Pseudo-spin Symmetry in Density-Dependent Relativistic Hartree-Fock theory

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OUTLINE

Pseudo-spin Symmetry

- Pseudo-spin Symmetry (PSS)
  - Important features of nuclear spectrum: $\left[(n, l, j = l + \frac{1}{2}),\ (n-1, l+2, j = l+3/2)\right]$
    A. Arima et al.(1969), K. Hecht et al.(1969); A. Bohr et al.(1982), T. Beuschel et al.(1997)
  - Reflecting the relativistic symmetry in Dirac equation: Ginocchio(1997)

$$f_a(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} iG_a(r)\mathscr{Y}^{l_a}_{j_a m_a}(\hat{\mathbf{r}}) \\ -F_a(r)\mathscr{Y}^{l'_a}_{j_a m_a}(\hat{\mathbf{r}}) \end{pmatrix} \chi_{\frac{1}{2}}(\tau_a) \tag{1}$$

Pseudo-orbit $\tilde{l}_a = l'_a$, pseudo-spin $\tilde{s} = s = \frac{1}{2}$ and then the total angular momentum $j_a = \tilde{l}_a \pm \tilde{s}$.

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- Realization of the PSS: Ginocchio(1997), Meng et al.(1998), Tanabe et al.(1998), Marcos et al.(2001)
  - Cancelation between the scalar and vector potentials

$$\Sigma_S + \Sigma_0 = 0 \tag{2}$$

  - Pseudo-centrifugal barrier (PCB) vs pseudo-spin orbital potential (PSOP)

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- Relativistic mean field (RMF) theory: Walecka (1974), Serot(1986), Reinhard(1989), Ring(1996)
  - Covariant formulation: Origin of the spin-orbit potential and PSS
  - Missing Fock terms

Progresses in the RHF approach

A. Relativistic Hartree-Fock (RHF) approach: Bouyssy et al.(1985, 1987)

$$\mathscr{L} = \mathscr{L}_{\text{RHF}}(\psi, \sigma, \omega, \rho, \pi, A) \tag{4}$$

Too large incompressibility, and the nuclei were not bound enough

B. RHF approach + σ self-interaction: Barnardos et al.(1993)

$$\mathscr{L} = \mathscr{L}_{\text{RHF}} - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \tag{5}$$

Without recovering the chiral symmetry

C. RHF approach + Zero-range self-interaction: Marcos et al.(2004)

$$\mathscr{L} = \mathscr{L}_{\text{RHF}} - \frac{1}{3}\beta_\sigma \left(\bar{\psi}\psi\right)^3 - \frac{1}{4}\gamma_\sigma \left(\bar{\psi}\psi\right)^4 \tag{6}$$

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D. Density-dependent Relativistic Hartree-Fock (DDRHF) theory: arXiv: nucl-th/0512086

$$\mathscr{L} = \mathscr{L}_{\text{RHF}} \qquad g = g(\hat{\rho}) \tag{7}$$

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d. Proper isospin and energy dependent behaviors of the effective mass
e. The role of π-N coupling

Lagrangian and Hamiltonian

- Starting point: Lagrangian density

$$\mathscr{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right) - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi$$
$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu$$
$$+\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad (8)$$

where $\Omega^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$, $\vec{R}^{\mu\nu} \equiv \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$, $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.

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  where $\Omega^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$, $\vec{R}^{\mu\nu} \equiv \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$, $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.
- Variational basis: Hamiltonian $\longrightarrow$ Energy functional

$$\mathscr{H} = \bar{\psi}\left(-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M\right)\psi + \frac{1}{2}\int d^4x_2 \sum_{i=\sigma,\omega,\rho,\pi,A}\bar{\psi}(x_1)\bar{\psi}(x_2)\Gamma_i D_i\psi(x_2)\psi(x_1), \tag{9}$$

  where D_i denote the meson propagator of Yukawa type and the interacting matrix Γ_i read as

$$\Gamma_\sigma(1,2) \equiv -\, g_\sigma(1)g_\sigma(2), \qquad \Gamma_\rho(1,2) \equiv +\left(g_\rho\gamma_\mu\vec{\tau}\right)_1\cdot\left(g_\rho\gamma^\mu\vec{\tau}\right)_2, \tag{10}$$

$$\Gamma_\omega(1,2) \equiv +\left(g_\omega\gamma_\mu\right)_1\left(g_\omega\gamma_\mu\right)_2, \quad \Gamma_\pi(1,2) \equiv -\left(\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_5\gamma_\mu\partial^\mu\right)_1\cdot\left(\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_5\gamma_\nu\partial^\nu\right)_2, \tag{11}$$

$$D_i(1,2) = \frac{1}{4\pi}\frac{e^{-m_i|\mathbf{x}_1-\mathbf{x}_2|}}{|\mathbf{x}_1-\mathbf{x}_2|} \qquad \Gamma_A(1,2) \equiv +\frac{e^2}{4}\left(\gamma_\mu(1-\tau_3)\right)_1\left(\gamma^\mu(1-\tau_3)\right)_2. \tag{12}$$

Density-dependent meson-nucleon couplings

- Density-dependent meson-nucleon couplings: Brockmann (1992), Lenske (1995), Fuchs (1995)

$$g_i(\rho_b) = g_i(0)e^{-a_i\xi}, \qquad i = \rho, \pi; \qquad g_i(\rho_b) = g_i(\rho_0)f_i(\xi), \qquad i = \sigma, \omega \tag{13}$$

where $\rho_b = \sqrt{j^\mu j_\mu}$, $\xi = \rho_b/\rho_0$, and

$$f_i(\xi) = a_i \frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2} \tag{14}$$

with five constraint conditions $f_i(1) = 1$, $f_i''(0) = 0$, $f_\sigma''(1) = f_\omega''(1)$.

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- Density-dependence $\longrightarrow$ rearrangement term Σ_R^μ

$$\Sigma \longrightarrow \Sigma + \gamma_\mu \Sigma_R^\mu, \tag{15}$$

where $\Sigma_R = \Sigma_{R,(\sigma)} + \Sigma_{R,(\omega)} + \Sigma_{R,(\rho)} + \Sigma_{R,(\pi)}$.

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- Effective interactions in DDRHF (8 free parameters)
  a. PKO1: $g_\rho(0)$ and $f_\pi(0)$ are taken as the experimental values: $g_\rho(0) = 2.629$, $f_\pi(0) = 1.0$.
  b. PKO2: Without π-meson, $g_\rho(0)$ free to be adjusted
  c. PKO3: Similar as PKO1, but $g_\rho(0)$ free, and a_π adjusted by hand

Root mean square deviations from the experimental data

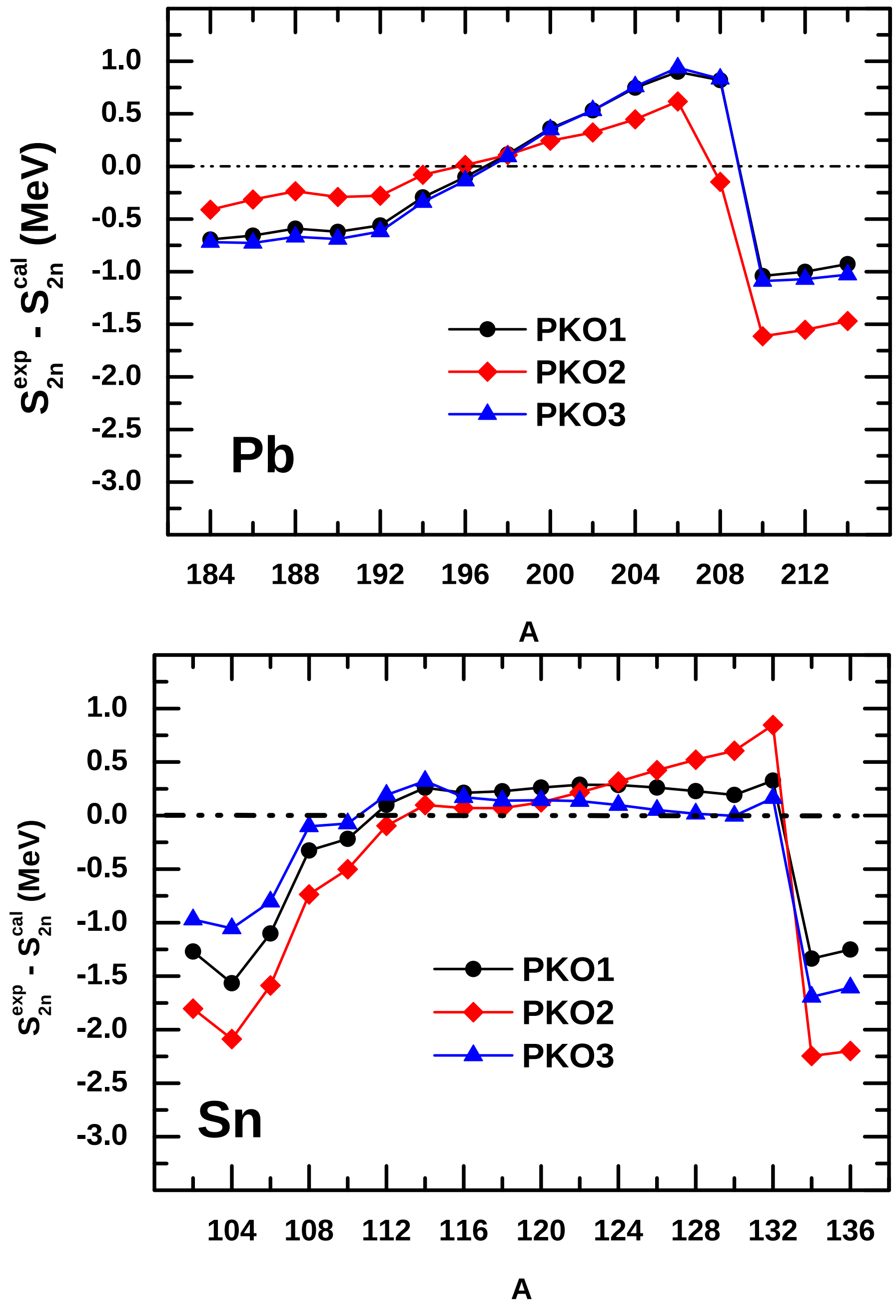
Selected Nuclei(S.N.): ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{58}Ni, ^{68}Ni, ^{90}Zr, ^{112}Sn, ^{116}Sn, ^{124}Sn, ^{132}Sn, ^{182}Pb, ^{194}Pb, ^{204}Pb, ^{208}Pb, ^{214}Pb, ^{210}Po

Table: RMS deviations Δ from the data for E_b of S.N., Pb and Sn isotopes, S_{2n}, r_c, isotope shift (I.S.) of Pb isotopes and spin-orbit (S.O.) splittings.

		PKO1	PKO2	PKO3	PK1	PKDD	NL3	DD-ME1
Δ_{E_b}	S.N.	1.6177	1.8745	2.0489	1.8825	2.3620	2.2506	2.7561
	Pb	1.8995	1.5797	1.5627	2.0336	2.7007	2.0021	2.1491
	Sn	1.2665	2.3136	1.5260	1.9552	2.4567	1.6551	0.9168
$\Delta_{S_{2n}}$	Pb	0.6831	0.7264	0.7262	0.9192	1.3139	0.9359	1.2191
	Sn	0.6813	1.0203	0.5867	0.7762	1.0629	0.8463	0.7646
Δ_{r_c}	S.N.	0.0269	0.0299	0.0225	0.0204	0.0188	0.0177	0.0163
	Pb	0.0056	0.0071	0.0061	0.0061	0.0060	0.0143	0.0150
$\Delta_{\text{I.S.}}$	Pb	0.0760	0.1122	0.0790	0.0784	0.0784	0.0679	0.0567
$\Delta_{\text{S.O.}}$		0.4143	0.5639	0.4005	0.5943	0.8097	0.6837	0.5848

PK1 and PKDD: Long(2004); NL3: Lalazissis(1997); DD-ME1: Niksic(2002)

Tow-neutron separation energies of Pb and Sn isotopes



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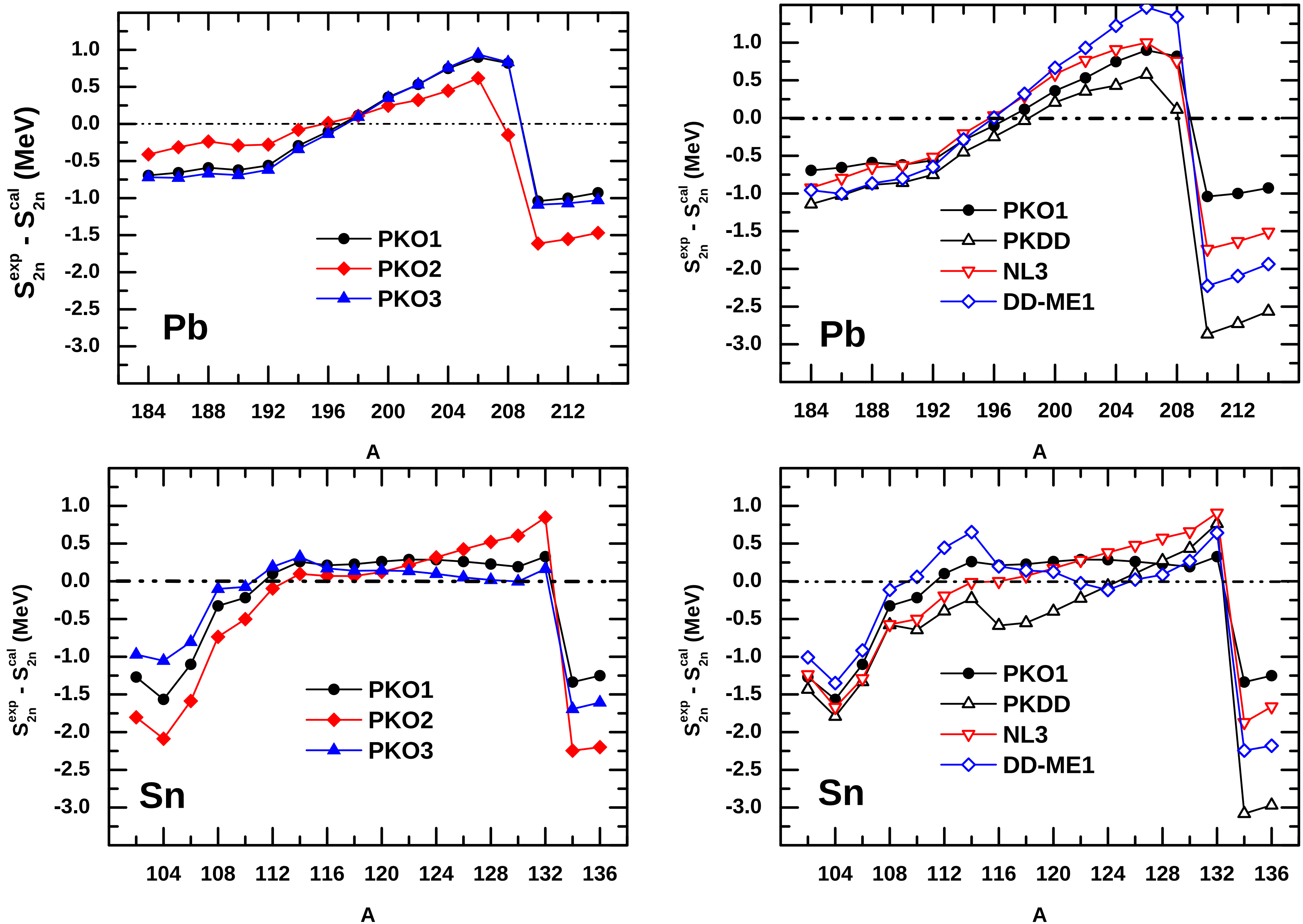

Radial Dirac Equations

- Radial Dirac Equations (Fock-related terms X_a and Y_a)

$$E_a G_a(r) = -\left[\frac{d}{dr} - \frac{\kappa_a}{r}\right] F_a(r) + \left[\Sigma_S(r) + \Sigma_0(r)\right] G_a(r) + Y_a(r), \quad (16a)$$

$$E_a F_a(r) = +\left[\frac{d}{dr} + \frac{\kappa_a}{r}\right] G_a(r) - \left[2M + \Sigma_S(r) - \Sigma_0(r)\right] F_a(r) + X_a(r) \quad (16b)$$

Example (σ-meson)

$$X_a^{(\sigma)} = -g_\sigma \sum_b \delta_{\tau_a\tau_b} \frac{\hat{j}_b^2}{4\pi} \sum_L{}' \left(C^{L0}_{j_a\frac{1}{2}j_b-\frac{1}{2}}\right)^2 \int dr' \left[g_\sigma \left(G_b G_a - F_b F_a\right)\right]_{r'} R_{LL}(m_\sigma; r, r') F_b(r)$$

$$Y_a^{(\sigma)} = +g_\sigma \sum_b \delta_{\tau_a\tau_b} \frac{\hat{j}_b^2}{4\pi} \sum_L{}' \left(C^{L0}_{j_a\frac{1}{2}j_b-\frac{1}{2}}\right)^2 \int dr' \left[g_\sigma \left(G_b G_a - F_b F_a\right)\right]_{r'} R_{LL}(m_\sigma; r, r') G_b(r)$$

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- Localization

$$X_a(r) = \frac{G_a(r)X_a(r)}{G_a^2 + F_a^2} G_a(r) + \frac{F_a(r)X_a(r)}{G_a^2 + F_a^2} F_a(r) \equiv X_{a,G_a}(r) G_a(r) + X_{a,F_a}(r) F_a(r), \tag{17a}$$

$$Y_a(r) = \frac{G_a(r)Y_a(r)}{G_a^2 + F_a^2} G_a(r) + \frac{F_a(r)Y_a(r)}{G_a^2 + F_a^2} F_a(r) \equiv Y_{a,G_a}(r) G_a(r) + Y_{a,F_a}(r) F_a(r), \tag{17b}$$

which lead to

$$\left[\frac{d}{dr} - \frac{\kappa_a}{r} - Y_{a,F_a}\right] F_a - \left[\Delta_a(r) - E_a\right] G_a = 0, \tag{18a}$$

$$\left[\frac{d}{dr} + \frac{\kappa_a}{r} + X_{a,G_a}\right] G_a + \left[V_a(r) - E_a\right] F_a = 0. \tag{18b}$$

where $\Delta_a \equiv \Delta^D + Y_{a,G_a}$, $V_a \equiv V^D + X_{a,F_a}$ and $\Delta^D \equiv \Sigma_S + \Sigma_0$, $V^D \equiv \Sigma_0 - \Sigma_S - 2M$.

The PCB and PSOP in DDRHF

- Schrödinger type equation for the lower component F, (subindex a is omitted)

$$\frac{d^2}{dr^2}F + V_1\frac{d}{dr}F + \left(V_{\rm PCB} + V_{\rm PSO} + V_2\right)F = -\left(V^D - E\right)\left(\Delta^D - E\right)F \qquad (19)$$

where the PCB $V_{\rm PCB}$, PSOP $V_{\rm PSO}$, potentials V_1 and V_2 read as

$$V_{\rm PCB} \equiv \frac{\kappa(1-\kappa)}{r^2}, \qquad (20a)$$

$$V_{\rm PSO} \equiv \frac{\kappa}{r}\left[\frac{1}{\Delta - E}\frac{d\Delta}{dr} - (X_G + Y_F)\right], \qquad (20b)$$

$$V_1 \equiv (X_G - Y_F) - \frac{1}{\Delta - E}\frac{d\Delta}{dr}, \qquad (20c)$$

$$V_2 \equiv Y_F\frac{1}{\Delta - E}\frac{d\Delta}{dr} - X_G Y_F - \frac{d}{dr}Y_F + Y_G\left(V^D - E\right) + X_F\left(\Delta - E\right). \qquad (20d)$$

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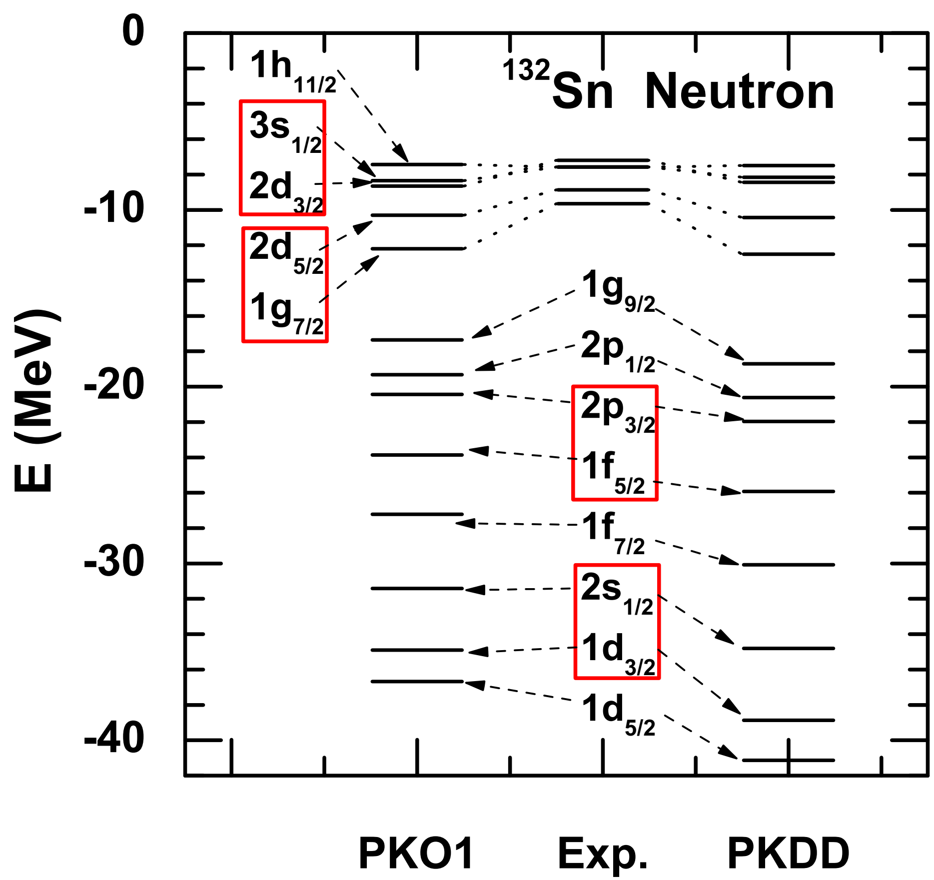
$$V_2 \equiv Y_F\frac{1}{\Delta - E}\frac{d\Delta}{dr} - X_GY_F - \frac{d}{dr}Y_F + Y_G\left(V^D - E\right) + X_F\left(\Delta - E\right). \tag{20d}$$

- Hartree and Fock terms of the PSOP and V_1

$$V_{\rm PSO}^D = \frac{\kappa}{r}\frac{1}{\Delta - E}\frac{d\Delta^D}{dr}, \qquad V_{\rm PSO}^E = \frac{\kappa}{r}\left[\frac{1}{\Delta - E}\frac{dY_G}{dr} - \left(X_G + Y_F\right)\right], \tag{21a}$$

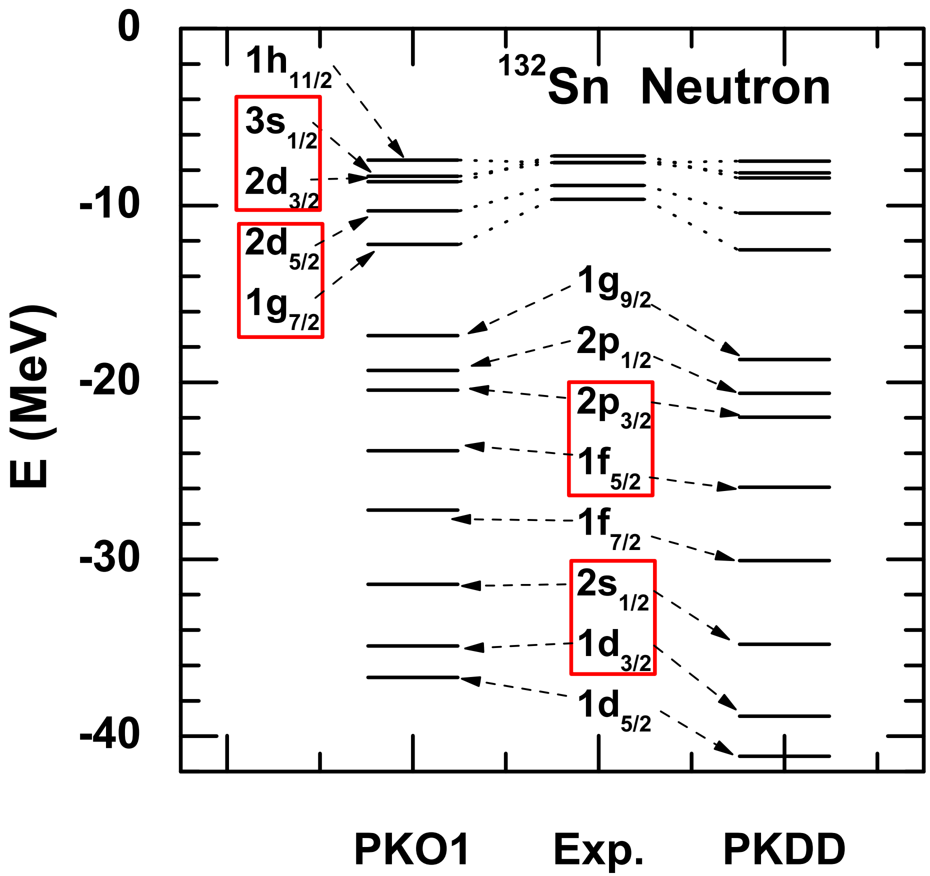
$$V_1^D = -\frac{1}{\Delta - E}\frac{d\Delta^D}{dr}, \qquad V_1^E = \left(X_G - Y_F\right) - \frac{1}{\Delta - E}\frac{dY_G}{dr}. \tag{21b}$$

Single particle energies and pseudo-spin orbital splitting



Neutron single particle energies of ^{132}Sn calculated by the DDRHF with PKO1 and the RMF with PKDD.

Single particle energies and pseudo-spin orbital splitting



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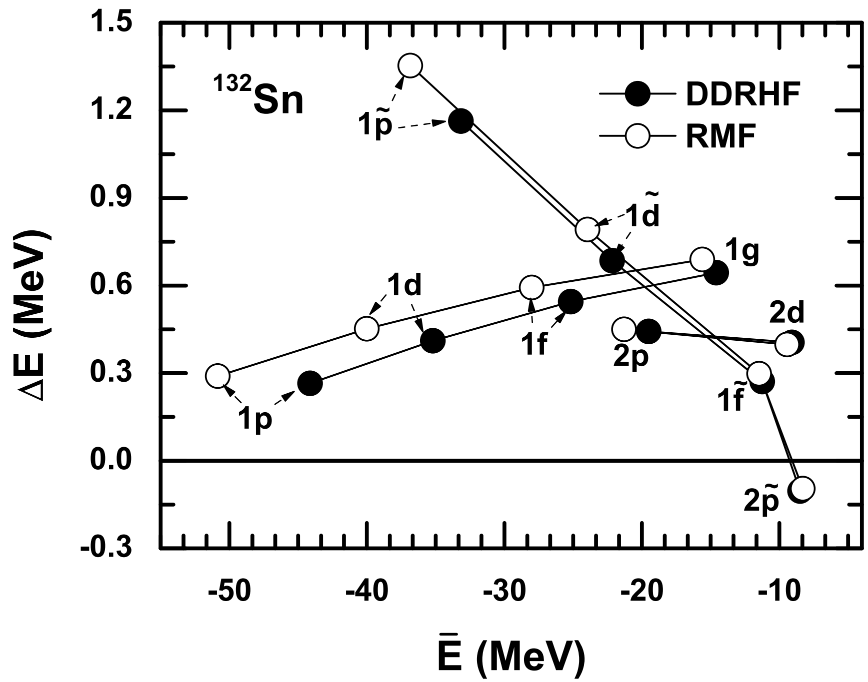
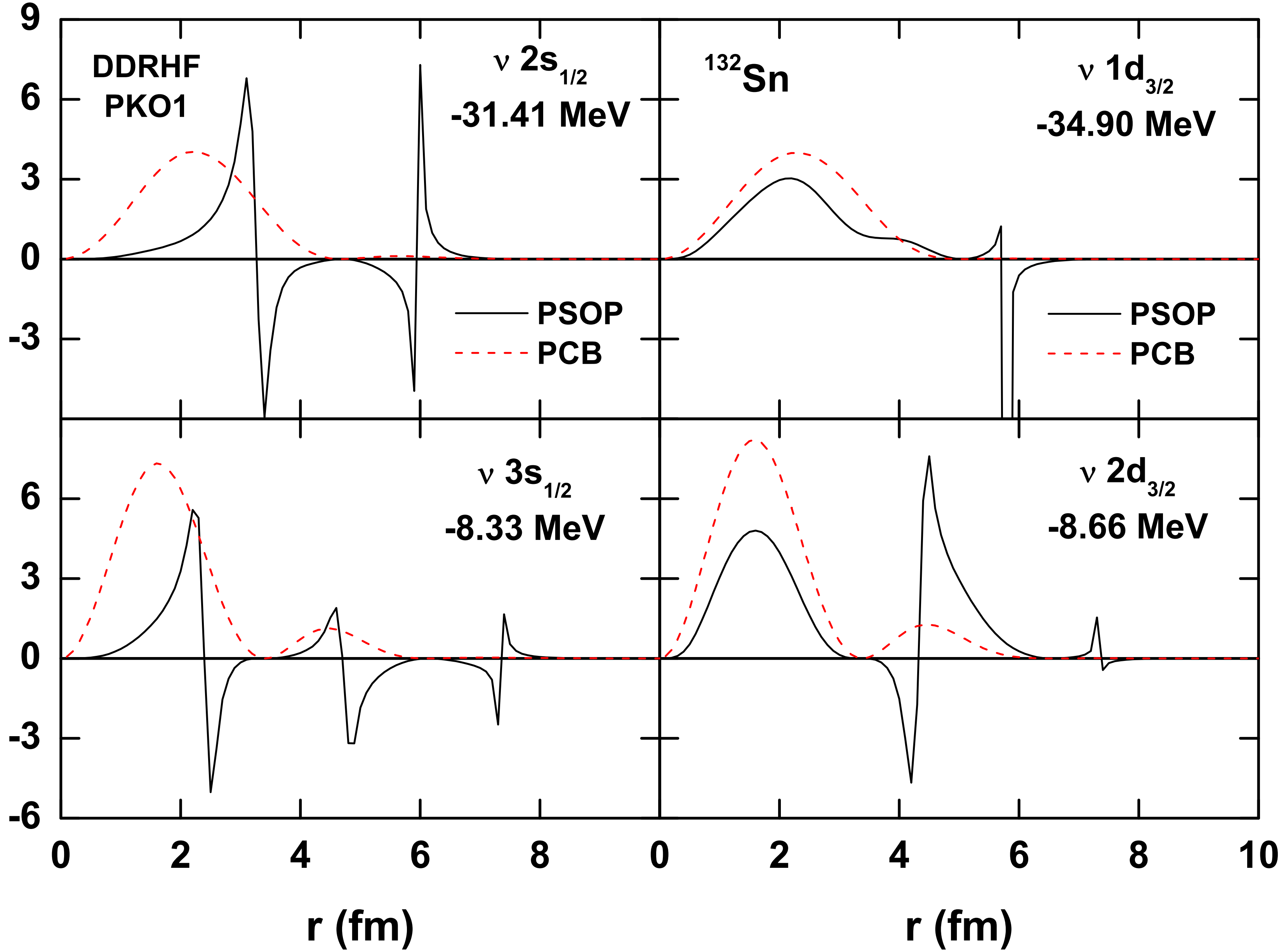


The pseudo-spin orbital splitting $\Delta E_{\rm PSO} = (E_{\tilde{l}j=\tilde{l}-1/2} - E_{\tilde{l}j=\tilde{l}+1/2})/(2\tilde{l}+1)$ versus the average binding energy $\bar{E}_{\rm PSO} = (E_{\tilde{l}j=\tilde{l}-1/2} + E_{\tilde{l}j=\tilde{l}+1/2})/2$ for the neutron states in ^{132}Sn. The spin-orbit splitting $\Delta E_{\rm SO} = (E_{lj=l-1/2} - E_{j=l+1/2})/(2l+1)$ are also given as a function of $\bar{E}_{\rm SO} = (E_{lj+l-1/2} + E_{lj=l+1/2})/2$.

Pseudo-spin orbital potential and pseudo-centrifugal barrier



The PCB and PSOP multiplied by the factor $F^2/\left(V^D - E\right)$.

Contributions from different terms to pseudo-spin orbital splitting

Equation for the lower component F

$$\frac{1}{V^D - E}\frac{d^2}{dr^2}F + \frac{1}{V^D - E}\left[V_{\rm PCB} + \hat{\mathcal{V}}^D + \hat{\mathcal{V}}^E\right] F + \Delta^D F = EF \tag{22}$$

where the operators $\hat{\mathcal{V}}^D$ and $\hat{\mathcal{V}}^E$ read as,

$$\hat{\mathcal{V}}^D = V_1^D\frac{d}{dr} + V^D_{\rm PSO}, \qquad \hat{\mathcal{V}}^E = V_1^E\frac{d}{dr} + V^E_{\rm PSO} + V_2. \tag{23}$$

Contributions from different terms to pseudo-spin orbital splitting

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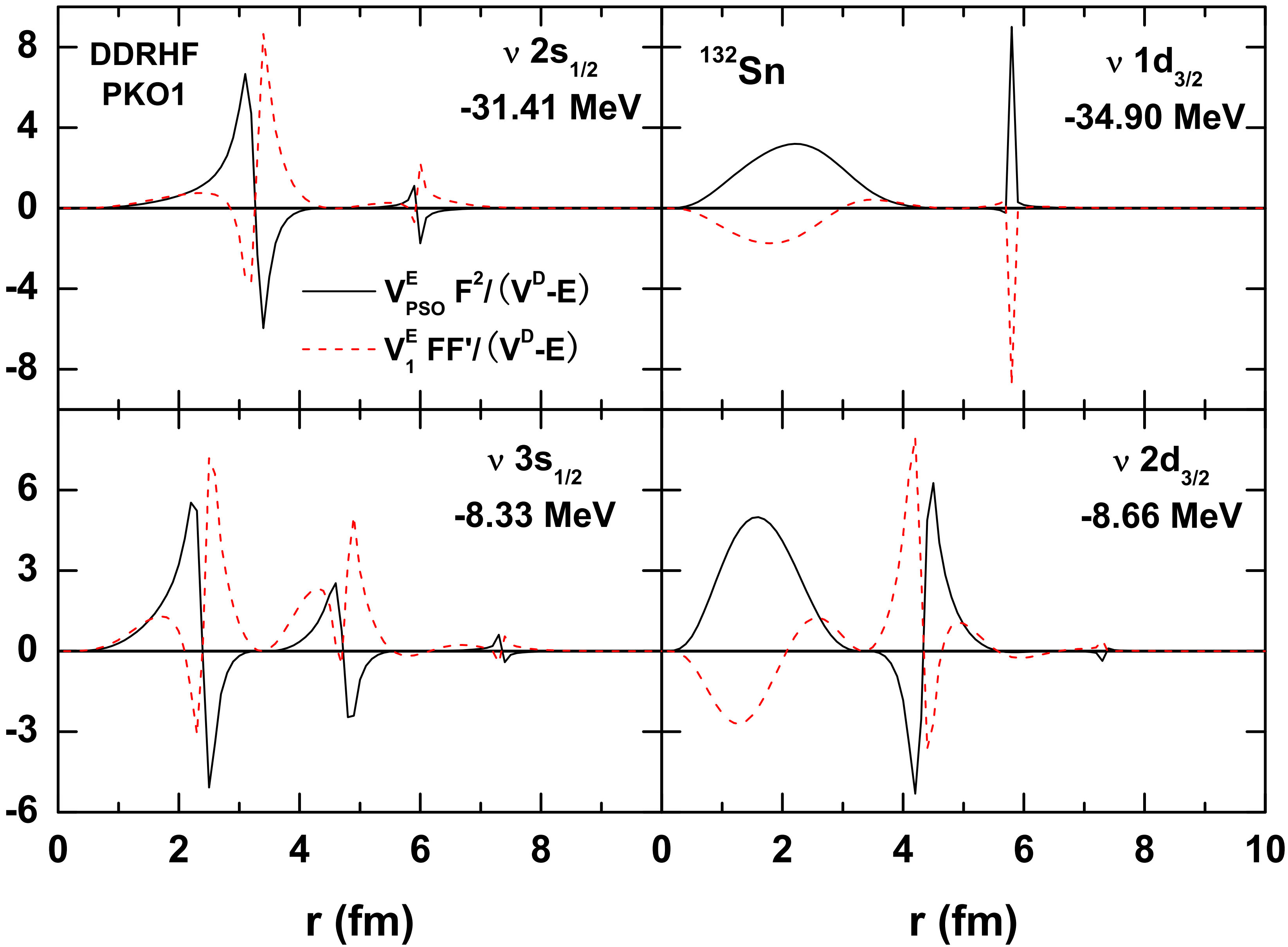
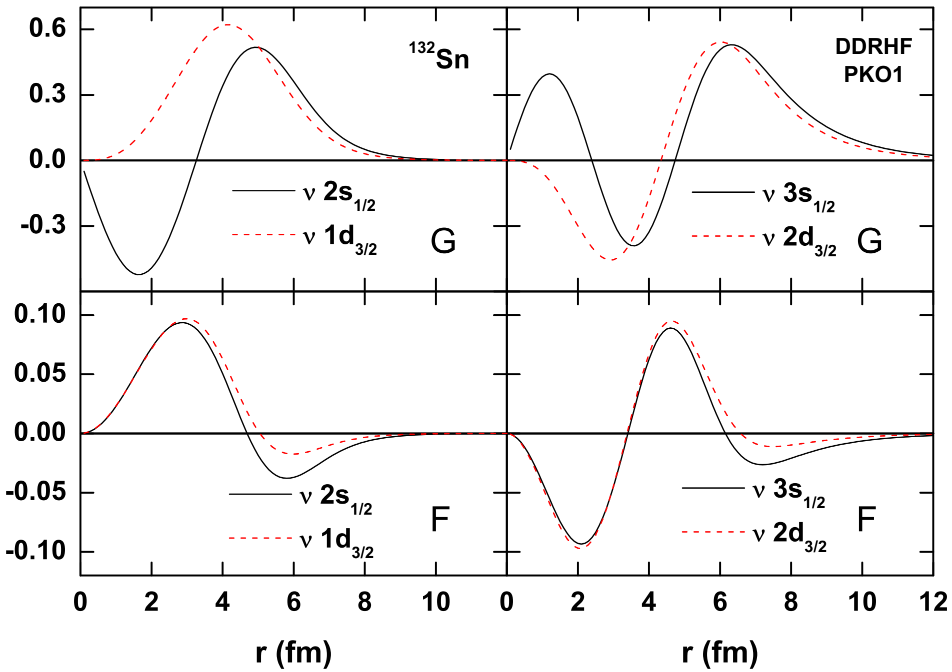
Table: The single particle energies E and the contributions from different terms in left hand of Eq. (22) given by the DDRHF with PKO1, in comparison with those by the RMF with PKDD. All units are in MeV.

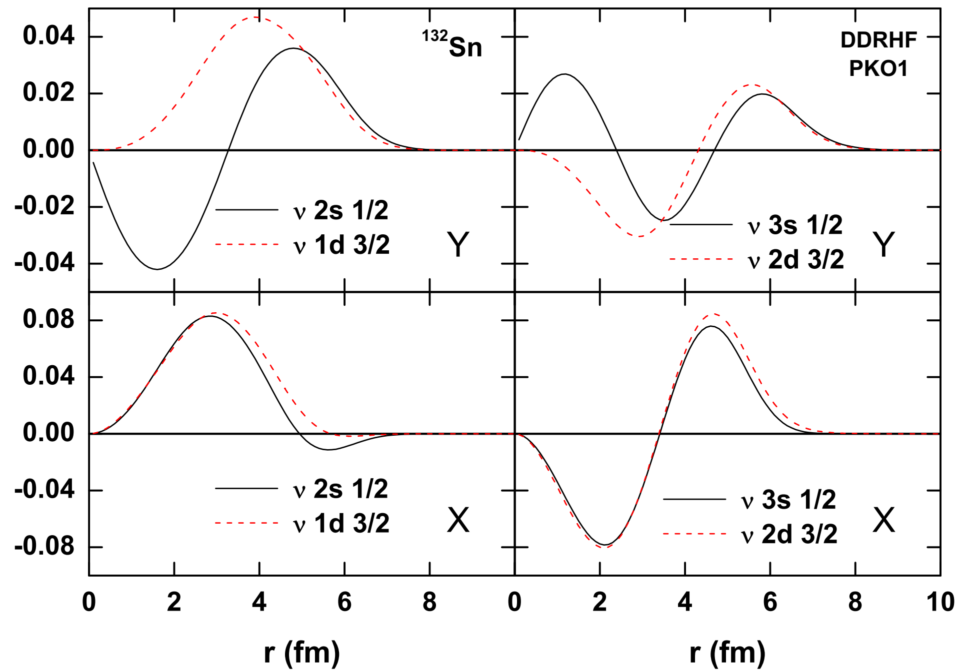
Model	Orbit	E	F''	Δ^D	$V_{\rm PCB}$	$\hat{\mathcal{V}}^D$	$\hat{\mathcal{V}}^E$
DDRHF PKO1	$\nu 2s_{1/2}$	-31.41	18.11	-75.35	9.30	-2.99	19.51
	$\nu 1d_{3/2}$	-34.90	14.87	-79.01	9.54	0.44	19.26
	$\nu 3s_{1/2}$	-8.33	34.25	-72.00	11.11	0.09	18.22
	$\nu 2d_{3/2}$	-8.66	31.93	-73.96	11.32	3.89	18.17
DDRMF PKDD	$\nu 2s_{1/2}$	-34.81	21.86	-64.65	11.04	-3.07	—
	$\nu 1d_{3/2}$	-38.87	18.17	-68.08	11.41	-0.37	—
	$\nu 3s_{1/2}$	-8.15	40.13	-61.97	13.02	0.67	—
	$\nu 2d_{3/2}$	-8.44	37.65	-63.75	13.36	4.30	—

Functions $V_{PSO}^{E}F^{2}/(V^{D}-E)$ and $V_{1}^{E}FF'/(V^{D}-E)$

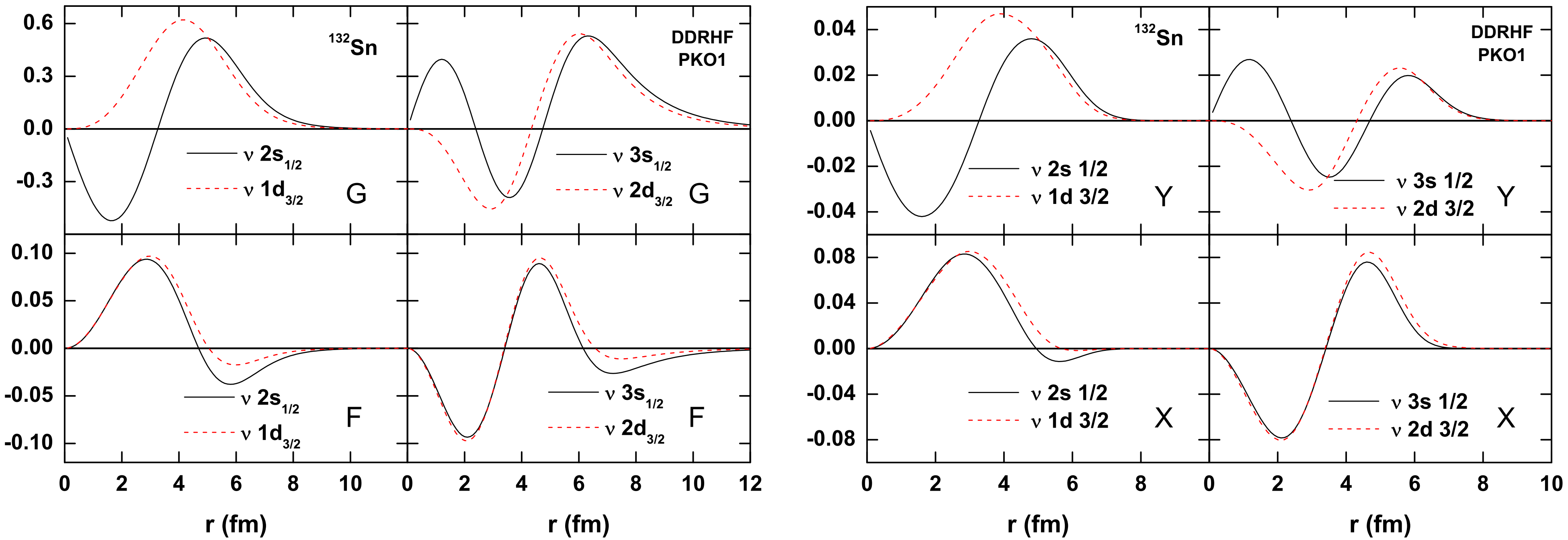


Fock-related terms and Dirac wave functions





Fock-related terms and Dirac wave functions



We might be able to validate the following relations,

$$X(r) \simeq X_0(r)F(r), \qquad Y(r) \simeq Y_0(r)G(r), \tag{24}$$

where X_0 and Y_0 are supposed to be state-independent potentials due to the Fock terms.

$$\left[\frac{d}{dr} - \frac{\kappa_a}{r}\right] F_a - \left[\Sigma_S + \Sigma_0 + Y_0 - E_a\right] G_a = 0, \tag{25a}$$

$$\left[\frac{d}{dr} + \frac{\kappa_a}{r}\right] G_a + \left[\Sigma_0 - \Sigma_S - 2M + X_0 - E_a\right] F_a = 0. \tag{25b}$$

Conclusions

- The PSS in the DDRHF theory was investigated in the doubly magic nucleus ^{132}Sn.
- The PSOP was derived by transforming the coupled radial Dirac equations into the Schrödinger type equation of the lower component properly taking account of the non-local Fock terms.
- The analyses of the single particle spectrum and the pseudo-spin orbital splitting indicate that the PSS is preserved as a good symmetry for the pseudo-spin partner $(\nu 3s_{1/2}, \nu 2d_{3/2})$ of ^{132}Sn in the DDRHF on the same level as RMF.
- Although the Fock terms bring substantial contributions to the PSOP, these contributions to the pseudo-spin orbital splitting, However, are canceled by the other terms due to the non-locality of the exchange potentials.
- The physical mechanism of these cancelations was discussed in relation to the similarity between the non-local terms and the Dirac wave functions.

Effective interaction for DDRHF theory

- Observable: Binding energies of ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{132}Sn, ^{182}Pb, ^{194}Pb, ^{208}Pb, ^{214}Pb and quantities of nuclear matter: $(K, J, \rho_{\rm sat})$
- Chi-square error:

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(y_i^{\rm cal}(\mathbf{a}) - y_i^{\rm exp}\right)^2}{\sigma_i^2} \tag{26}$$

- How to choose 8 free parameters: Contributions of Fock term and role of π-meson
  a. PKO1: $g_\rho(0)$ and $f_\pi(0)$ are taken as the experimental values: $g_\rho(0) = 2.629$, $f_\pi(0) = 1.0$.
  b. PKO2: Without π-meson, $g_\rho(0)$ free to be adjusted
  c. PKO3: Similar as PKO1, but $g_\rho(0)$ free, and a_π adjusted by hand

	m_σ	g_σ	g_ω	$g_\rho(0)$	$f_\pi(0)$	a_ρ	a_π	$\rho_{\rm sat.}$
PKO1	525.7691	8.8332	10.7299	2.6290	1.0000	0.0768	1.2320	0.1520
PKO2	534.4618	8.9206	10.5506	4.0683	–	0.6316	–	0.1510
PKO3	525.6677	8.8956	10.8027	3.8325	1.0000	0.6353	0.9341	0.1530

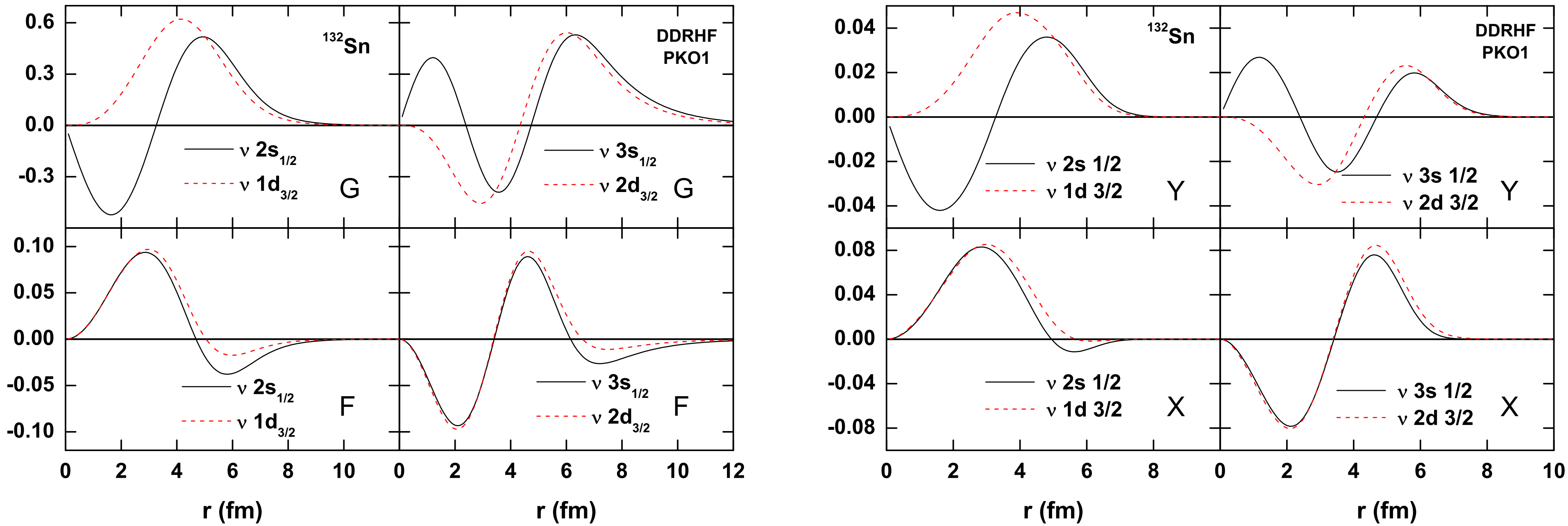
	a_σ	b_σ	c_σ	d_σ	a_ω	b_ω	c_ω	d_ω
PKO1	1.3845	1.5132	2.2966	0.3810	1.4033	2.0087	3.0467	0.3308
PKO2	1.3758	2.0644	3.0524	0.3305	1.4514	3.5744	5.4784	0.2467
PKO3	1.2446	1.5667	2.0746	0.4008	1.2457	1.6458	2.1771	0.3913

Description of symmetric nuclear matter

	K(MeV)	$\rho_{\text{sat.}}$(fm^{-3})	J(MeV)	E/A(MeV)	$M_S(p_f)/M$	$M^*_{\text{NR}}(p_f)/M$	$M^*_{\text{R}}(p_f)/M$
PKO1	250.24	0.1520	34.371	-15.996	0.5900	0.7459	0.7272
PKO2	249.60	0.1510	32.492	-16.027	0.6025	0.7636	0.7447
PKO3	262.47	0.1530	32.987	-16.041	0.5862	0.7416	0.7229
PK1	282.69	0.1482	37.641	-16.268	0.6055	0.6811	0.6642
PK1R	283.67	0.1482	37.831	-16.274	0.6052	0.6812	0.6639
TM1	281.16	0.1452	36.892	-16.263	0.6344	0.7074	0.6900
NL3	271.73	0.1483	37.416	-16.250	0.5950	0.6720	0.6547
PKDD	262.18	0.1500	36.790	-16.267	0.5712	0.6507	0.6334
TW99	240.27	0.1530	32.767	-16.247	0.5549	0.6371	0.6198
DD-ME1	244.76	0.1520	33.069	-16.202	0.5779	0.6574	0.6403
Set e	465.00	0.1484	28.000	-15.750	0.5600	—	—
HFSI	250.00	0.1400	35.000	-15.750	0.6100	—	—
ZRL1	250.00	0.1550	35.000	-16.390	0.5800	—	—

Set e: *Bouyssy(1987)*; HFSI: *Bernardos(1993)*; ZRL1: *Marcos(2004)*; PK series: *Long(2004)*;
NL3: *Lalazissis(1997)*; TM1: *Sugahara(1994)*; TW99: *Typel(1999)*; DD-ME1: *Niksic(2002)*

Dirac wave functions and exchange potentials



Dirac wave functions and exchange potentials