Pairing in Nuclear Matter

- General theory: Gorkov equations, BCS approximation
- In-medium interaction: Polarization effects
- nn pairing gaps in neutron matter
- np pairing in asymmetric nuclear matter
- Transition to Bose-Einstein (deuteron) condensation
- Application: Pairing gaps in neutron stars

Collaboration with

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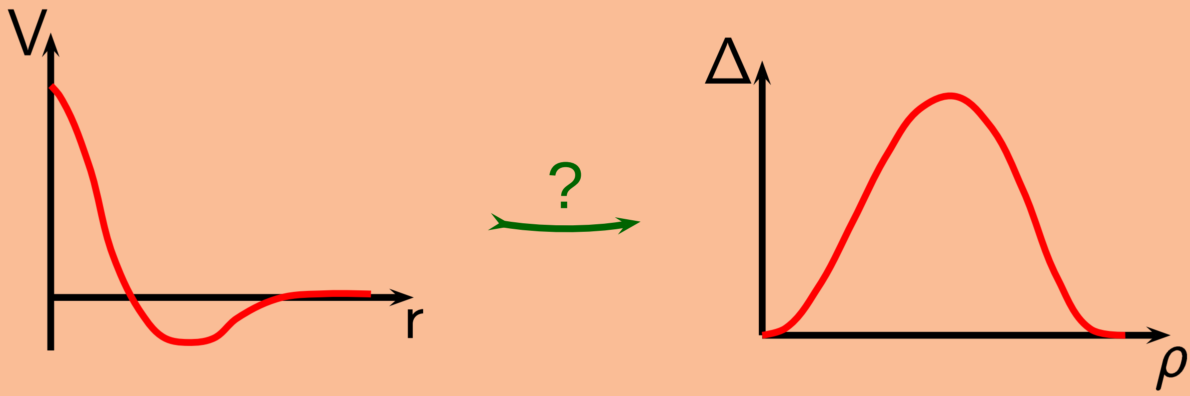
Zuo Wei & Xian-Rong Zhou & ... ; China
M. Baldo & U. Lombardo ; Catania
J. Cugnon & A. Lejeune ; Liège
P. Schuck ; Orsay

Motivation:

- Theoretical goal:

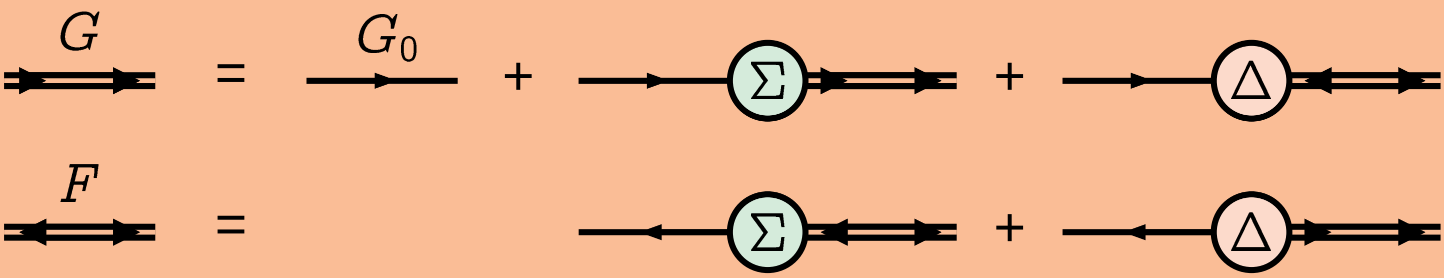
  Microscopic calculation of pairing gaps from fundamental interaction (bare potential):



- Experimental relevance:
  - Electron systems
  - T=1 nn,pp pairing:
    - Pairing force in finite (halo) nuclei
    - Superfluidity in neutron stars: glitches, cooling
  - T=0 np pairing:
    - Relevance for finite nuclei ($N \approx Z$) ?
    - Deuteron correlations, production
  - Pairing in trapped atomic gases
  - Color superconductivity

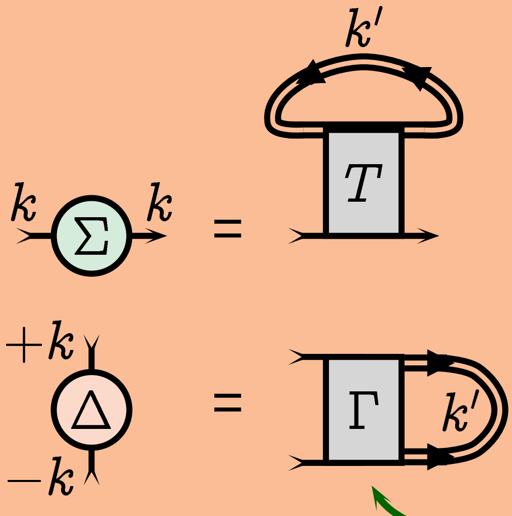
Superfluid Fermi Systems:

- General Framework: Gorkov Equations:



Generalization of Dyson equation:
Gap function Δ is analog of self-energy Σ

- Gap Equation (4-dim):



$$\Sigma(k) = i \int \frac{d^4k'}{(2\pi)^4} \langle k, k'|T|k, k'\rangle G(k')$$

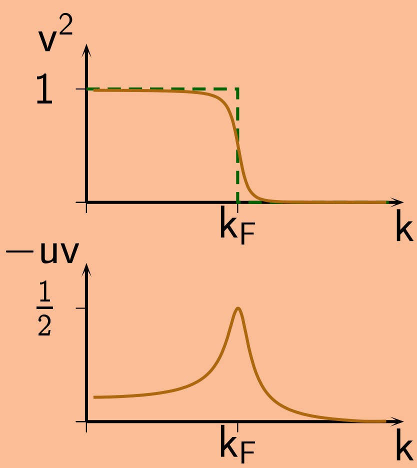
$$\Delta(k) = i \int \frac{d^4k'}{(2\pi)^4} \langle k, -k|\Gamma|k', -k'\rangle F(k')$$

Irreducible interaction kernel

- Simplest (BCS) approximation: $\Gamma = V$ (bare potential):

$$\Sigma(\boldsymbol{k}) = \sum_{k'} v_{k'}^2 \, \langle \boldsymbol{k}, \boldsymbol{k}'|V|\boldsymbol{k}, \boldsymbol{k}'\rangle_a$$

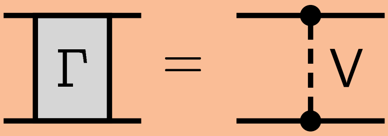
$$\Delta(\boldsymbol{k}) = \sum_{k'} (uv)_{k'} \underbrace{\langle +\boldsymbol{k}', -\boldsymbol{k}'|V| +\boldsymbol{k}, -\boldsymbol{k}\rangle_a}_{\langle \boldsymbol{k}'|V|\boldsymbol{k}\rangle}$$

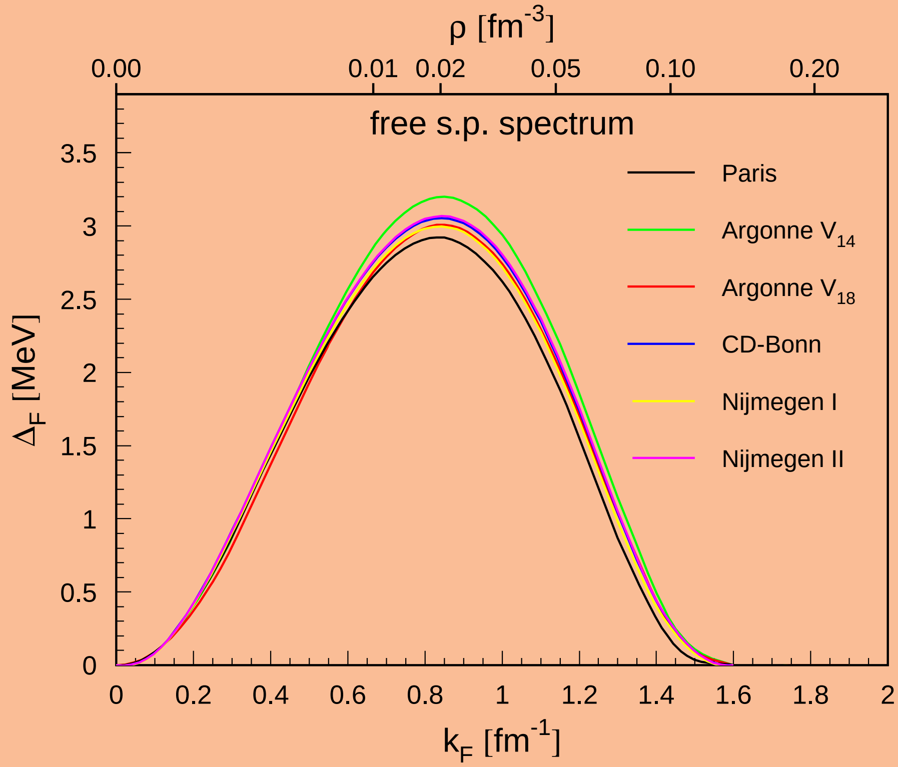


Mean field approximation !

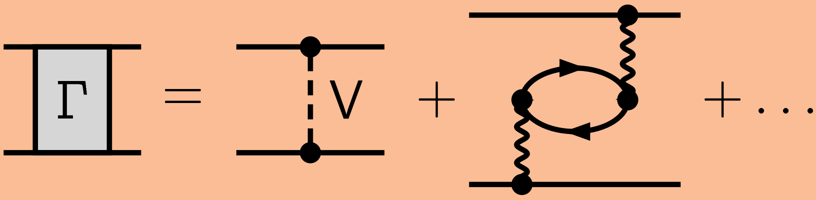
$^1S_0$ nn Gap with and without Polarization Effects:

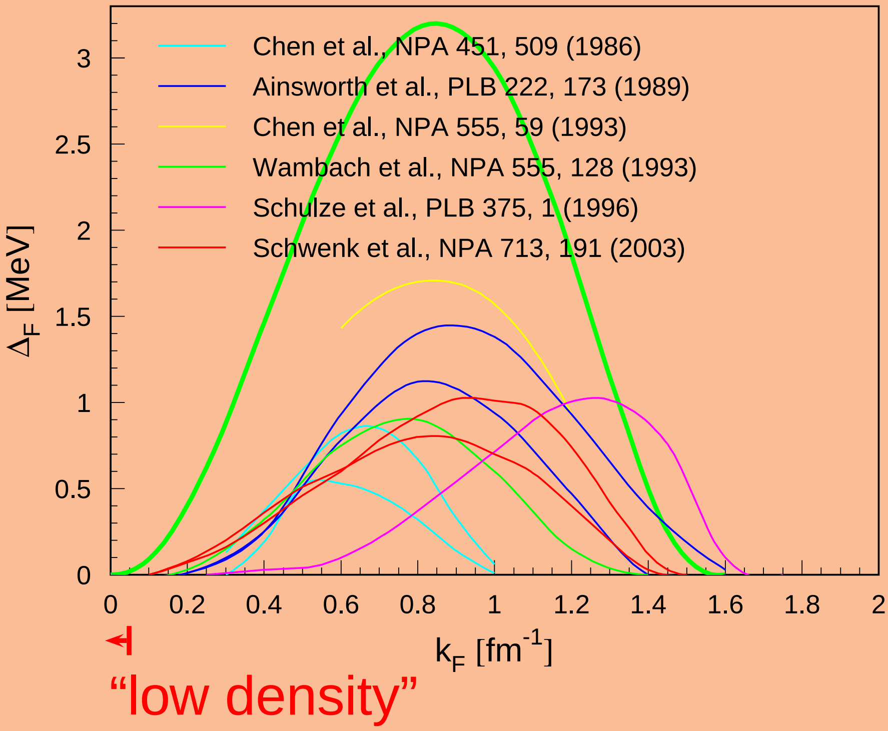
- Free potential:



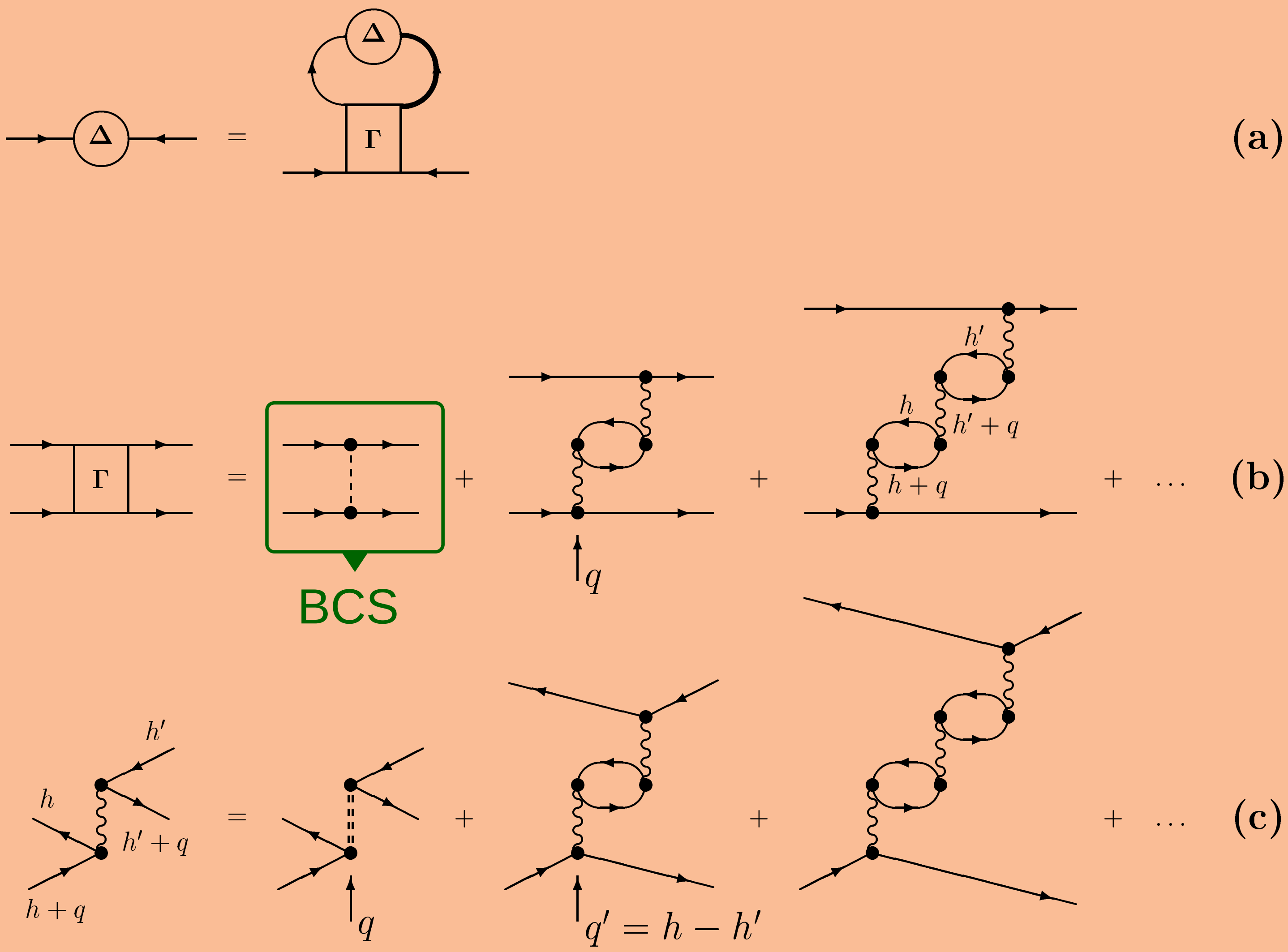


- Including polarization:





Beyond First Order: Babu-Brown Approach:



H.-J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, U. Lombardo; PLB 375, 1 (1996)

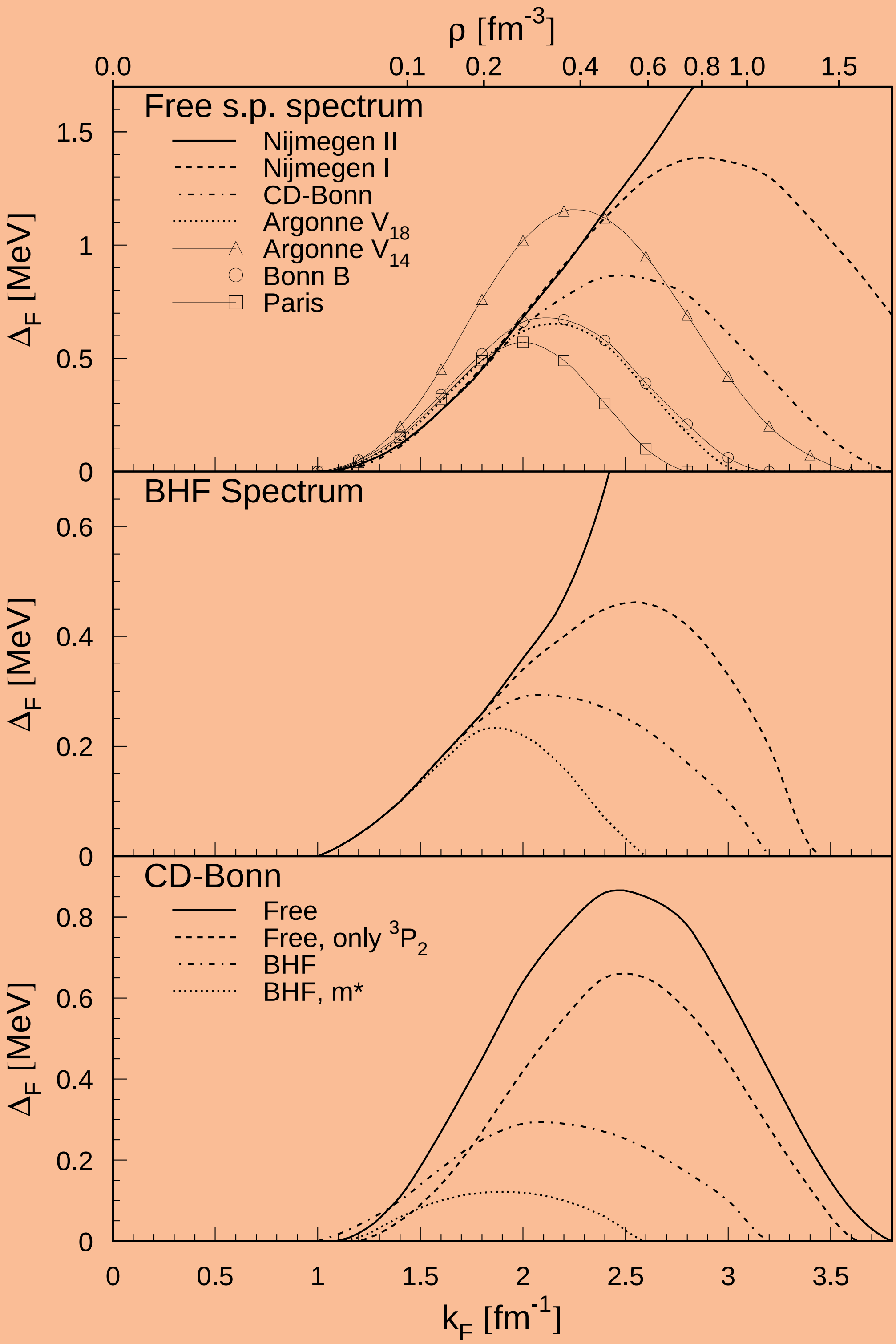
Too difficult to solve exactly:
Strong approximations necessary

↪ Large uncertainty of results

$^3PF_2$ nn Gap in Neutron Matter:

- BCS results with bare nn potentials:

M. Baldo, Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, H.-J. Schulze; PRC 58, 1921 (1998)

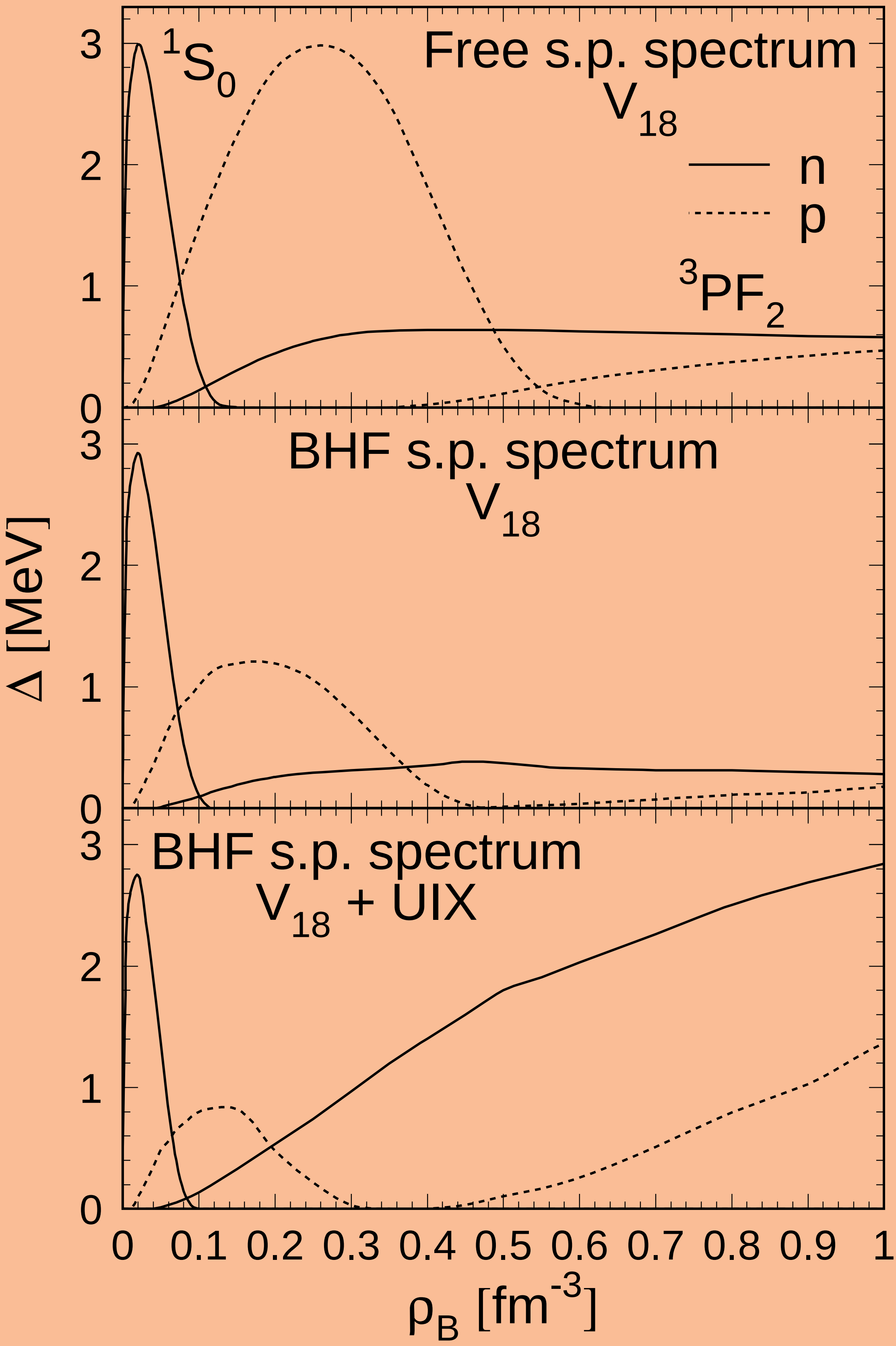


- Not constrained by phase shifts above $k_F \approx 2\ \mathrm{fm}^{-1}$
- Self-energy effects are large
- $P-F$ coupling is important
- Polarization effects are unknown
- TBF are important

(Schwenk & Friman, PRL 92: $\Delta_{^3P_2} < 10^{-2}$ MeV)

Gaps in Neutron Star Matter:

EOS: BHF (V18 + UIX + NSC89)



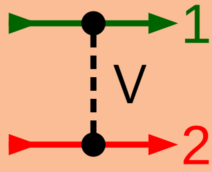
➥ Self-energy effects suppress gaps

TBF suppress pp $^1S_0$ but strongly enhance $^3PF_2$ gaps !

X.-R. Zhou, H.-J. Schulze, E.-G. Zhao, Feng Pan, J.P. Draayer; PRC 70, 048802 (2004)

Pairing in Asymmetric Matter:

- Principal equations:



$$\Delta_{k'} = -\sum_k V_{kk'} \frac{\Delta_k}{2E_k} \left[1 - f(E_k^+) - f(E_k^-)\right]$$

$$\rho_1 + \rho_2 = \sum_k \left[1 - \frac{\epsilon_k}{E_k}\left[1 - f(E_k^+) - f(E_k^-)\right]\right]$$

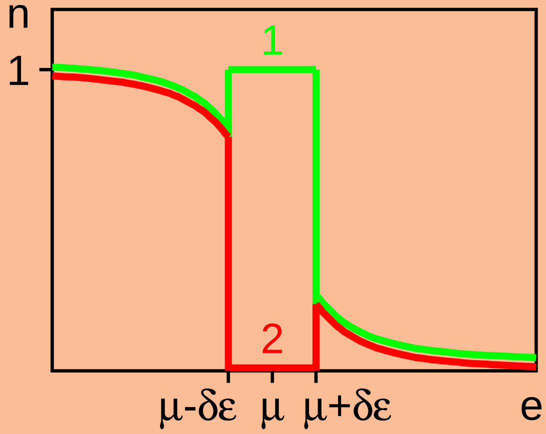
$$\rho_1 - \rho_2 = \sum_k \left[f(E_k^-) - f(E_k^+)\right]$$

$$\mu = (\mu_1 + \mu_2)/2$$
$$\delta\mu = (\mu_1 - \mu_2)/2$$
$$E_k^\pm = E_k \pm \delta\mu$$

- At zero temperature: $f(E_k^+) = 0\ ,\ f(E_k^-) = \theta(\delta\mu - E_k)$:

Unpaired particles concentrated in region around μ, Pauli-blocking the gap equation

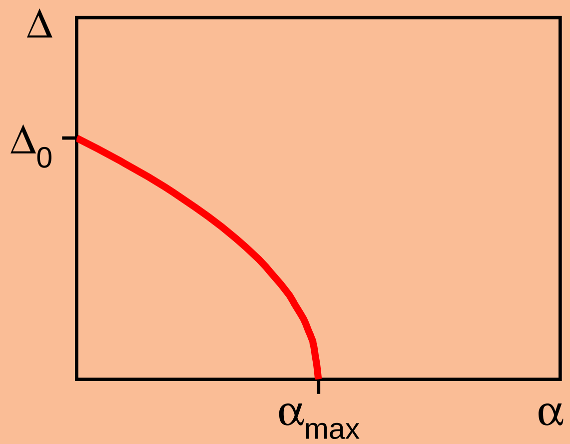
→ Strong suppression of the gap with asymmetry



- Solution in weak-coupling approximation $\Delta \ll \mu$:

$$\frac{\Delta_\alpha}{\Delta_0} = \sqrt{1 - \frac{\alpha}{\alpha_{\text{max}}}}\ ,\quad \alpha = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\alpha_{\text{max}} = \frac{3\Delta_0}{4\mu} = \frac{6}{e^2} \exp\left[\frac{\pi}{2k_F a}\right]$$



- Very small maximal asymmetry allowing pairing !

Transition to Bose-Einstein Condensation:

- In case of strong attraction with a bound state:

$$\mu \quad \xrightarrow{\rho \to 0} \quad \mu_B = -E_B/2 < 0$$

- Two equations for Δ_k and μ:

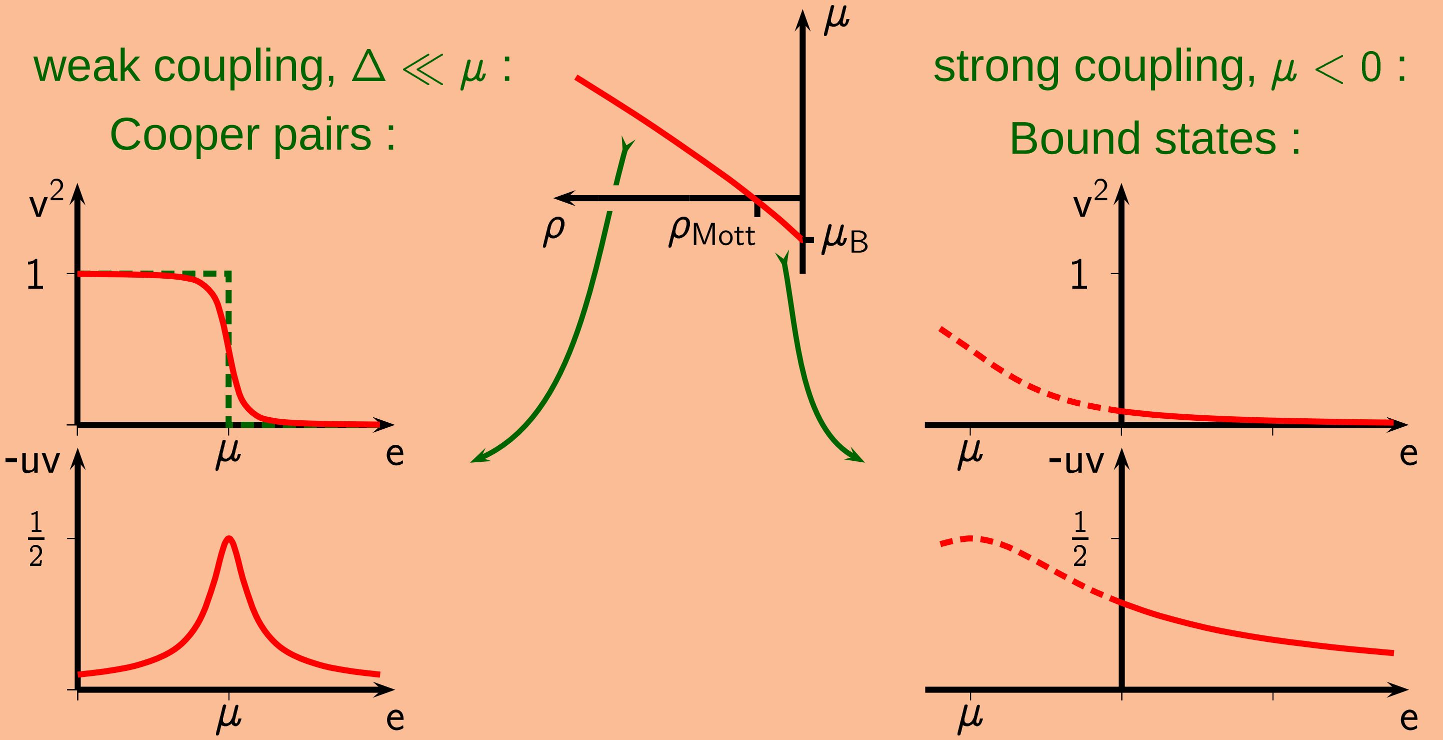
$$\Delta_{k'} = \sum_k (uv)_k V(k,k') \qquad (uv)_k = \frac{-\Delta_k}{2E_k} \quad , \quad E_k^2 = \epsilon_k^2 + \Delta_k^2$$

$$\rho = 2\sum_k v_k^2 \qquad v_k^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k}{E_k}\right) , \quad \epsilon_k = \frac{k^2}{2m} - \mu$$

Combination:

$$\frac{k^2}{m}\psi(k) + (1 - v_k^2)\sum_{k'} V(k,k')\psi(k') = \overbrace{2\mu}^{\text{eigenvalue : binding energy}} \psi(k) \; , \; \psi(k) \sim (uv)_k$$

Schrödinger equation with Pauli-blocking $\longrightarrow \mu(\rho)$:



- Interpretation: Formation and Bose-Einstein condensation of bound states below the Mott density

Example: n-p pairing in the ${}^3SD_1$ channel $\longrightarrow$ deuteron . . .

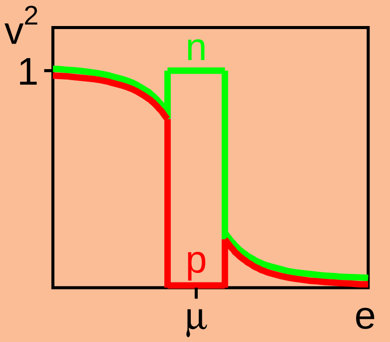
Deuteron Condensation in Asymmetric Matter:

U. Lombardo, P. Nozières, P. Schuck, H.-J. Schulze, A. Sedrakian; PRC 46, 064314

- Blocking windows:

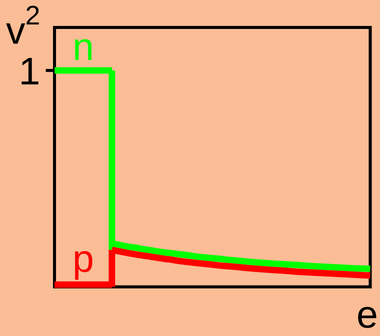
$\mu \gg \Delta$:

Cooper pairs



$\mu \to -E_D/2$:

Bound states



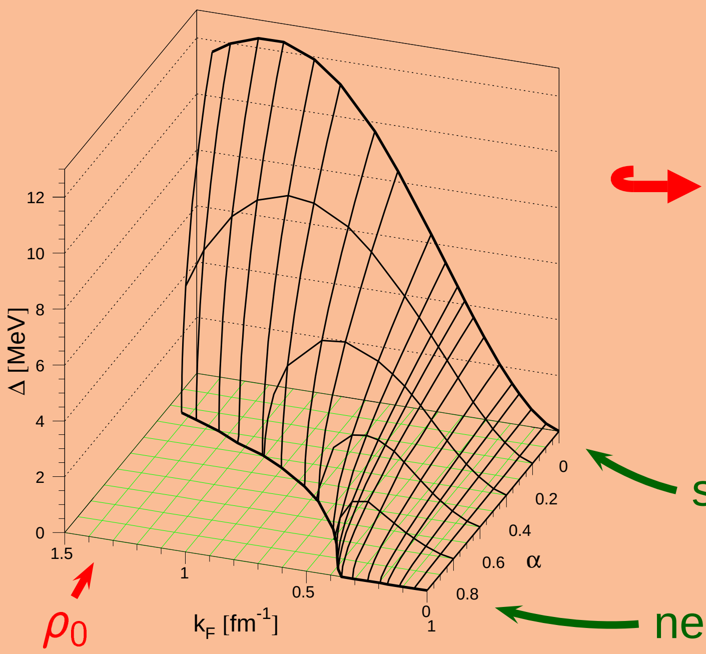
- Interpretation: Bose-condensed deuterons and a Fermi sea of excess neutrons coexist
  Pauli-blocking due to the excess neutrons becomes negligible in the dilute limit

- Analytical low-density result ($E_D = 2.2$ MeV):

$$\frac{\Delta}{E_D}(\rho,\alpha) = \sqrt{\frac{\rho(1-\alpha)}{\rho_\mu}}\,, \qquad \rho_\mu \equiv \frac{(mE_D)^{3/2}}{2\pi} \approx 0.0020\,\mathrm{fm}^{-3}$$

→ Much stronger than weak-coupling pairing
Superfluid at any asymmetry

- Numerical calculation (bare potential, free s.p. spectrum):



→ $\alpha_{\max}(\rho_0) \approx 0.2$

$\rho_{\rm Mott} \approx \rho_0/100$

Summary

- BCS is never valid:
  - Low density ($k_F \ll 1/|a|$): $\Delta \xrightarrow{k_F \to 0} \Delta_{\text{BCS}}/(4e)^{1/3}$ approached from below !
  - Higher density: Polarization diagrams in pp and ph channels are important $\to$ Babu-Brown approach
  - Result: polarization suppresses the BCS $^1S_0$ gap

- Asymmetry destroys pairing rapidly: $\alpha_{\text{max}} = 3\Delta_0/4\mu$

- Transition to BE (deuteron) condensation:
  - Strong $^3SD_1$ pairing at any asymmetry

- Future Problem: Reliable calculation at normal density:
  - Polarization interaction beyond first order
  - 4-dim gap equation

- Future Applications:
  - Gaps in neutron stars: cooling, glitches
  - Microscopic pairing forces in finite nuclei