we call collective subspace hereafter. These collective phonon excitations form a collection of harmonic oscillators distributed around the most collective excitation, which corresponds to the resonance peak within RPA. The mean deviation from the center of this distribution is called the Landau width (Landau damping) of GDR. Other solutions are noncollective ph states, in which one ph component dominates in the RHS of Eq. (2). The Hamiltonian (5) is nothing but the PDM Hamiltonian, where the first two terms at the RHS represent the noninteracting single-particle and phonon fields, respectively, while the last term is the coupling between these two fields. It is this coupling that causes the damping of phonons beyond RPA. The quantities $\tilde{\omega}_\nu$ and $F_{kl}^{(\nu)}$, which enter PDM as parameters ω_q and $F_{ss'}^{(q)}$ [1,2], respectively, are now calculated microscopically from the RPA energy ω_ν, the transition matrix elements f_{kl}, the coupling constant k, and the RPA amplitudes $X_{ph}^{(\nu)}$ and $Y_{ph}^{(\nu)}$. All the results of PDM [1,2], therefore, are applicable using Eq. (5). As the GDR damping within PDM is caused by the coupling of the RPA phonons to all ph, pp, and hh configurations beyond RPA, it is not the Landau damping discussed above. The equations of PDM-1 are used in the present work.

In realistic calculations, the formalism developed above allows us to use the single-particle energies obtained within a self-consistent method such as the Hartree-Fock one, or within a mean-field potential such as the Woods-Saxon one. The matrix elements f_{kl} can be calculated from the mean-field potential or by using different kinds of effective forces such as the Skyrme interactions. The constant k remains then the only parameter of the model. In the present work, as an illustration, we prefer to carry out the numerical analysis within a schematic microscopic model similar to what proposed in Ref. [11]. This model consists of L single-particle levels with a degeneracy equal to 2, which have an equal spacing E_0 except for a shell gap Δ above the $L_{\rm gap}$-th level. Thus, the single-particle energies measured from the Fermi energy $E_{\rm F}$ are $E_l = (l-1)E_0 - E_{\rm F}$ for $1 \leqslant l \leqslant L_{\rm gap}$, and $E_l = (l-2)E_0 + \Delta - E_{\rm F}$ for $L_{\rm gap} < l \leqslant L$. The Fermi energy $E_{\rm F}$ is defined at $T \neq 0$ from the condition $2\Sigma_k^L n_k = N$. In order to mimic the GDR, we choose the following values for the parameters: $L=200$, $L_{\rm gap}=100$, $E_0=0.15$ MeV, $\Delta=6.62$ MeV, and $k=-0.18$ MeV^{-1}. The negative value of k corresponds to the repulsive force, which takes place in the case of GDR. With this parameter's selection, the Fermi level E at $T=0$ is located in the middle of the shell gap, i.e., between 100th and 101st levels. The schematic picture of energy levels is similar to Fig. 1(a) of Ref. [11]. The values of the matrix element f_{kl} are chosen to be $f_{ph}=0.18$ MeV for all ph transitions, and $f_{pp'}=f_{hh'}=0.4961$ MeV for all pp and hh ones. We assume that the giant resonance state, which we refer to hereafter as the GDR, is generated within RPA in the collective subspace $\{C\}$, which consists of p and h levels whose distance $E_h - E_p$ is equal to $E=12$ MeV. The RPA phonon energy ω is then found analytically as

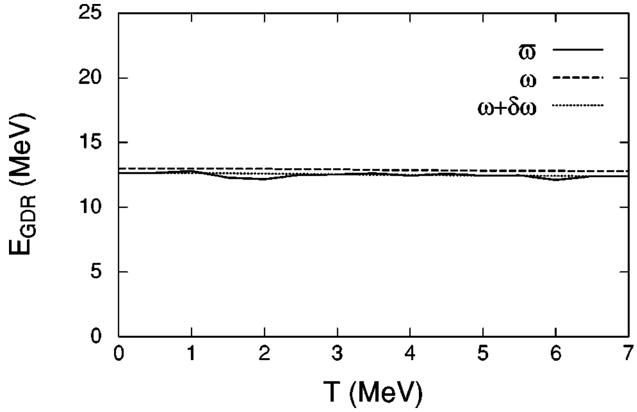

FIG. 1. The GDR energy as a function of temperature. The solid line is energy $\overline{\omega}$ obtained from Eq. (10). The dashed line shows the RPA energy ω (8), while the dotted line stands for the shifted RPA energy $\tilde{\omega}$ (6).

$$\omega = E\sqrt{1-8k\sum_{ph} f_{ph}^2 D_{ph}/E}, \quad (8)$$

where $D_{kl}=n_l-n_k$ with the single-particle number n_k given by the Fermi-Dirac distribution $n_k=\{\exp[(E_k-E_{\rm F})/T]+1\}^{-1}$. The RPA amplitudes X_{ph} and Y_{ph}, with $n_h>n_p$ are given as

$$X_{ph}=\frac{1}{\mathcal{N}}\frac{f_{ph}\sqrt{D_{ph}}}{E-\omega}, \quad Y_{ph}=\frac{1}{\mathcal{N}}\frac{f_{ph}\sqrt{D_{ph}}}{E+\omega},$$
$$\mathcal{N}=\frac{2\sqrt{E\omega\sum_{ph} f_{ph}^2 D_{ph}}}{|E^2-\omega^2|}. \quad (9)$$

Using Eq. (9), we find that the RPA energy ω is shifted down to $\tilde{\omega}$ by $\delta\omega=-2k(E/\omega)\Sigma_{ph}f_{ph}^2\sqrt{D_{ph}}/\Sigma_{ph}D_{ph}$ as given by Eq. (6). The Landau damping within this model is zero as there is only one phonon state, which generates the GDR. The resonance centroid energy $\overline{\omega}$ is found within PDM as the solution of equation for the pole of the Green function for phonon propagation [1], which takes a simple form in this case

$$\overline{\omega}-\tilde{\omega}-P(\overline{\omega})=0, \quad P(\overline{\omega})=-8k\sum_{kl}\frac{f_{kl}^2 D_{kl}(E_l-E_k)}{(E_l-E_k)^2-\overline{\omega}^2}. \quad (10)$$

The strength function $S(\omega)$ and the phonon damping $\gamma(\omega)$ are calculated according to Ref. [1] as

$$S(E_\gamma)=\frac{1}{\pi}\frac{\gamma(E_\gamma)}{(E_\gamma-\overline{\omega})^2+[\gamma(E_\gamma)]^2},$$
$$\gamma(E_\gamma)=\pi k\sum_{kl}(f_{kl})^2 D_{kl}\delta(E_\gamma-E_l+E_k). \quad (11)$$

The calculations have been carried out at $T=0$–7 MeV. The δ function in the expression for the damping $\gamma(E_\gamma)$ in Eq. (11) is smoothed using $\delta(x)=(2\pi i)^{-1}[(x-i\varepsilon)^{-1}+(x+i\varepsilon)^{-1}]$ with $\varepsilon=0.2$ MeV in the calculations. The Fermi energy $E_{\rm F}$ is found to be almost temperature independent. The RPA energy ω, the shifted RPA energy $\tilde{\omega}$, and the GDR energy $\overline{\omega}$ are shown in Fig. 1 as a function of T. The shift $\delta\omega$ is 0.37 – 0.38 MeV, which is only around 3% of the GDR energy. The RPA energy ω and the shifted RPA energy $\tilde{\omega}$ are found to be almost independent of T. The GDR energy $\overline{\omega}$ deviates from $\tilde{\omega}$ by at most ~ 0.7 MeV at $T\simeq 1.5$–2 MeV. In general, this figure shows that the GDR energy is rather stable against varying T. This also leads to a very weak increase of the matrix element (7) with increasing T so that they can be considered to be temperature independent, and equal to $F_{ph}\equiv F_1\simeq 0.67\times 10^{-1}$ MeV, and $F_{pp}=F_{hh}\equiv F_2\simeq 0.17$ MeV.

Shown in Fig. 2 are the calculated strength function $S(E_\gamma)$ at several temperatures, and the full width at the half maximum (FWHM) $\Gamma=2\gamma(E_\gamma=\overline{\omega})$ as a function of T. The width Γ is the total width caused by coupling of GDR phonon to all ph, pp, and hh configurations [solid line in Fig. 2(b)]. The quantal width $\Gamma_{\rm Q}$ due to coupling of GDR phonon to only ph configurations, and the thermal width Γ_T due to coupling to only pp and hh configurations at $T\neq 0$ are also shown in Fig. 2(b) as the dashed and dotted lines, respectively. The total width Γ increases sharply with increasing T up to $T\simeq 3$–4 MeV, and saturates at $T>4$ MeV in agreement with the trend observed in experiments [7]. This feature

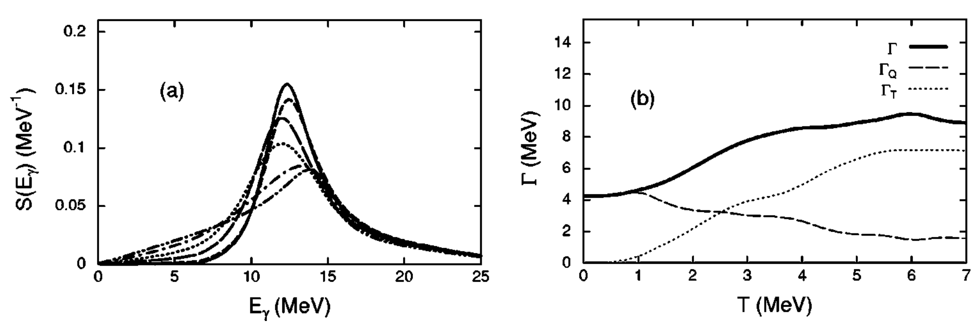

FIG. 2. Temperature dependence of the strength functions (a) and width (b) of GDR. In (a) solid, short-dashed, long-dashed, dotted, dash-dotted, and dash–double-dotted lines are results at $T=0$, 1, 1.5, 2, 3, and 5 MeV, respectively. (b) The thick solid, dashed, and dotted lines are the total width Γ, the quantal width Γ_Q, and the thermal width Γ_T, respectively, as described in the text.

is mostly dictated by the thermal width Γ_T. The quantal width Γ_Q decreases with increasing *T*. The behavior of the quantal, thermal, and total widths as a function of *T* is consistent with the results reported and discussed previously within PDM [1,2]. Figure 2(a) also shows that the low-energy part of the GDR shape ($E_\gamma \leqslant 8-10$ MeV) becomes enhanced strongly, while the high-energy tail ($E_\gamma > 17$ MeV) is depleted weakly with increasing *T*. As *T* approaches the value of the shell gap Δ, the shape of the GDR peak becomes slightly skewed toward a higher energy, but the centroid remains nearly unchanged (see also Fig. 1). We also found that the maximal absolute value for the shift δE_s of the single-particle energies in Eq. (4) is 2.124×10^{-7} MeV at $T \geqslant 5.5$ MeV (with $\lambda=1$ and the degeneracy $2j_s+1=2$). At lower *T* the shift is much smaller. Such a negligible shift practically causes no effect on the GDR width.

In conclusion, we have shown how the phonon operators, which PDM uses to generate GDR, can be constructed from RPA. This allows the PDM Hamiltonian to be derived from a microscopic Hamiltonian with a two-body residual interaction, confirming the microscopic foundation of PDM. The numerical analysis within a schematic model shows that the energy shift due to exclusion of double counting is not significant for GDR as it amounts to only few percents of the resonance energy obtained within RPA. This has been actually taken into account in Refs. [1,2] by adjusting the unperturbed energy ω_q of the structureless phonon so that the calculated GDR energy coincides with the value extracted in experiments. The numerical results within this simple schematic model also confirm the general features of GDR as a function of temperature. They include a position of the GDR, which is rather stable against *T*, and the behavior of the GDR width, which increases sharply with increasing *T* up to $T \simeq 3-4$ MeV, and saturates at higher *T*. It is also confirmed that the quantal width Γ_Q due to coupling of GDR to only *ph* configurations does not stay constant, but decreases with increasing *T*.

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