Comment on Parametrizations of Nuclear Mean-Field for Triaxial Deformation

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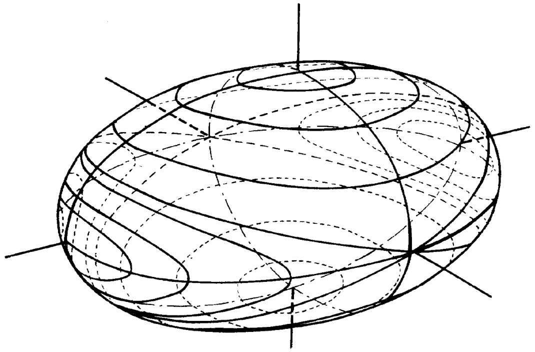
Triaxiality of nucleus — A long standing issue

triaxial body



- Davydov-Filippov model
- CoulEx. sum rule method:

  $$\langle [[E2 \times E2] \times E2]_0 \rangle \propto \cos 3\gamma$$

- $E_{\rm ex}$ or $M1/E2$ staggering
  at high-spins in odd or odd-odd nuclei

recently

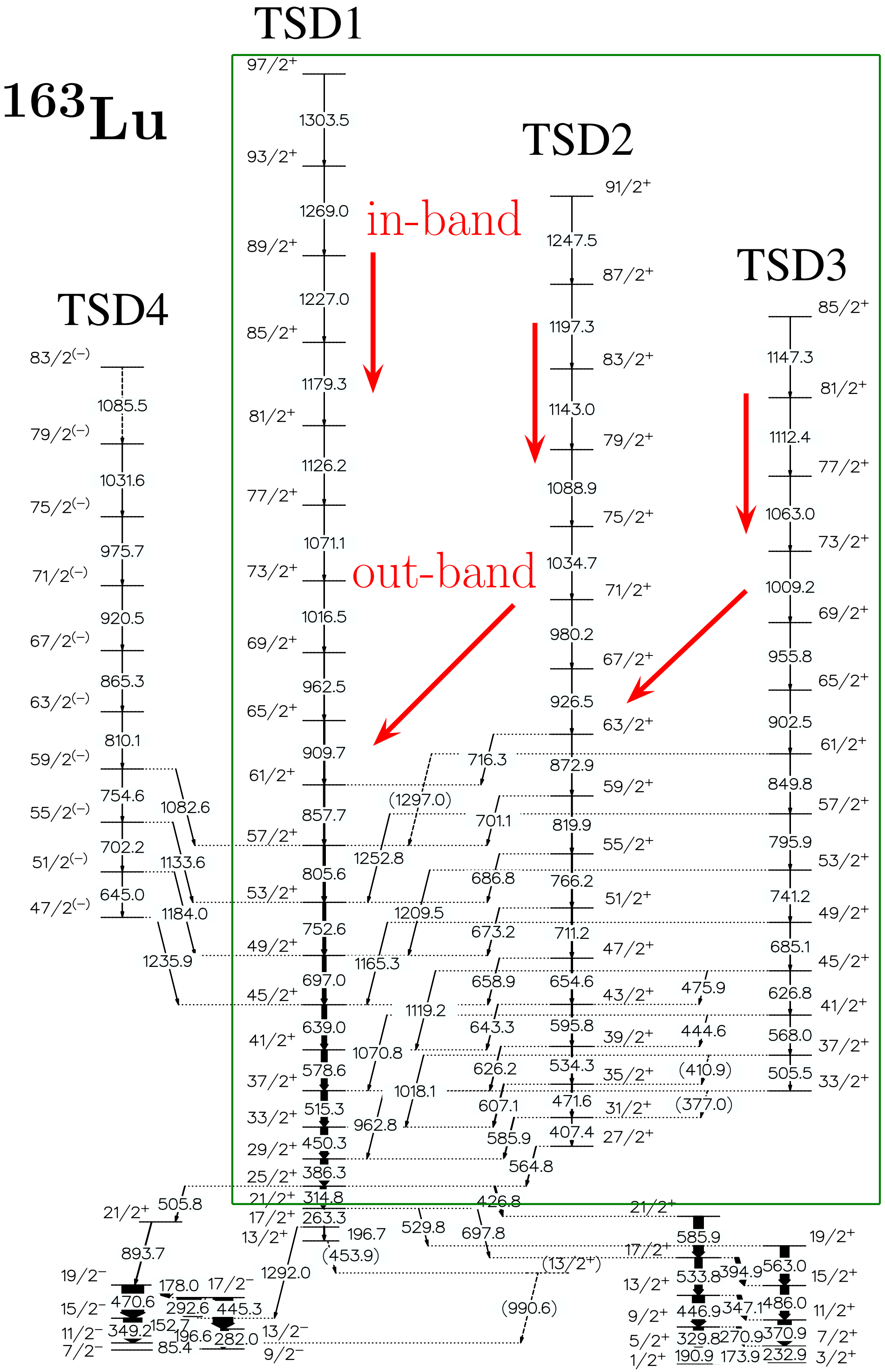
- **Wobbling band !**
  in ^{161,163,165,167}Lu nuclei

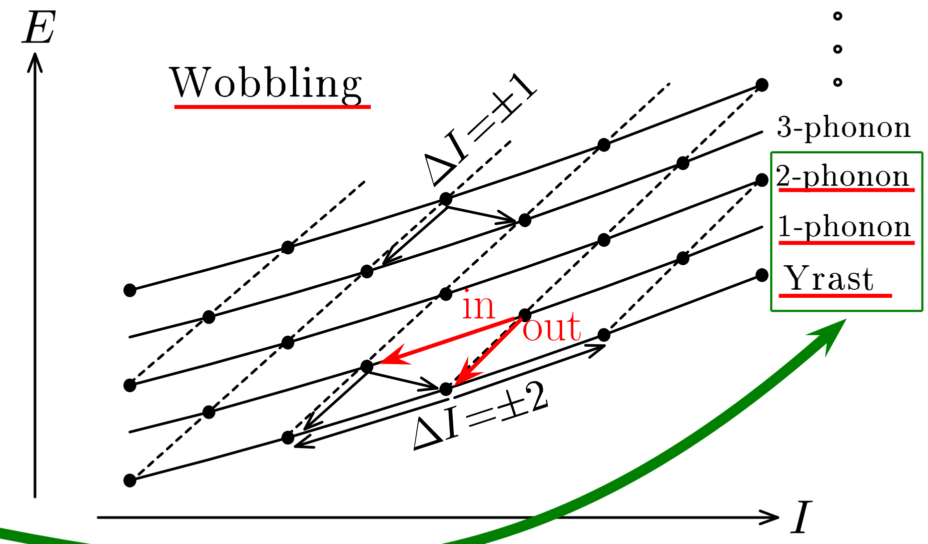
Wobbling Spectra

D. R. Jensen at al., Eur. Phys. J. **A19** (2004), 173.

First identified by Ødegård et al. (2001)

^{163}Lu





$B(E2)_{out}/B(E2)_{in}$
sensitive to
triaxiality γ !
(Hamamoto-Hagemann)

$B(E2)_{\mathrm{out}}/B(E2)_{\mathrm{in}}$ in the Rotor Model

Model parameters

Energy spectra ⟷ moments of inertia $\mathcal{J}_x, \mathcal{J}_y, \mathcal{J}_z$

(more in $h_{\mathrm{s.p.}}$ if a particle coupled)

$B(E2)$'s ⟷ two Q-moments

$$\begin{cases} Q_{20} \propto \Sigma(2z^2 - x^2 - y^2) \\ Q_{22} \propto \Sigma(x^2 - y^2) \end{cases}$$

$$\tan \gamma = -\frac{\sqrt{2}\,Q_{22}}{Q_{20}}$$

(Lund convention of sign)

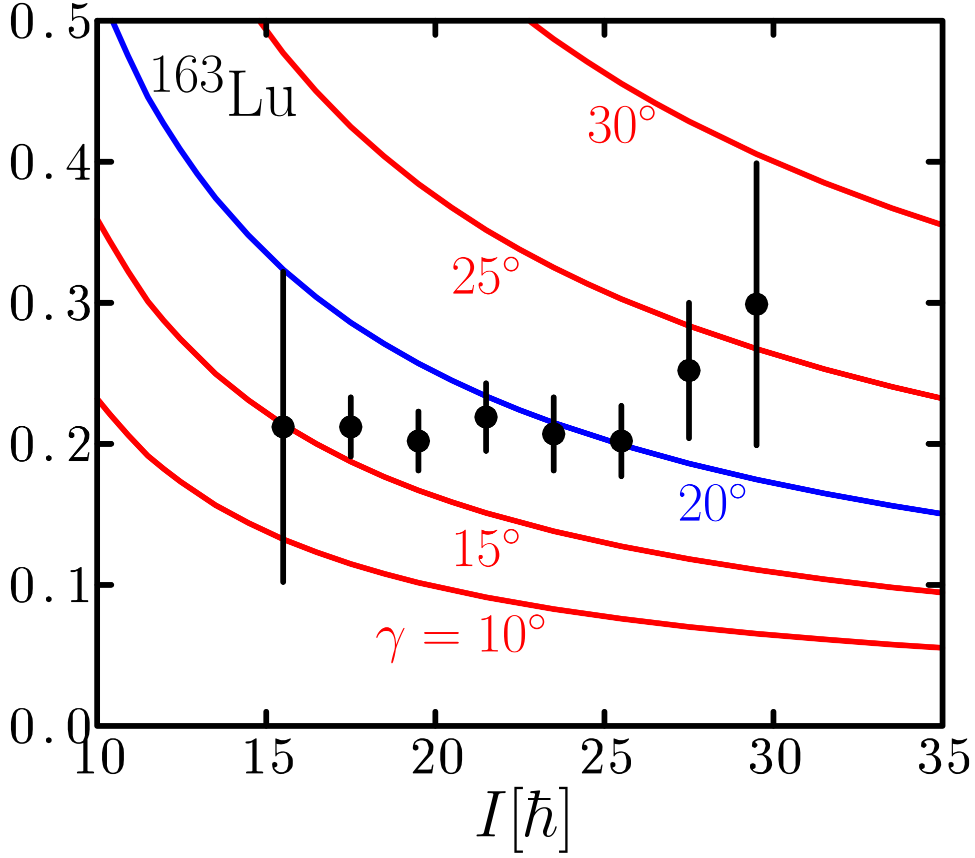
c.f. if $\mathcal{J}_y = \mathcal{J}_z$

$$B(E2)_{\mathrm{out}}/B(E2)_{\mathrm{in}} \propto \tan^2(\gamma + 30^\circ)$$

Particle-Rotor Model Cal. by

Hamamoto-Hagemann, PRC**67**, 014319 (2003).

$\mathcal{J}_x : \mathcal{J}_y : \mathcal{J}_z = 145 : 135 : 50$



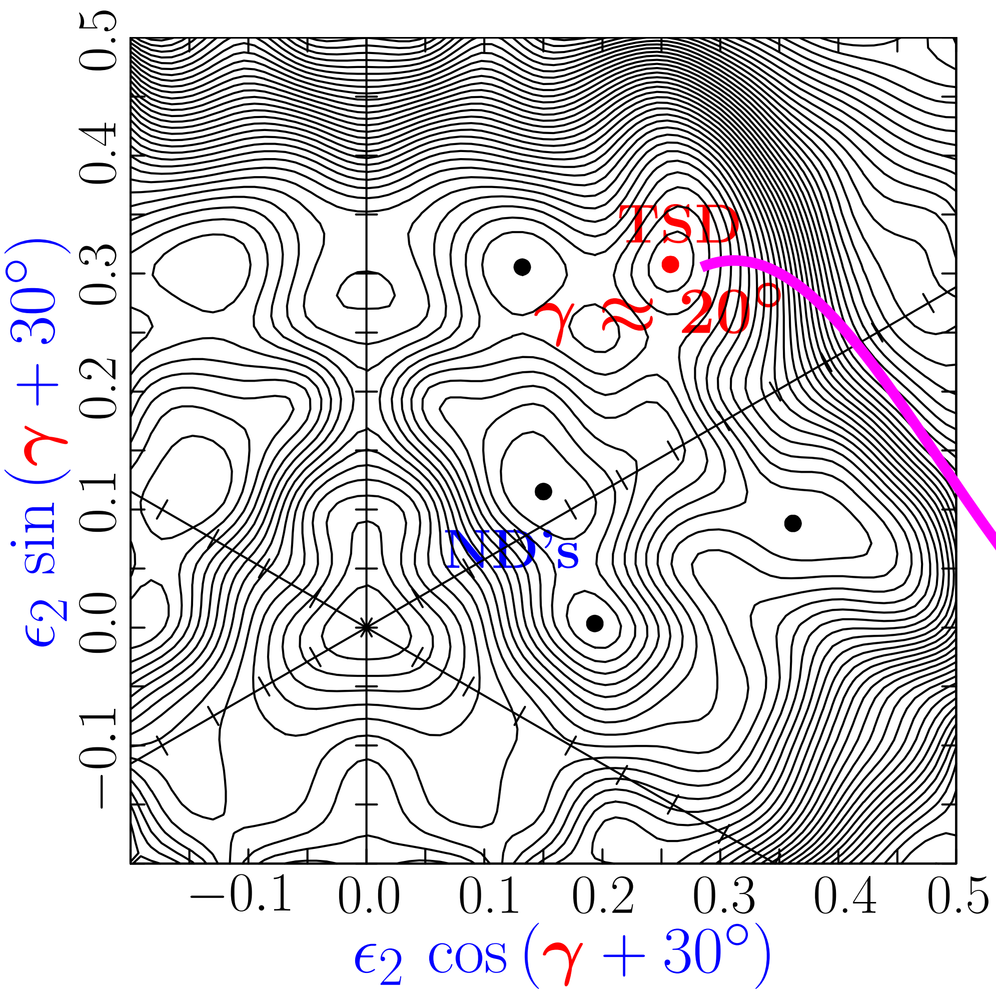
Microscopic cranked mean-field plus RPA (1)

good correspondence

Rotor Model

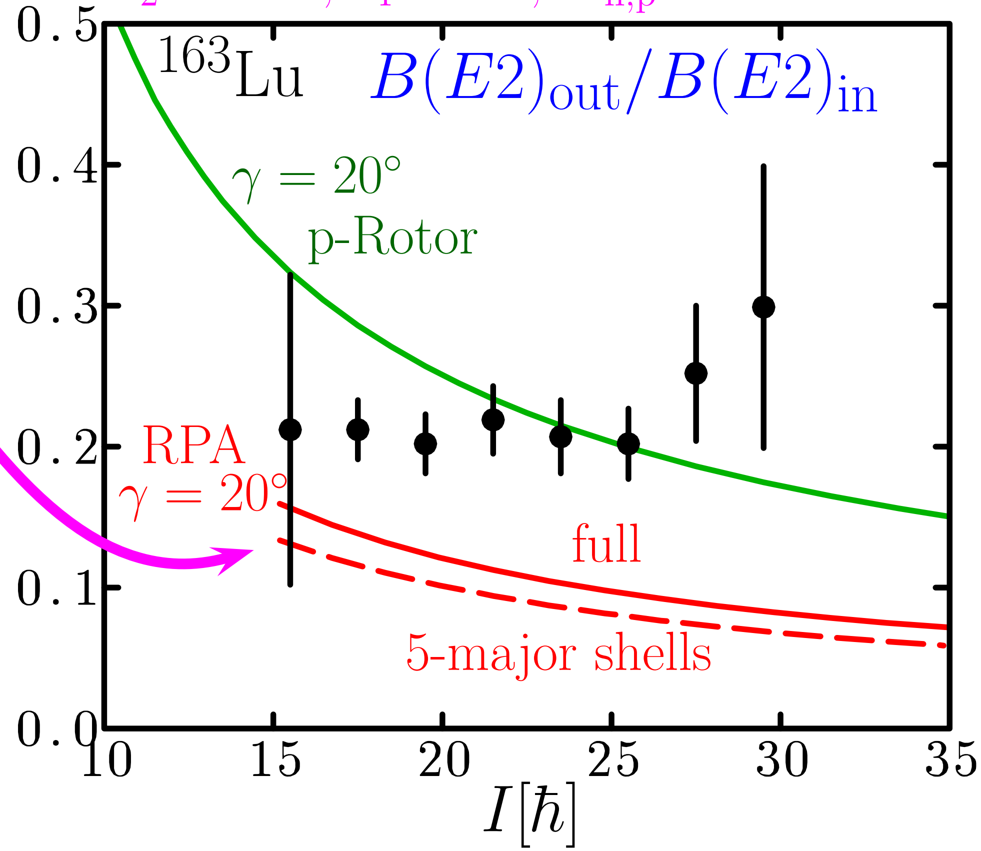
Mean-field parameters

Nilsson potential: $(\epsilon_2, \gamma, \epsilon_4, \ldots, \Delta_{n,p})$ ↔ determined selfconsistently (Strutinsky calculations)



Matsuzaki-Shimizu-Matsuyanagi, PRC**65**, 041303(R) (2002).

$\epsilon_2 = 0.43$, $\epsilon_4 = 0.0$, $\Delta_{n,p} = 0.3$ MeV



Microscopic cranked mean-field plus RPA (2)

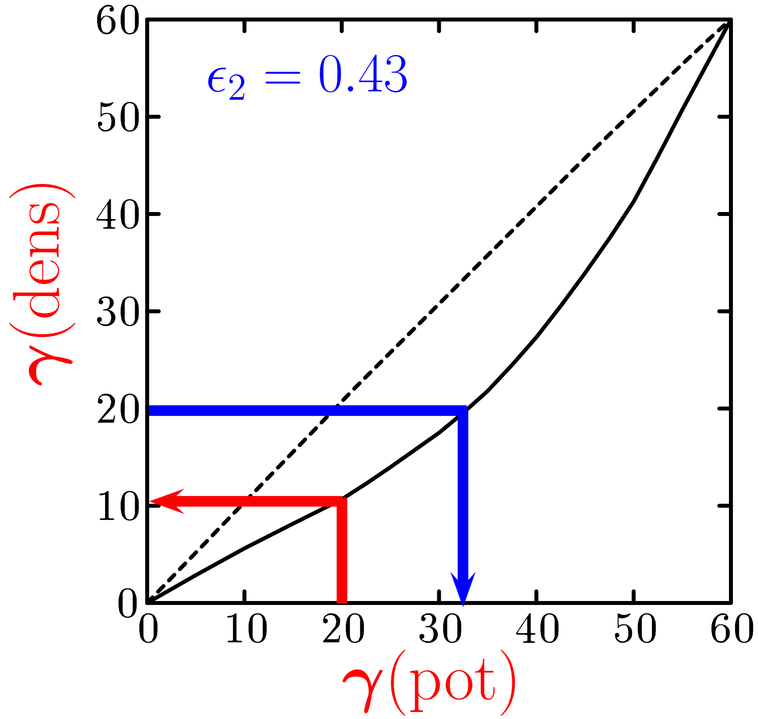
Difference in the definition of γ

- "γ(pot)" in Nilsson potential

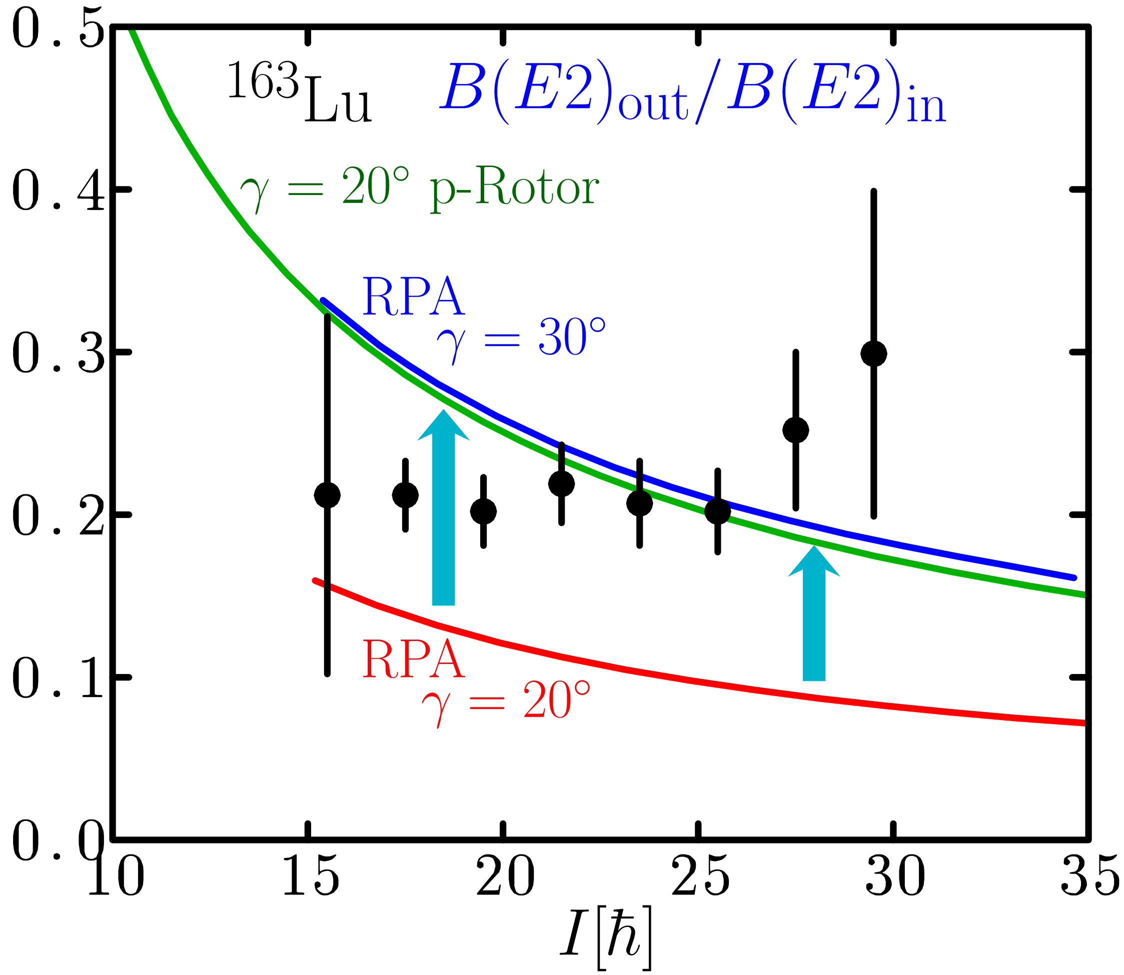
  v.s.

- "γ(dens)"$= \tan^{-1}\left(-\frac{\langle\sqrt{2}\,Q_{22}\rangle}{\langle Q_{20}\rangle}\right)$

  (⟷ Rotor Model)



Good correspondence to Rotor Model by changing γ(pot)$= 20°$ to $30°$!



Parametrization of triaxial deformation (1)

- **$\boldsymbol{\gamma}$(dens)** ⟷ density distribution

$$\boldsymbol{\gamma}(\mathrm{dens}) = \tan^{-1}\left(-\frac{\langle\sqrt{2}\,Q_{22}\rangle}{\langle Q_{20}\rangle}\right), \qquad \langle O\rangle = \int O(\boldsymbol{r})\rho(\boldsymbol{r})d^3\boldsymbol{r}$$

- **$\boldsymbol{\gamma}$(shape)** ⟷ potential shape (geometry)

ρ_{LD} : liquid-drop like sharp-cutoff density

$$\boldsymbol{\gamma}(\mathrm{shape}) = \tan^{-1}\left(-\frac{\langle\sqrt{2}\,Q_{22}\rangle_{\mathrm{LD}}}{\langle Q_{20}\rangle_{\mathrm{LD}}}\right), \qquad \rho \to \rho_{\mathrm{LD}} = \begin{cases} \rho_0 & \text{in } \Sigma \\ 0 & \text{out of } \Sigma \end{cases}$$

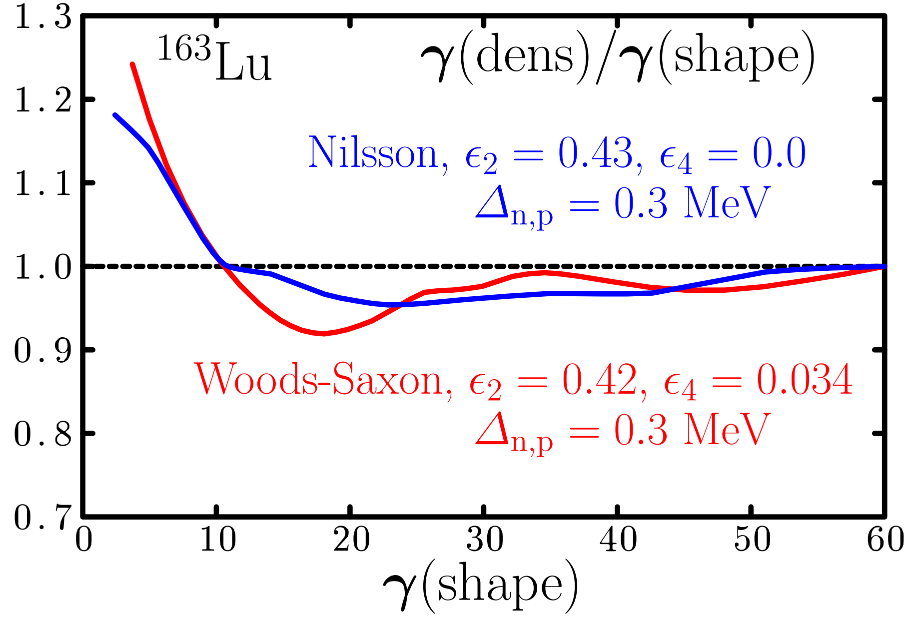
Σ : nuclear surface

where the nuclear surface Σ is defined by $V(\boldsymbol{r};\mathrm{def.}) = \mathrm{const.}$ (e.g. $\frac{1}{2}V_0$)

$$\Sigma : r = R(\Omega)$$

(neglecting velocity-dependent part)

nuclear selfconsistency
$\boldsymbol{\gamma}(\mathrm{dens}) \approx \boldsymbol{\gamma}(\mathrm{shape})$
(in good approximation)



Parametrization of triaxial deformation (2)

- $\boldsymbol{\gamma}$**(pot)** ⟷ parametrization in each mean-field potential

Nilsson: $(\epsilon_2, \gamma, \epsilon_4, \ldots)$ defined in stretched coordinate (r', Ω') by $(x' = \sqrt{\omega_x/\omega_0}\, x, \ldots)$

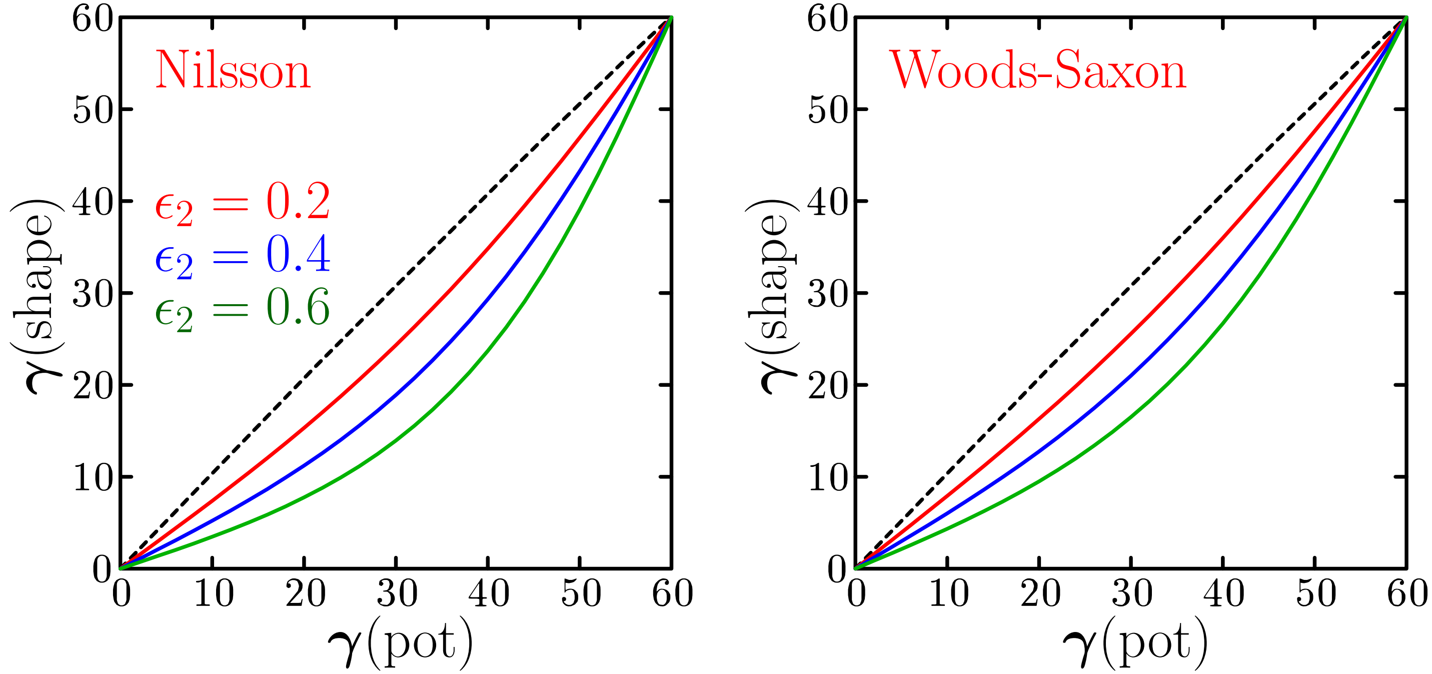
$$V(\boldsymbol{r}) = \frac{1}{2} M \omega_{\rm v}^2 r'^2 \left(1 - \sqrt{\frac{16\pi}{45}}\epsilon_2[\cos\gamma\, Y_{20}(\Omega') - \sin\gamma\, \frac{1}{\sqrt{2}}(Y_{22}(\Omega') + Y_{2-2}(\Omega'))] + \epsilon_4 \times \cdots\right)$$

$$\Sigma : R(\Omega) = R_{\rm v}\left[(\cdots)(1 - \sqrt{\frac{16\pi}{45}}\epsilon_2[\cos\gamma\, Y_{20}(\Omega) - \sin\gamma\, \frac{1}{\sqrt{2}}(Y_{22}(\Omega) + Y_{2-2}(\Omega))])\right]^{-1/2}$$

Woods-Saxon: $(\beta_2, \gamma, \beta_4, \ldots)$

$$V(\boldsymbol{r}) = V_0[1 + \exp(D_\Sigma(\boldsymbol{r})/a)]^{-1}, \qquad D_\Sigma(\boldsymbol{r}) = \text{distance between } \boldsymbol{r} \text{ and surface } \Sigma$$

$$\Sigma : R(\Omega) = R_{\rm v}\left(1 + \beta_2[\cos\gamma\, Y_{20}(\Omega) - \sin\gamma\, \frac{1}{\sqrt{2}}(Y_{22}(\Omega) + Y_{2-2}(\Omega))] + \beta_4 \times \cdots\right)$$



Parametrization of triaxial deformation (3)

(Harmonic oscillator potential : Nilsson with $\epsilon_4 = 0$)

$$V(\boldsymbol{r}) = \frac{1}{2}M\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

$\Updownarrow$

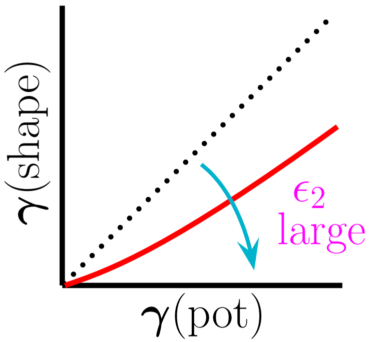
surface Σ: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$: Ellipsoid

here $a : b : c = \omega_x^{-1} : \omega_y^{-1} : \omega_z^{-1}$

parametrization of Nilsson pot. (ϵ_2, γ)

$$\begin{cases} \omega_x = \omega_{\rm v}\left(1 + \frac{1}{3}\epsilon_2(\cos\gamma + \sqrt{3}\sin\gamma)\right) \\ \omega_y = \omega_{\rm v}\left(1 + \frac{1}{3}\epsilon_2(\cos\gamma - \sqrt{3}\sin\gamma)\right) \\ \omega_z = \omega_{\rm v}\left(1 - \frac{2}{3}\epsilon_2\cos\gamma\right) \end{cases}$$

$$\tan\gamma(\text{shape}) = -\frac{\langle\sqrt{2}\,Q_{22}\rangle_{\rm LD}}{\langle Q_{20}\rangle_{\rm LD}} = \frac{\sqrt{3}\,\langle\sum(y^2 - x^2)\rangle_{\rm LD}}{\langle\sum(2z^2 - x^2 - y^2)\rangle_{\rm LD}}$$



$$\gamma(\text{shape}) = \tan^{-1}\left(\frac{\sqrt{3}\,(\omega_y^{-2} - \omega_x^{-2})}{2\omega_z^{-2} - \omega_x^{-2} - \omega_y^{-2}}\right) \Leftrightarrow \gamma(\text{pot}) = \tan^{-1}\left(\frac{\sqrt{3}\,(\omega_y - \omega_x)}{2\omega_z - \omega_x - \omega_y}\right)$$

if $\epsilon_2, |\gamma| \ll 1$, $\quad \gamma(\text{shape}) \approx \underline{\left(1 - \frac{3}{2}\epsilon_2\right)}\,\gamma(\text{pot})$

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c.f. For the spherical H.O. plus $Q \cdot Q$ force (usually restricted in one-two major shells)

$$\left.\begin{array}{l} V(\boldsymbol{r}) = \frac{1}{2}M\omega_0^2 \boldsymbol{r}^2 - \alpha_{20}Q_{20} - \alpha_{22}(Q_{22} + Q_{2-2}) \\ \qquad\qquad\qquad \alpha_{2\mu} = \chi\langle Q_{2\mu}\rangle \end{array}\right\} \longrightarrow \gamma(\text{dens}) = \tan^{-1}\left(\frac{\sqrt{3}\,(\omega_y^2 - \omega_x^2)}{2\omega_z^2 - \omega_x^2 - \omega_y^2}\right)$$

(Introducing δ_2 corresponding to ϵ_2) if $\delta_2, |\gamma| \ll 1$, $\quad \gamma(\text{shape}) \approx \underline{(1 - 2\delta_2)}\,\gamma(\text{dens})$

Selfconsistency, $\gamma(\text{shape}) \approx \gamma(\text{dens})$, is severely broken!

Triaxiality suggested by experimental $B(E2)$'s in ^{163}Lu

γ-dependence in Rotor Model ⟷ Microscopic RPA cal. (Talk by T. Shoji)

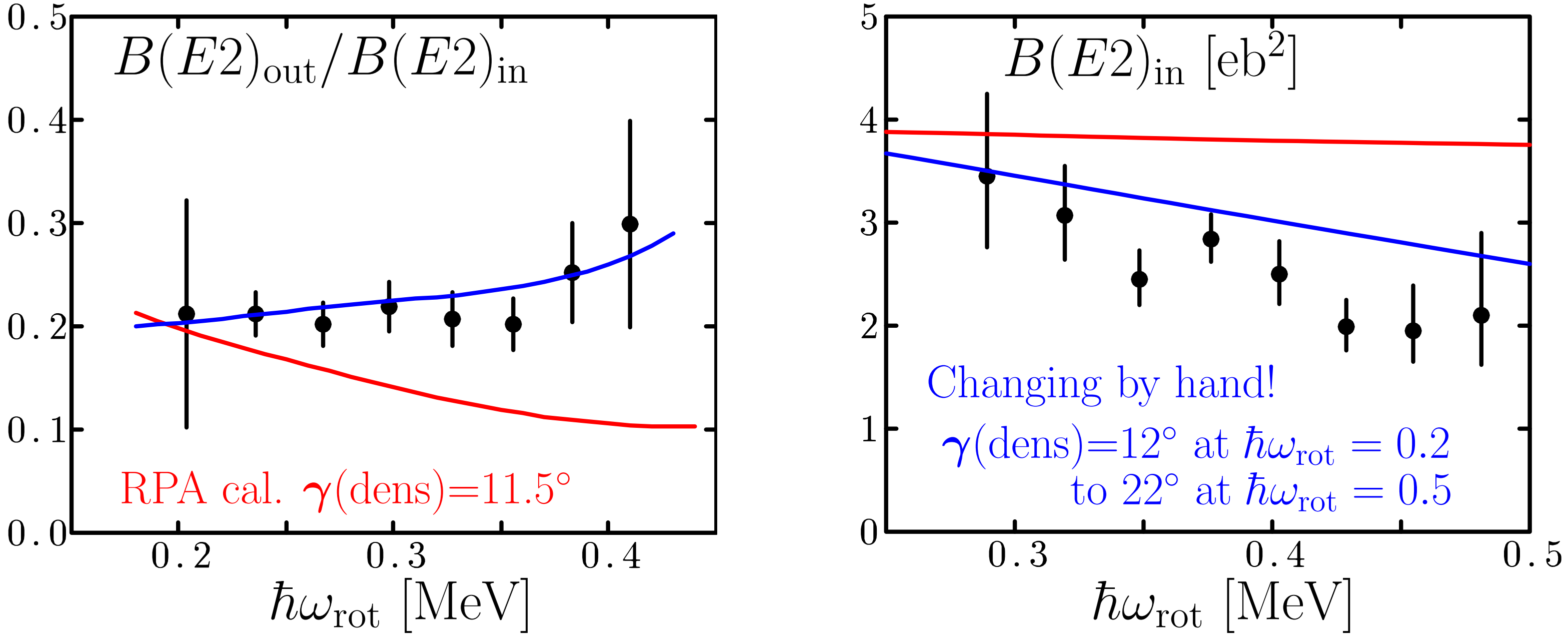
$$B(E2)_{\rm in} \approx \frac{15}{32\pi}\langle\Sigma(y^2 - z^2)\rangle^2 \propto \cos^2(\gamma + 30^\circ),$$ Here γ is γ(dens)

$$B(E2)_{\rm in}/B(E2)_{\rm out} \approx \frac{2}{I}\left(\frac{(w_z/w_y)^{1/4}\sin(\gamma+60^\circ) + (w_y/w_z)^{1/4}\sin\gamma}{\cos(\gamma+30^\circ)}\right)^2 \quad \begin{cases} w_y = 1/\mathcal{J}_z - 1/\mathcal{J}_x \\ w_z = 1/\mathcal{J}_y - 1/\mathcal{J}_x \end{cases}$$

Can we understand $B(E2)$'s by changing γ based on Microscopic RPA cal.?

Woods-Saxon RPA cal. using γ(pot)=18°(γ(dens)=11.5°)

→ An example: changing γ(dens)=12° to 22° linearly according to the formula above,



Summary

- $B(E2)_{\rm out}/B(E2)_{\rm in}$ of wobbling band is sensitive to the triaxiality γ,
  in both Rotor Model and Microscopic RPA calculation.
  $\longrightarrow$ can be used to probe intrinsic triaxial deformation!

- Different definitions of the triaxiality parameter γ

$$\begin{cases} \gamma(\text{dens}) & \longleftarrow \text{ density distribution} \\ \gamma(\text{shape}) & \longleftarrow \text{ shape of potential} \\ \gamma(\text{pot}) & \longleftarrow \text{ parametrization in each potential} \end{cases}$$

e.g. $\gamma(\text{dens}) = \tan^{-1}\left(-\frac{\langle\sqrt{2}\,Q_{22}\rangle}{\langle Q_{20}\rangle}\right)$

  γ(dens)$\approx\gamma$(shape) in good approximation (nuclear selfconsistency),
  but γ(pot) is quite different especially for large ϵ_2 deformation (Nilsson/Woods-Saxon).

  e.g. γ(dens)$\approx\gamma$(shape)$\approx 20^\circ \longleftrightarrow \gamma$(pot)$\approx 30^\circ$ at $\epsilon_2 \approx 0.4$ (TSD) !

- Suggestion by experimental data of $B(E2)$'s in ^{163}Lu
  Almost constant $B(E2)_{\rm out}/B(E2)_{\rm in}$ and decreasing $B(E2)_{\rm in}$ can be understood
  by increasing γ(dens) by about 10° (e.g. 12° to 22°) in the observed spin range.
  BUT Strutinsky calculation gives almost const. and small γ(dens) (e.g. 11° to 13°)!!

NEED MORE STUDY!!