Effects of time-odd components in mean-field on large-amplitude collective dynamics

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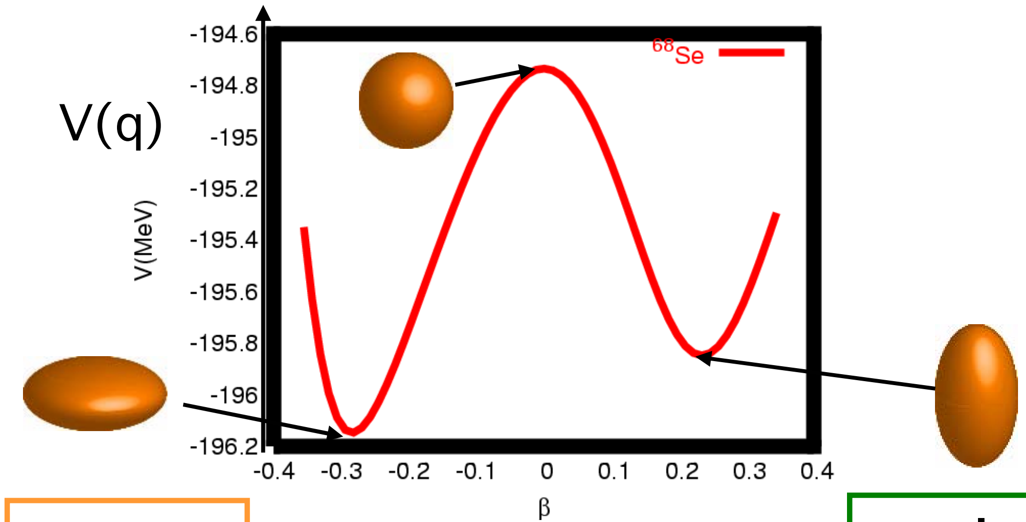
- nucl-th/0511086, Prog. Theor. Phys. 115 (2005) in press.

Shape Coexistence

- Existence of many HFB solutions (local minima)

  ⇨
  - interaction between mean fields (shape mixing)
  - quantum many-body tunneling effect

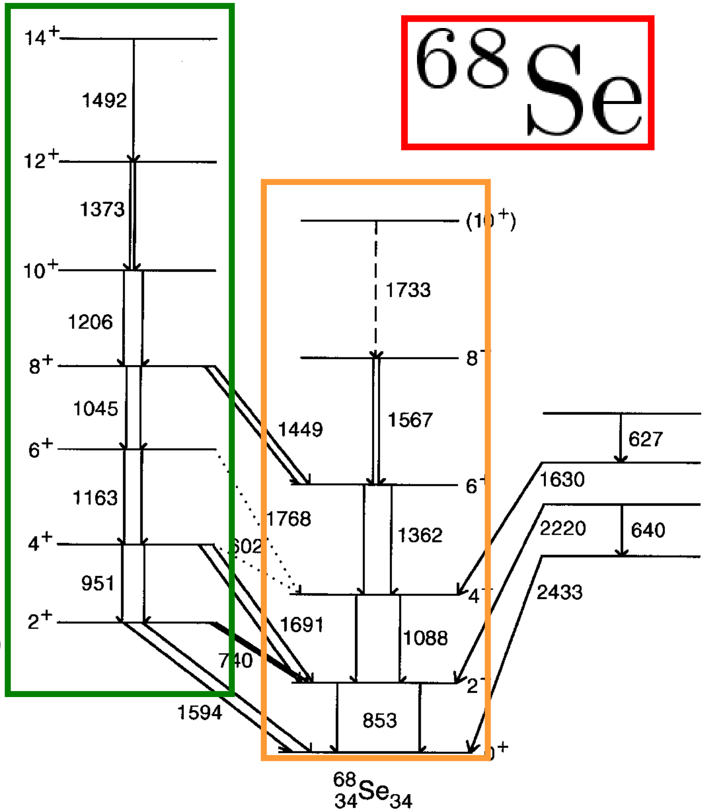
Beyond mean-field theory is required



oblate

prolate

^{68}Se



- How to determine the collective path ?
  (collective degrees of freedom)
- How to evaluate the mixing ?

Microscopic Approaches to describe Shape Coexistence Phenomena

□ Large-Scale Shell Model Calculation

- □ Exact diagonalization of many-body Hamiltonian.
- □ Matrix dimension becomes too large for medium-heavy nuclei (10^{13} dim for ^{80}Zr) → Too hard to perform !

□ Generator Coordinate Method

- □ How to choose the generator coordinate ? (The axially symmetric deformation is usually taken as GC) ← The triaxial deformation is ignored.

□ Time-Dependent Hartree-Fock

- □ The correlation beyond mean-field is taken into account by time-dependence of the mean-field.
  - □ Adiabatic TDHF theory (1976-)
  - □ Self-consistent Collective Coordinate (SCC) method (1980-)

Adiabatic SCC Method

M. Matsuo, T. Nakatsukasa, and K. Matsuyanagi, Prog. Theor. Phys. **103** (2000) 959.

- Framework based on Time-Dependent Hartree-Fock-Bogoliubov.
- Extract the collective path (parametrized by q,p)from TDHFB space
- Collective motion and non-collective motion are required to be maximally decoupled
- Decoupling collective motion with spurious(number fluctuate) motion.
- Assume collective motion to be slow, and expand the SCC basic equation up to second order of collective momentum.
- No expansion for collective coordinate

→ possible to describe large-amplitude collective motion

SCC basic equations

$$|\phi(t)\rangle = |\phi(q(t), p(t), \varphi(t), N(t))\rangle = e^{-i\varphi\hat{N}}|\phi(q,p,N)\rangle$$

- Equation of collective path

$$\delta\langle\phi(q,p,N)|\,\hat{H} - \frac{\partial\mathcal{H}}{\partial p}P - \frac{\partial\mathcal{H}}{\partial q}Q - \frac{\partial\mathcal{H}}{\partial N}\hat{N}\,|\phi(q,p,N)\rangle = 0$$

Collective mode — Number fluctuation mode

- Canonical variable conditions

Adiabatic expansion $|\phi(q,p,N)\rangle = e^{-ip\hat{Q}(q)+in\hat{\Theta}(q)}|\phi(q)\rangle$

ASCC Basic Equations

Moving-frame HFB equation (from 0-th order in p)

$$\delta \langle \phi(q)| \hat{H}_M(q) |\phi(q)\rangle = 0$$

moving-frame Hamiltonian

$$\hat{H}_M(q) = \hat{H} - \lambda(q)\hat{N} - \frac{\partial V}{\partial q}\hat{Q}(q)$$

Not included in HFB

Local harmonic equations (moving-frame QRPA equations)

$$\delta \langle \phi(q)| [\hat{H}_M(q), \hat{Q}(q)] - \frac{1}{i}B(q)\hat{P}(q) |\phi(q)\rangle = 0$$ (from 1st-order in p)

$$\delta \langle \phi(q)| [\hat{H}_M(q), \frac{1}{i}\hat{P}(q)] - C(q)\hat{Q}(q) - \frac{\partial \lambda}{\partial q}\hat{N}$$ (from 2nd-order in p)

$$- \frac{1}{2B(q)}[[\hat{H}_M(q), (\hat{H} - \lambda(q)\hat{N})_{aa,\, a^\dagger a^\dagger \text{ part}}], \hat{Q}(q)] |\phi(q)\rangle = 0$$

Terms not included in QRPA

$$C(q) = \frac{\partial^2 V}{\partial q^2} + \frac{1}{2B(q)}\frac{\partial B}{\partial q}\frac{\partial V}{\partial q}$$

$$\hat{P}(q)|\phi(q)\rangle = i\frac{\partial}{\partial q}|\phi(q)\rangle$$

Collective Hamiltonian

$$\begin{aligned} \mathcal{H}(q,p,N) &= \langle \phi(q,p,N)| \hat{H} |\phi(q,p,N)\rangle \\ &= V(q) + \frac{1}{2}B(q)p^2 + \lambda(q)n \end{aligned}$$

Canonical variable conditions

$$\begin{cases} \langle \phi(q)| [\hat{Q}(q), \hat{P}(q)] |\phi(q)\rangle = i \\ \langle \phi(q)| [\hat{\Theta}(q), \hat{N}] |\phi(q)\rangle = i \end{cases}$$

Basic scheme of the ASCC method

1st Step: Find collective path by solving ASCC basic equations.

Double iteration for each collective coordinate q

Moving-frame HFB Eq.

$$\delta\langle\phi(q)|\,\hat{H}_M(q)\,|\phi(q)\rangle = 0$$

$$\hat{H}_M(q) = \hat{H} - \lambda(q)\hat{N} - \frac{\partial V}{\partial q}\hat{Q}(q)$$

canonical variable condition

$$\langle\phi(q)|\,\hat{Q}(q-\delta q)\,|\phi(q)\rangle = \delta q$$

$|\phi(q)\rangle$ →

$\lambda(q)$

← $\hat{Q}(q)$

Local Harmonic Eq.

$$\delta\langle\phi(q)|\,[\hat{H}_M(q),\hat{Q}(q)] - \frac{1}{i}B(q)\hat{P}(q)\,|\phi(q)\rangle = 0$$

$$\delta\langle\phi(q)|\,[\hat{H}_M(q),\frac{1}{i}\hat{P}(q)] - C(q)\hat{Q}(q) - \frac{\partial\lambda}{\partial q}\hat{N}$$

$$-\frac{1}{2B(q)}[[\hat{H}_M(q),(\hat{H}-\lambda(q)\hat{N})_{aa,\,a^\dagger a^\dagger\ \mathrm{part}}],\hat{Q}(q)]\,|\phi(q)\rangle = 0$$

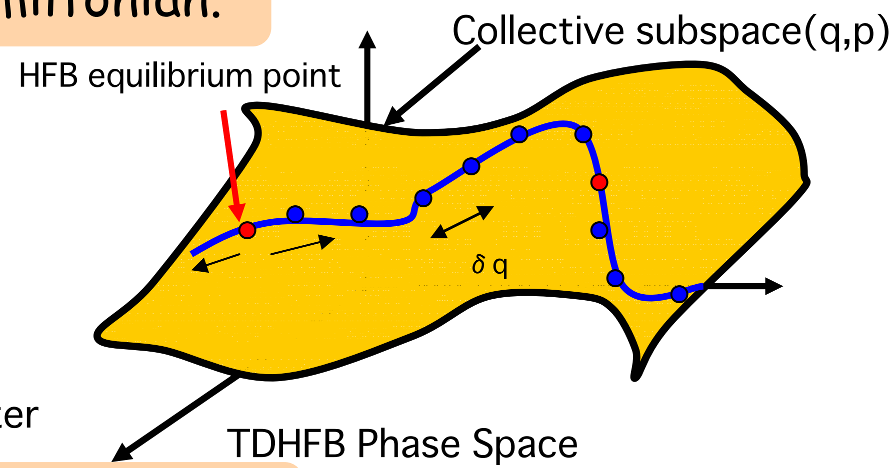
2nd Step: Calculate collective Hamiltonian.

Collective Hamiltonian

$$\mathcal{H}(q,p,N) = \langle\phi(q,p,N)|\,\hat{H}\,|\phi(q,p,N)\rangle$$

$$= V(q) + \frac{1}{2}B(q)p^2 + \lambda(q)n$$

Collective Potential — $V(q)$; Inverse Mass Parameter — $B(q)$



3rd Step: Requantize collective Hamiltonian.

Role of pairing correlations in large-amplitude collective dynamics

We focus on time-odd effects of the pairing on the collective mass.

time-odd component of mean-field

$$|\phi(q,p)\rangle = e^{ip\hat{Q}(q)}|\phi(q)\rangle$$

time-even mean-field

1st order in p -> time-odd mean-field
2nd order in p -> time-even mean-field

Collective Mass (inertia function)

hopping mass — pairing plays a central role

F. Barranco, G.F. Bertsch, G.A. Broglia, and E.Vigezzi, Nucl. Phys. **A512** (1990) 253.
G. F. Bertsch, Nucl. Phys. **A574** (1994), 169c.

Cranking mass — time-odd contribution to the inertial mass is ignored

ASCC Mass — both time-even and time-odd components are included

The time-odd effects of the pairing interactions in the large-amplitude collective dynamics is an interesting open problem.

Evaluation of Mass Parameter Using Solvable Multi-O(4) Model

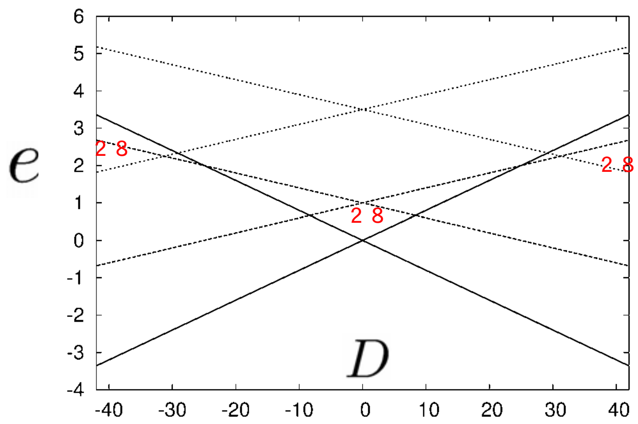
N. Hinohara, *et al.* nucl-th/0511086

Multi-O(4) model

$$\hat{H} = \hat{h}_0 - \frac{1}{2}G_0(A^\dagger A + AA^\dagger) - \frac{1}{2}G_2(B^\dagger B + BB^\dagger) - \frac{1}{2}\chi\hat{D}^2$$

- $-\frac{1}{2}G_0(A^\dagger A + AA^\dagger)$: monopole pairing interaction
- $-\frac{1}{2}G_2(B^\dagger B + BB^\dagger)$: quadrupole pairing interaction
- $-\frac{1}{2}\chi\hat{D}^2$: quadrupole interaction

single-particle energy



One-dimensional deformation

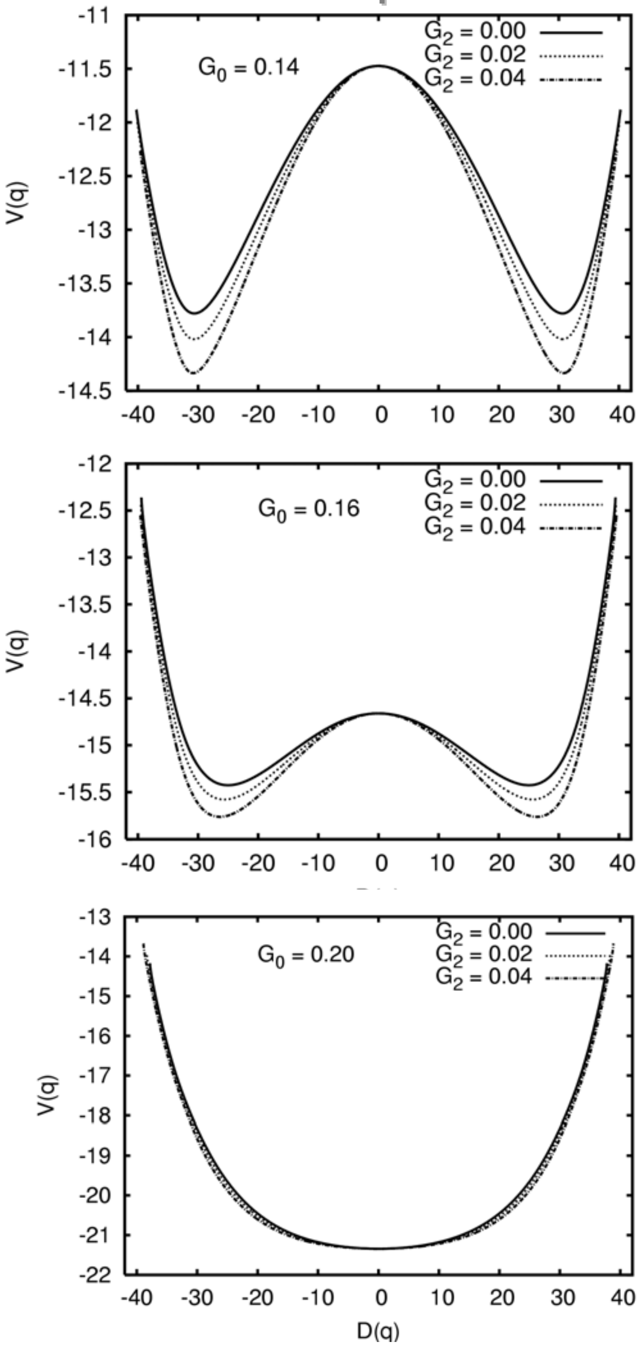
$$A^\dagger = \sum_j \sum_{m>0} c^\dagger_{jm} c^\dagger_{j-m} \qquad \hat{D} = \sum_j d_j \sum_m \sigma_{jm} c^\dagger_{jm} c_{jm}$$

$$B^\dagger = \sum_j d_j \sum_m \sigma_{jm} \hat{c}^\dagger_{jm} \hat{c}^\dagger_{j-m} \qquad \hat{h}_0 = \sum_j e^0_j \sum_m c^\dagger_{jm} c_{jm}$$

- particle number: N = 28
- Shell Model basis (SU(2)×SU(2)) : 1896
- χ = 0.04

similar model is used in
P.O. Arve and G.F. Bertsch, Phys. Lett. **B215** (1988) 1.

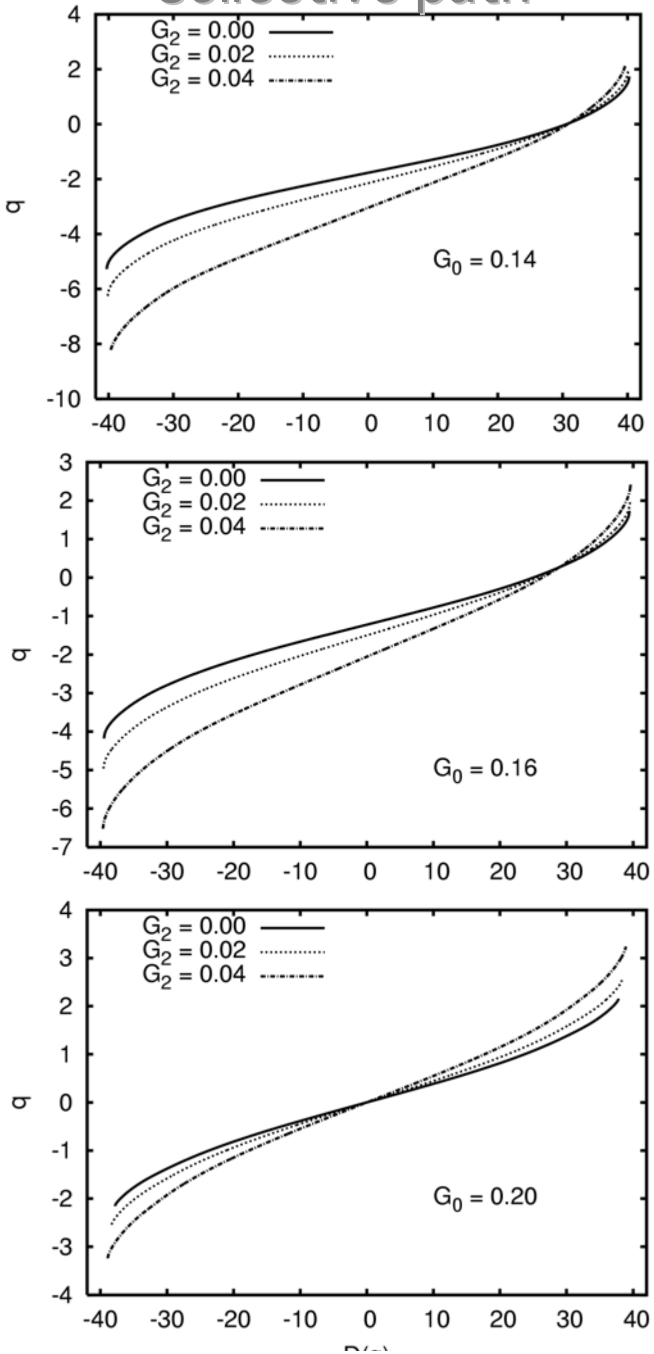
Collective potentials



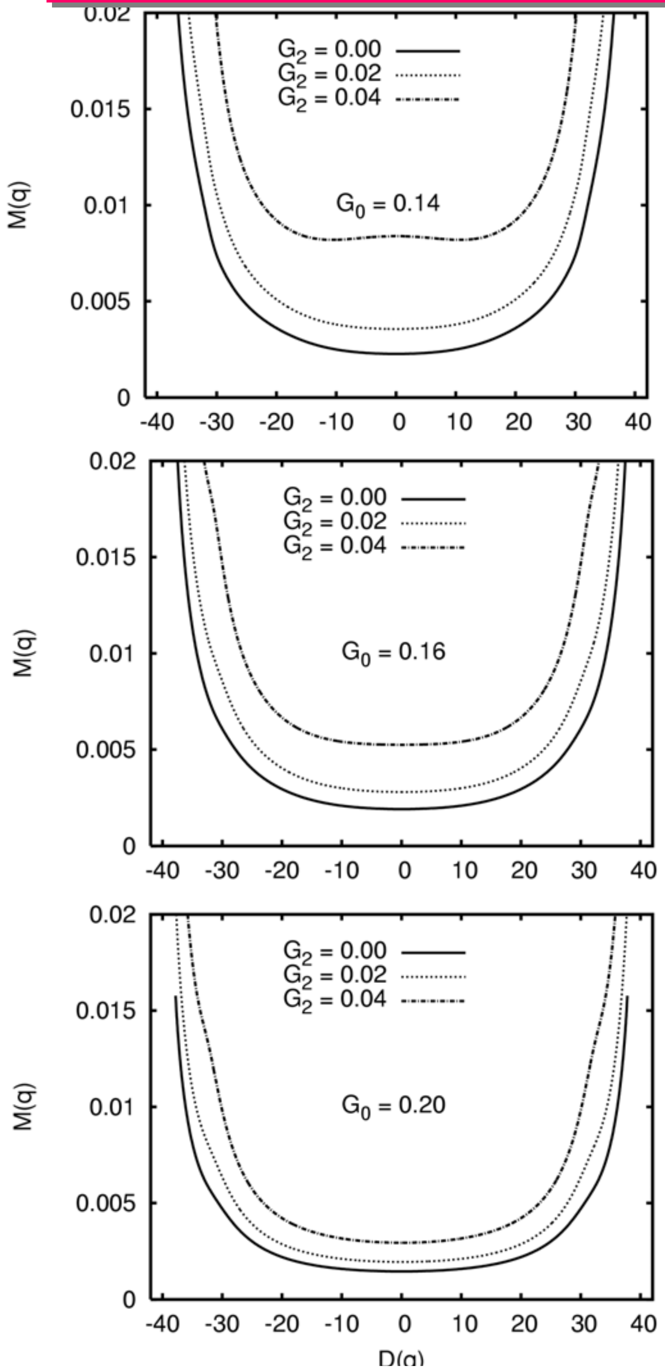
double well

anharmonic

Collective path



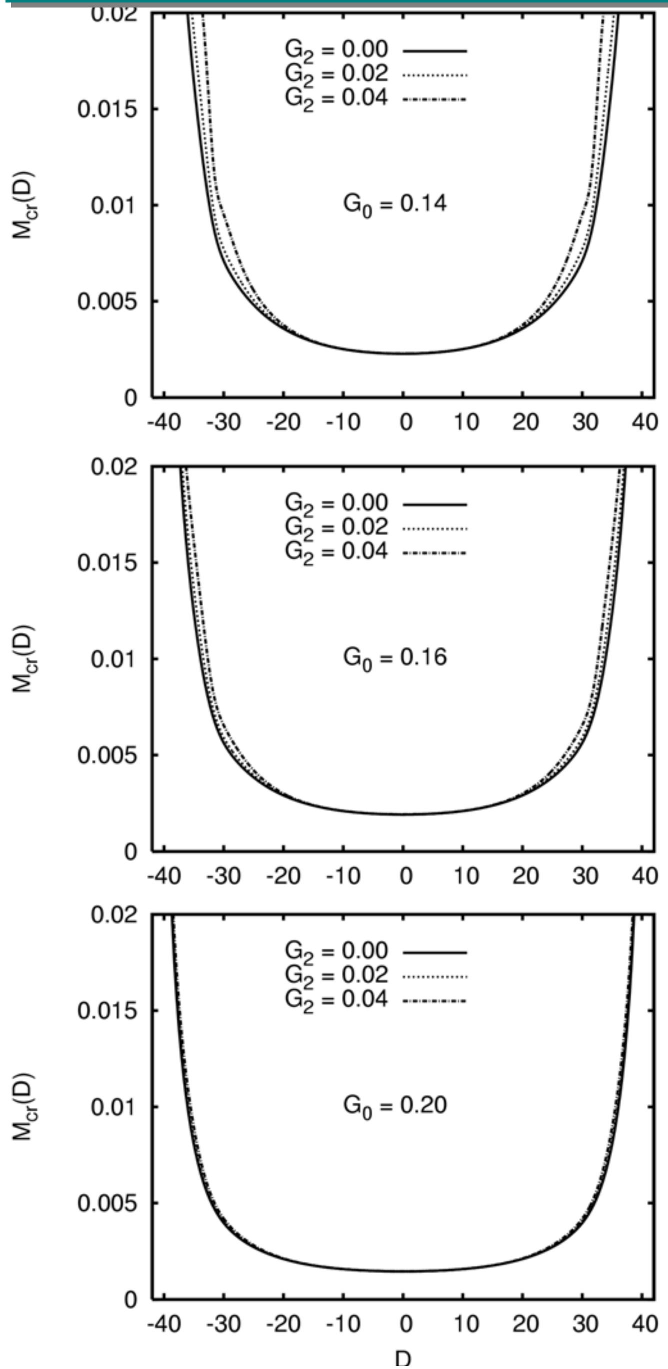
ASCC Mass



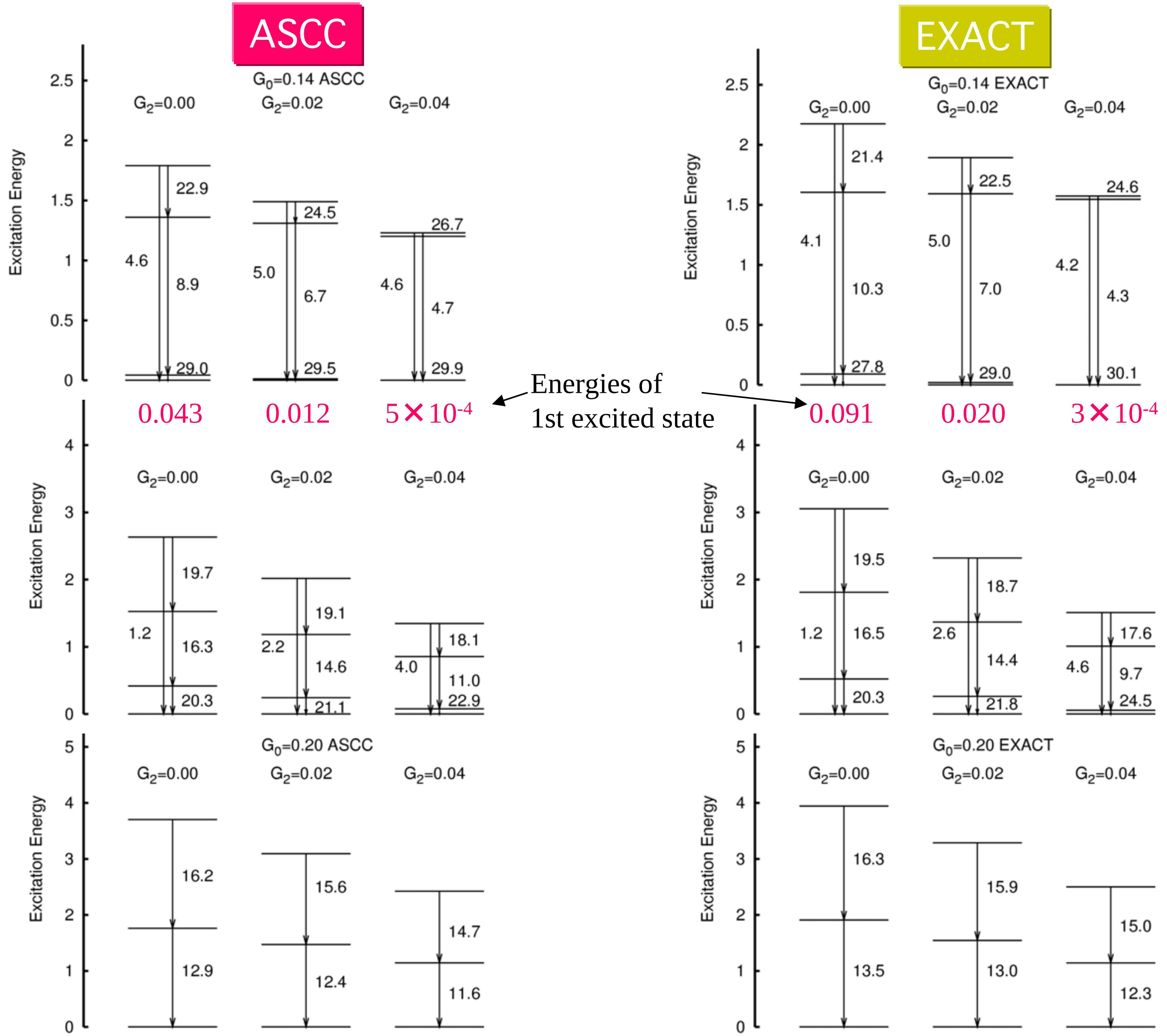
double well

anharmonic

Cranking Mass



Requantization of the collective Hamiltonian

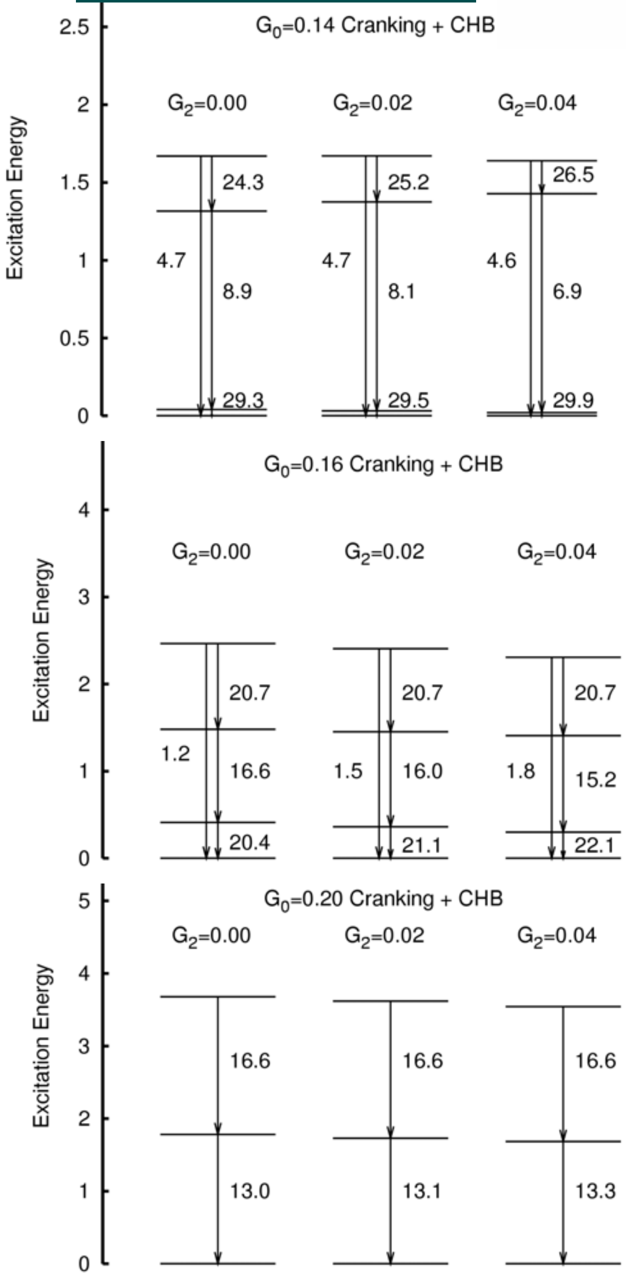


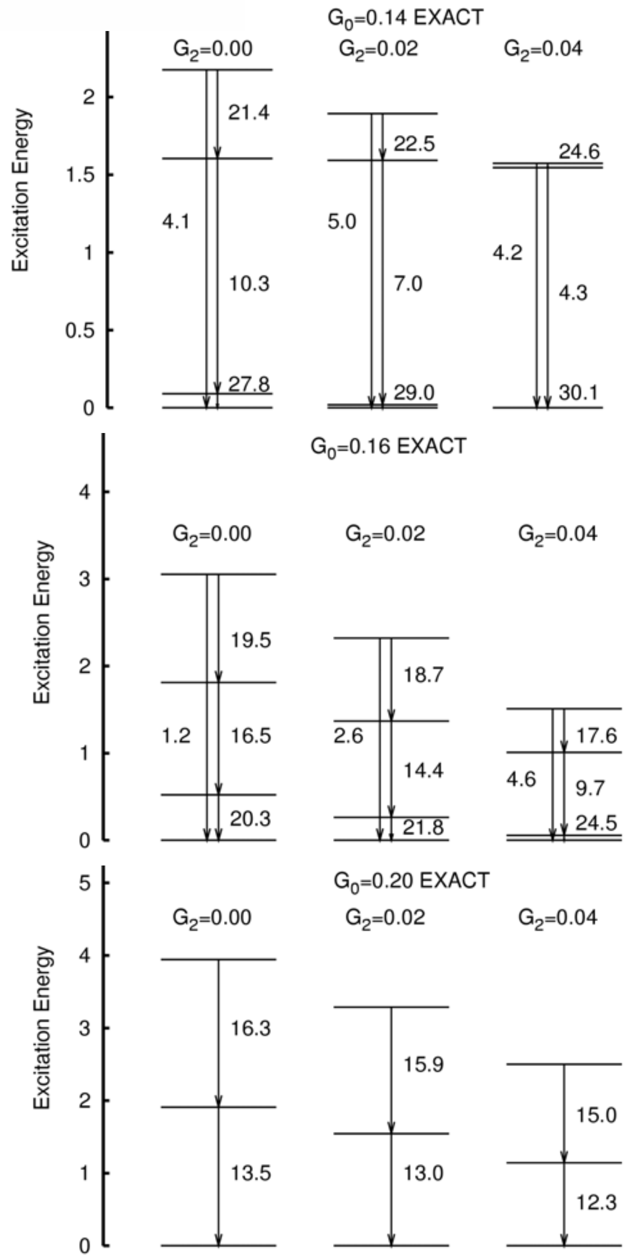
CHB + Cranking

$$\hat{H} = -\frac{1}{2M(D)^{1/4}}\frac{\partial}{\partial D}\frac{1}{\sqrt{M(D)}}\frac{\partial}{\partial D}\frac{1}{M(D)^{1/4}} + V(D)$$

$$\hat{H}\Phi_k = E_k\Phi_k$$

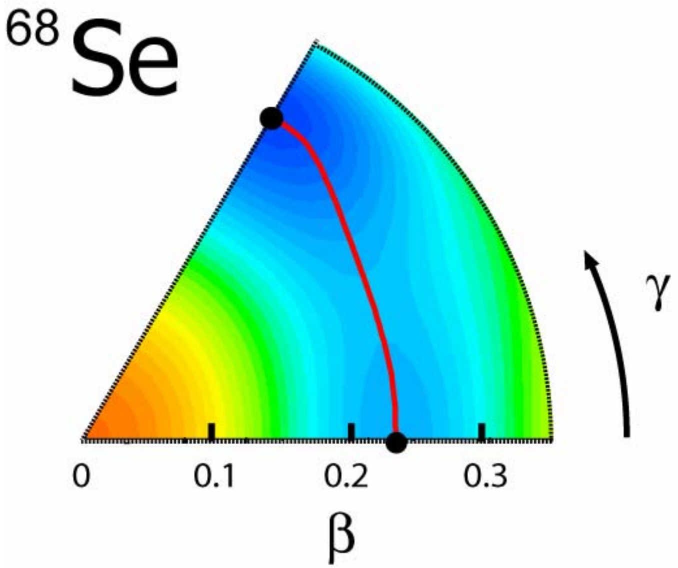
EXACT

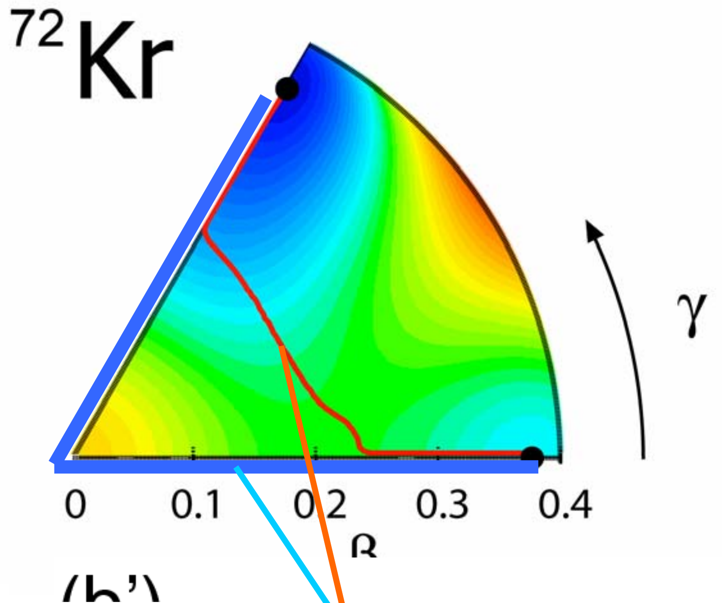


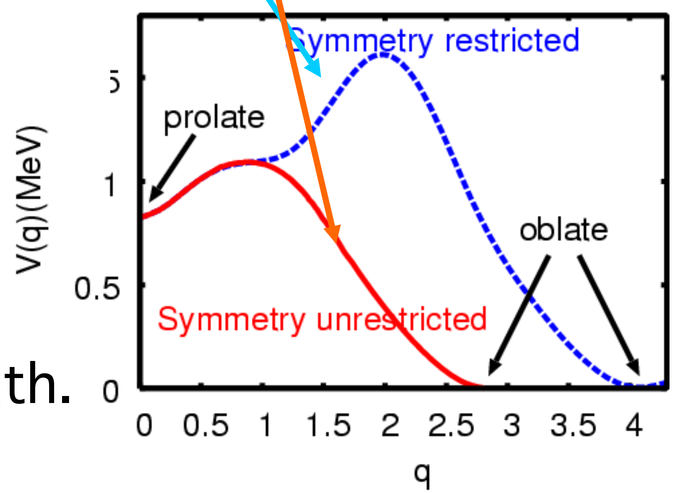


Collective paths for ^{68}Se and ^{72}Kr

Kobayasi *et al.* Prog. Theor. Phys. **112** (2004), 363.
Prog. Theor. Phys. **113** (2005), 129.







In symmetry restricted case,
Large-amplitude dynamics takes place
penetrating much higher barrier

… and collective mass also depends on path.

Conclusions

- Using the multi-O(4) model Hamiltonian including the quadrupole pairing type interaction, we have demonstrated the importance of the quadrupole pairing interaction for the mass parameters of the large-amplitude collective motion through the barrier between the "oblate" and "prolate" local minima. This is related to the time-odd component of the mean-field which is ignored in the cranking mass.

- Using the Pairing + Quadrupole Hamiltonian, we have succeeded in determining the collective path running through triaxial deformed region and connecting the oblate and prolate minima.

- Using the Pairing + Quadrupole + Quadrupole pairing Hamiltonian, we are now evaluating the mass parameters for the shape coexistence dynamics in ^{68}Se and ^{72}Kr.

- Quantizing the collective Hamiltonian, we shall compare the mixing properties of the oblate and prolate shapes for different collective paths and collective masses.