Cranked-RPA description of wobbling motion in triaxially deformed nuclei

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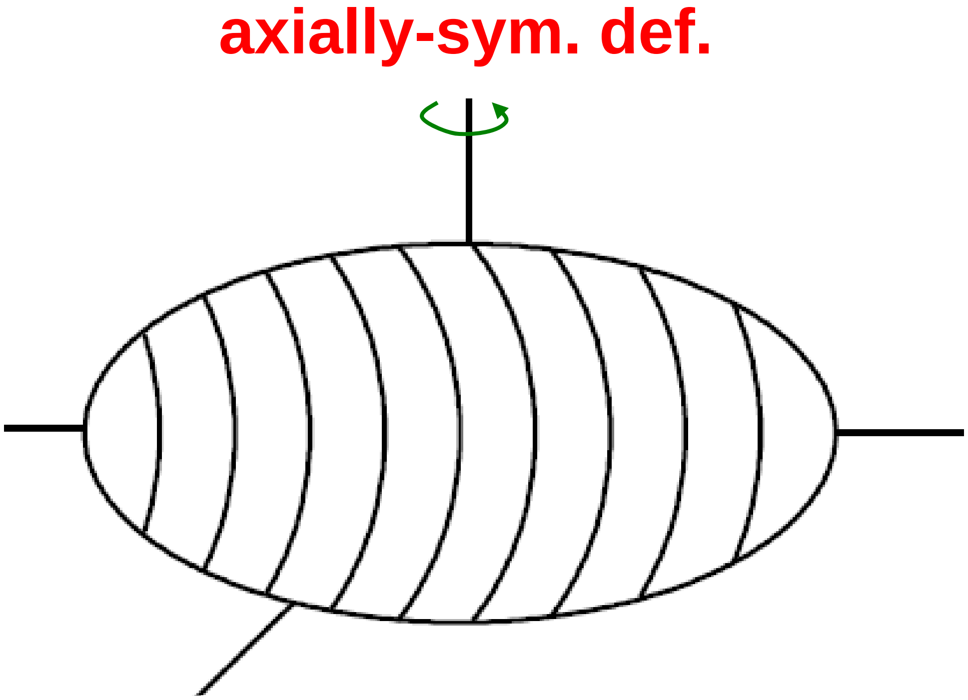
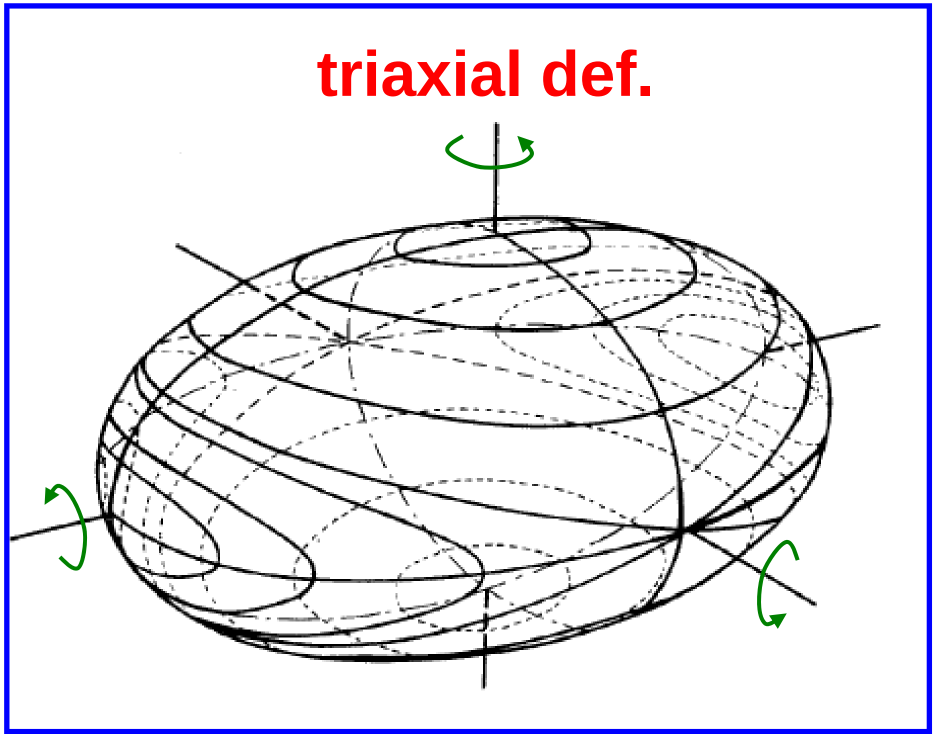




Wobbling motion (triaxially deformed)

Bohr – Mottelson vol.II

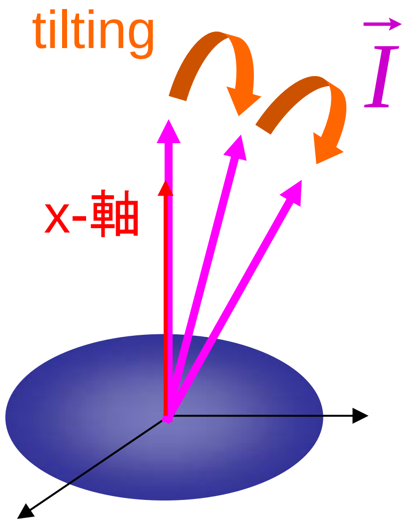
Rotor Model

$$H_{\mathrm{rot}} = \frac{I_x^2}{2\mathcal{J}_x} + \frac{I_y^2}{2\mathcal{J}_y} + \frac{I_z^2}{2\mathcal{J}_z}$$

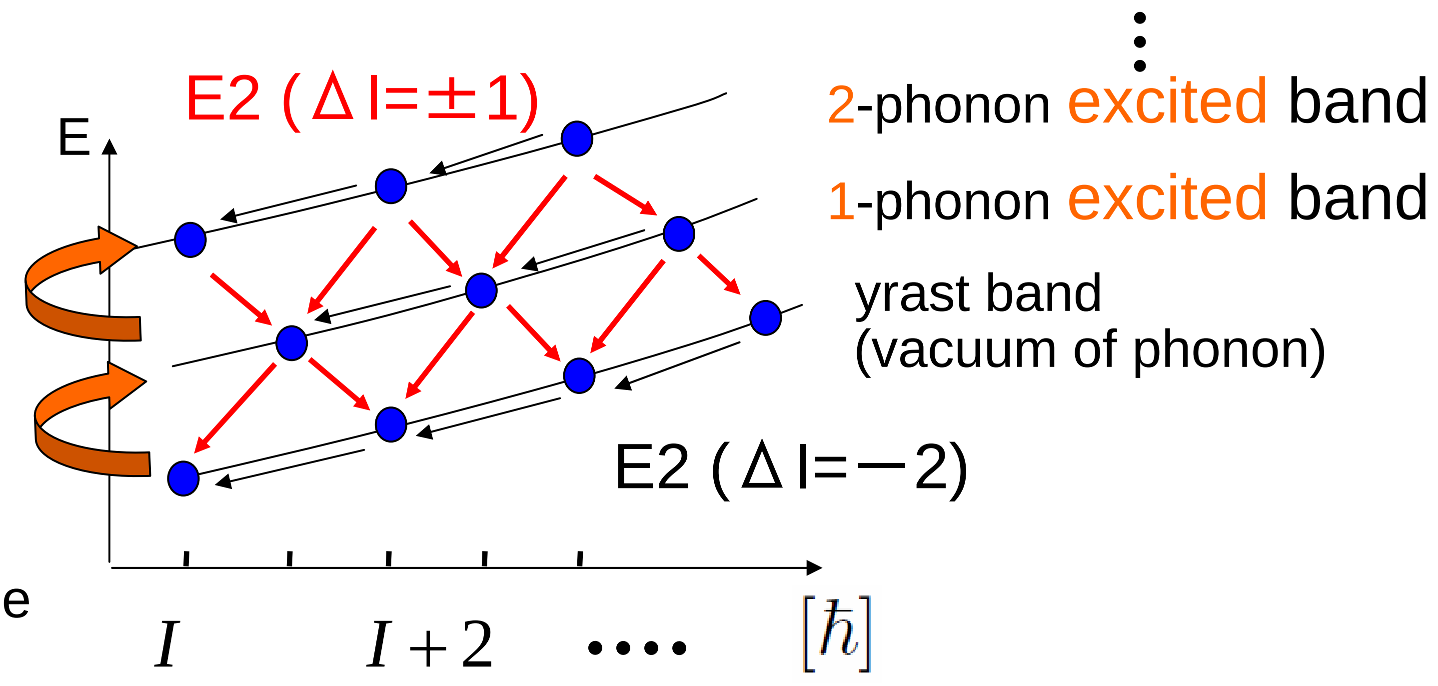
$I \gg 1$

$$E(I,n) = \frac{\hbar^2 I(I+1)}{2\mathcal{J}_x} + \hbar\omega_{\mathrm{wob}}\left(\frac{1}{2}n + 1\right)$$

$$\hbar\omega_{\mathrm{wob}} = \frac{\hbar^2 I}{\mathcal{J}_x}\sqrt{\left(\frac{\mathcal{J}_x}{\mathcal{J}_y} - 1\right)\left(\frac{\mathcal{J}_x}{\mathcal{J}_z} - 1\right)}$$

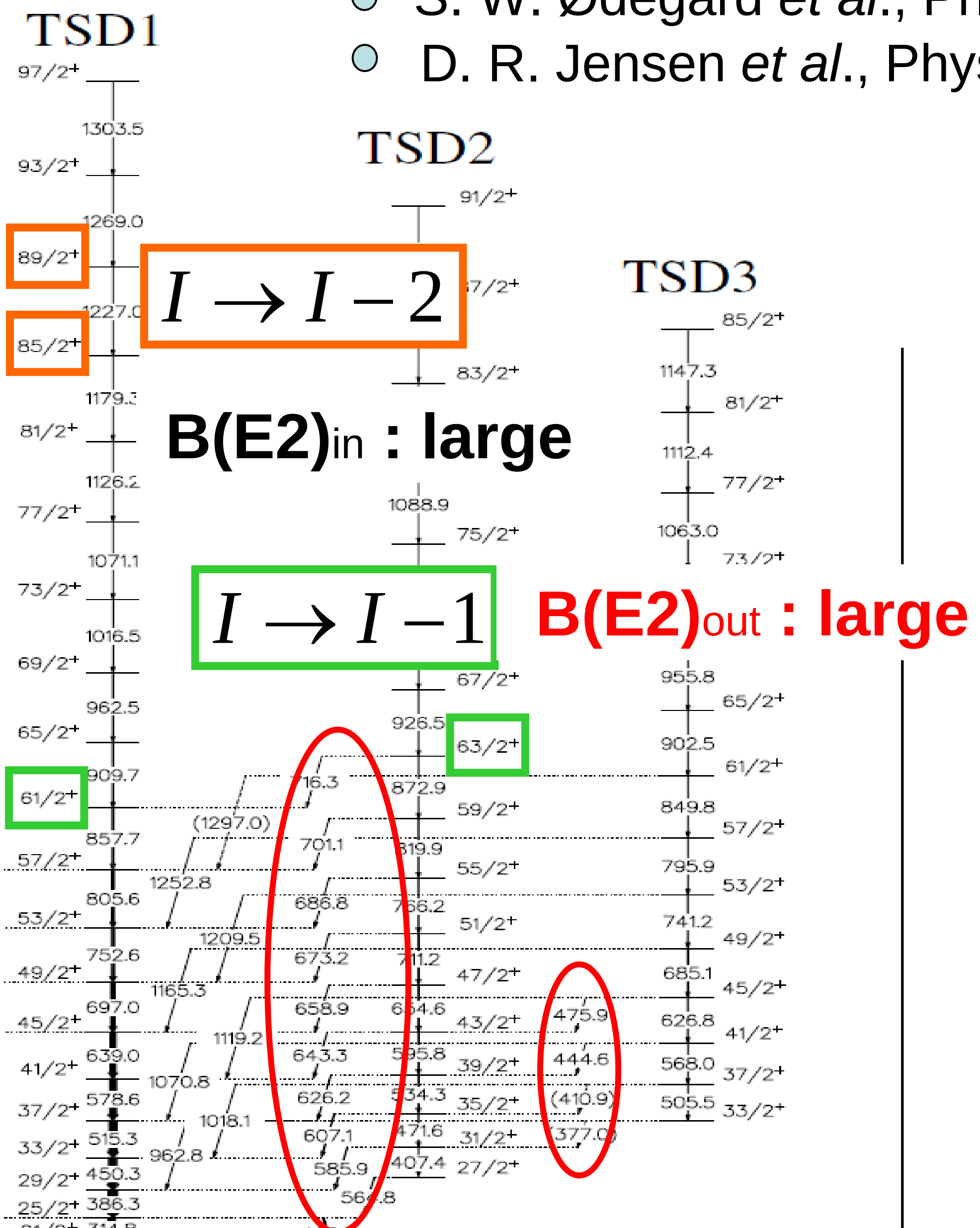


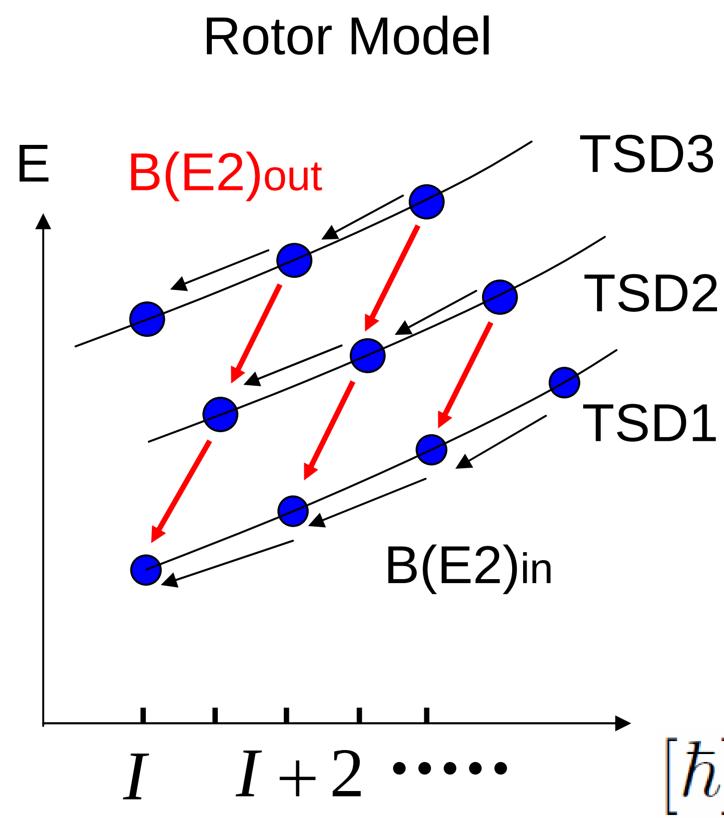
Body-fixed frame



$^{163}_{71}$Lu Evidence of wobbling motion

- S. W. Ødegård *et al*., Phys. Rev. Lett. 85 (2001) 5866
- D. R. Jensen *et al*., Phys. Rev. Lett. 89 (2002) 142503





Triaxial deformation in the rotor model

Triaxial deformation γ ⟺ Two intrinsic Q-moments

$$\begin{cases} Q_{20} \propto \langle \sum (2z^2 - x^2 - y^2)_a \rangle \\ Q_{22} \propto \langle \sum (x^2 - y^2)_a \rangle \end{cases}$$

$$\tan\gamma = -\frac{\sqrt{2}Q_{22}}{Q_{20}}$$

Three moments of inertia

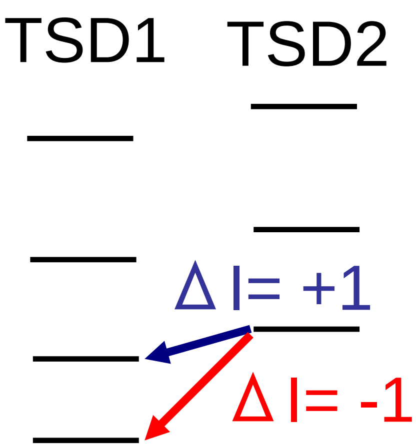
$$\mathcal{J}_x, \mathcal{J}_y, \mathcal{J}_z$$

B(E2)out (ΔI= -1) ⟹ $\gamma > 0$ in the Lund convention

⇓ irrotational moments of inertia

✗ $\mathcal{J}_y > \mathcal{J}_x > \mathcal{J}_z$

$$\hbar\omega_{\mathrm{wob}} \propto \sqrt{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}$$

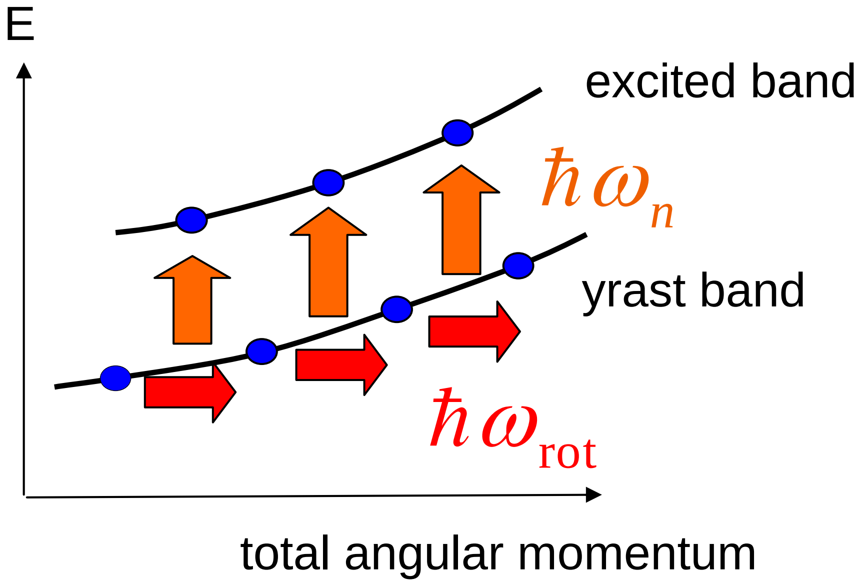


Microscopic study of wobbling motion

Cranked mean field

+

Random Phase Approximation (RPA)



$$h' = h_{\rm def} - \omega_{\rm rot} J_x$$

$$H' = h' + V_{\rm int}$$

$$[H', X_n^\dagger] = \hbar\omega_n X_n^\dagger$$

Microscopic Calculation ⟺ Rotor Model Picture

(Interpretation)

Microscopic wobbling theory

E. R. Marshalek, Nucl. Phys. A331 (1979) 429

$$H = h_{\mathrm{sph.}} + P + QQ$$

general def.

$$H = h_{\mathrm{def}} + V_{\mathrm{res}}$$

symmetry (J) restoring int.

Realistic Calculation

M. Matsuzaki, Nucl. Phys. A509 (1990) 269

Y. R. Shimizu, M. Matsuzaki, Nucl. Phys. A588 (1995) 559

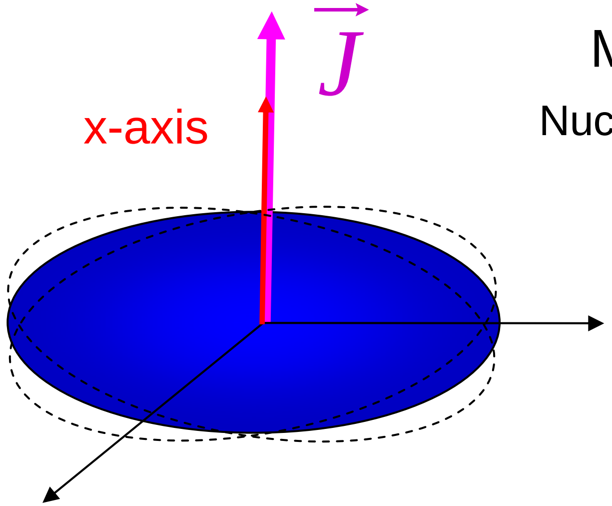
Recent Calculation for Lu, Hf nuclei

M. Matsuzaki, Y. R. Shimizu, K. Matsuyanagi, Phys. Rev. C 65,041303(R)

Phys. Rev. C 69,034325(2004)

using → Nilsson potential as a mean field

Uniformly-Rotating frame



Marshalek
Nucl. Phys. A331 (1979) 429

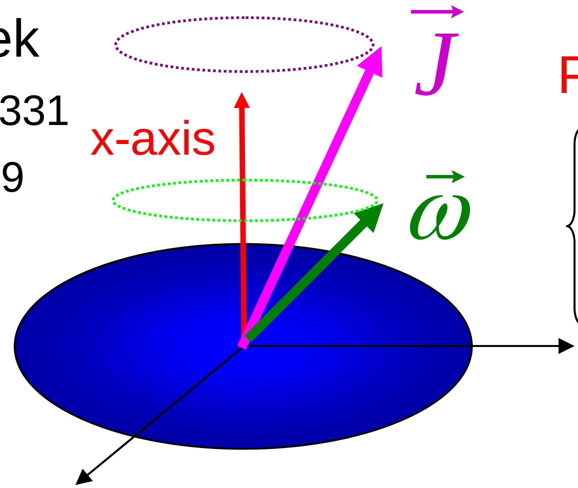
Shape fluctuation

$$h^{\mathrm{UR}}(t) = h_{\mathrm{def}} - \omega_{\mathrm{rot}} J_x$$
$$-\kappa_{zx} \langle Q_{zx} \rangle (t) Q_{zx} - \kappa_{xy} \langle Q_{xy} \rangle (t) Q_{xy}$$

Quadrupole tensor

$$Q_{ij} = \sqrt{\frac{5}{16\pi}} \sum_{a=1}^{A} [x_i x_j - r^2 \delta_{ij}]_a$$

Principal-Axis frame



PA-conditions

$$\begin{cases} \langle Q_{zx} \rangle^{\mathrm{PA}} = 0 \\ \langle Q_{xy} \rangle^{\mathrm{PA}} = 0 \end{cases}$$

Angular momentum fluctuation

$$h^{\mathrm{PA}}(t) = h_{\mathrm{def}} - \vec{\omega}(t) \cdot \vec{J}$$

moment of inertia

$$\langle J_{y,z} \rangle^{(n)}(t) = \mathcal{J}_{y,z}^{(n)} \, \omega_{y,z}^{(n)}(t)$$

$$\langle J_x \rangle \sim \hbar I \quad \mathcal{J}_x = \langle J_x \rangle / \omega_{\mathrm{rot}} \sim \hbar I / \omega_{\mathrm{rot}}$$

$$\hbar\omega_n = \frac{\hbar^2 I}{\mathcal{J}_x} \sqrt{\left(\frac{\mathcal{J}_x}{\mathcal{J}_y^{(n)}} - 1 \right) \left(\frac{\mathcal{J}_x}{\mathcal{J}_z^{(n)}} - 1 \right)}$$

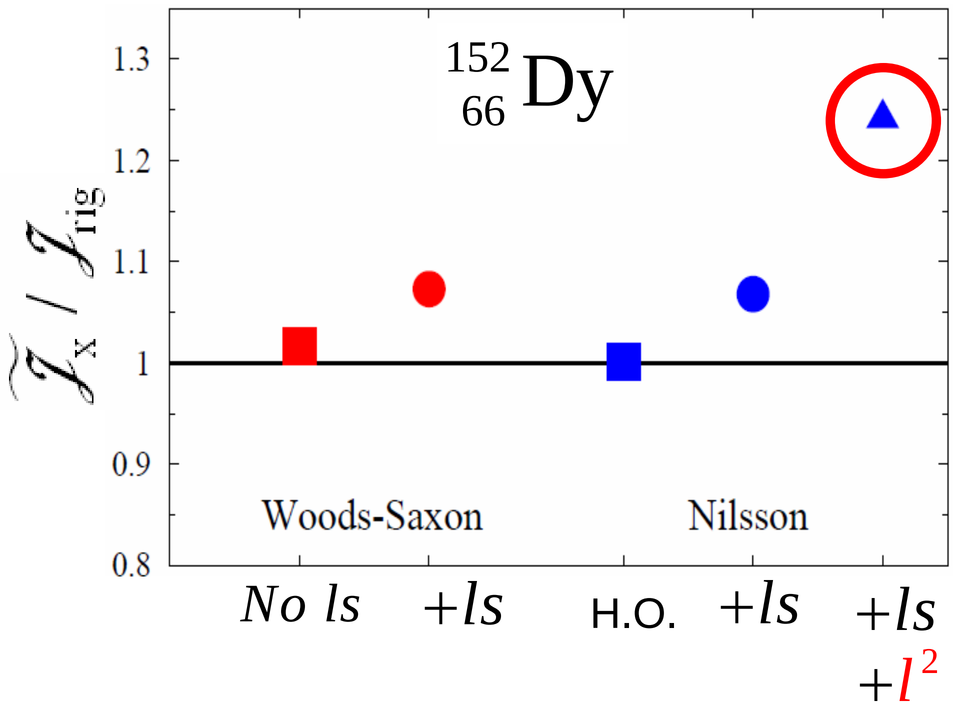
Mean-field (Woods-Saxon potential)

Nilsson potential

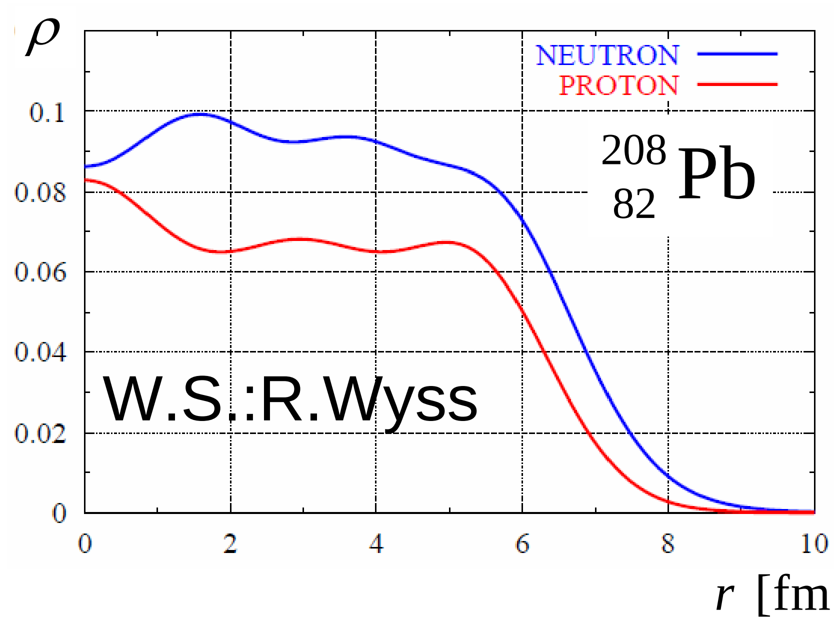
$\mathcal{J}_x$ is too large due to l^2 term

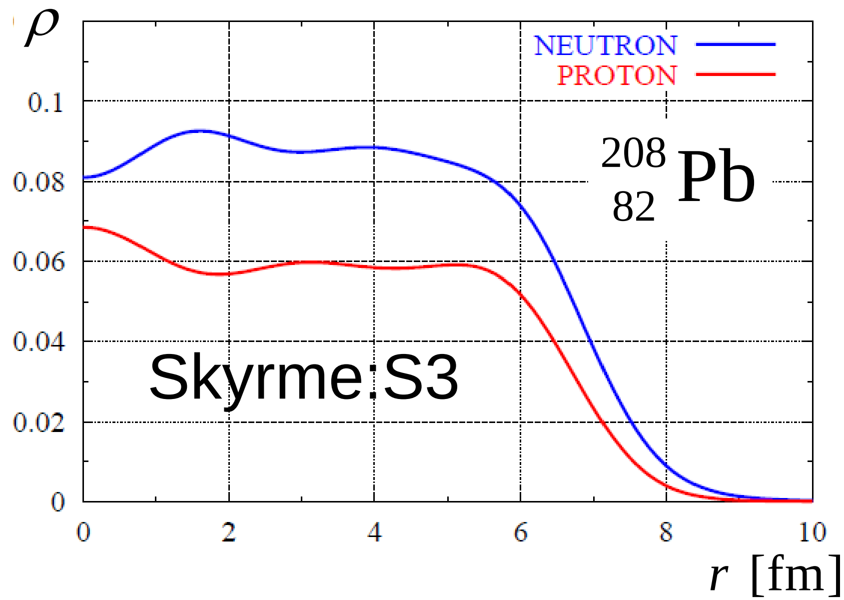
Woods-Saxon potential

good parametrization



Density distribution $\rho(r)$

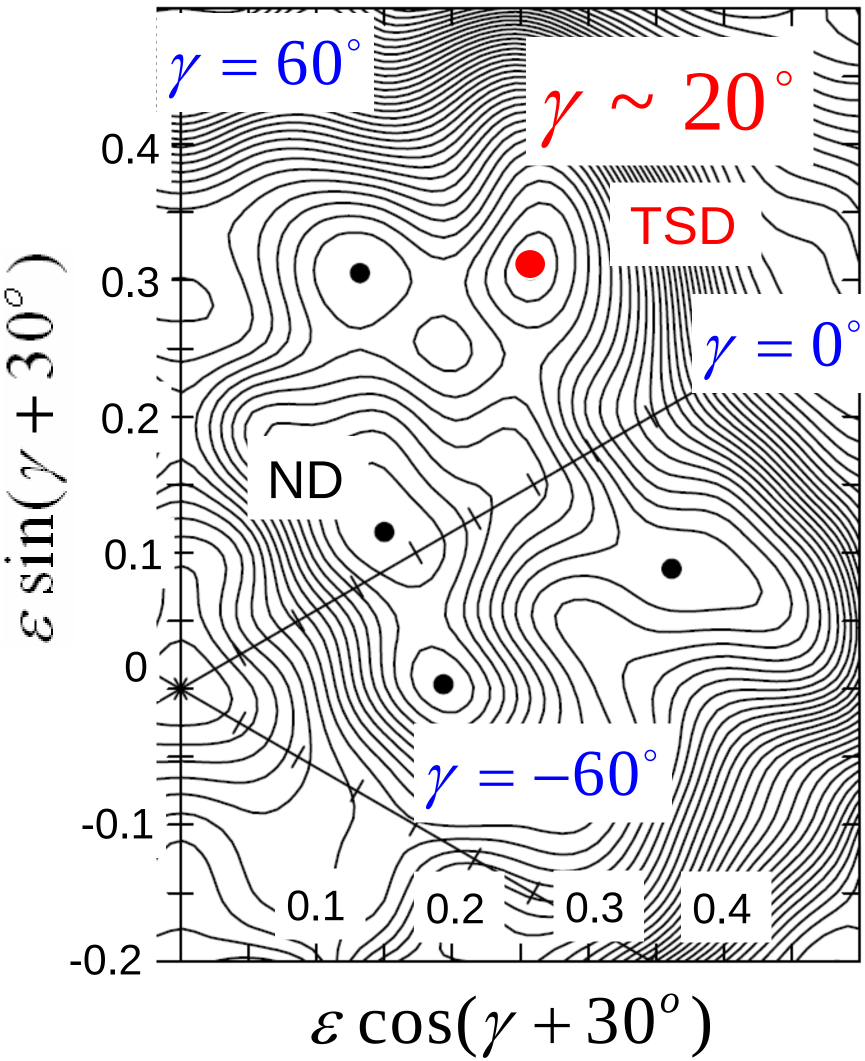




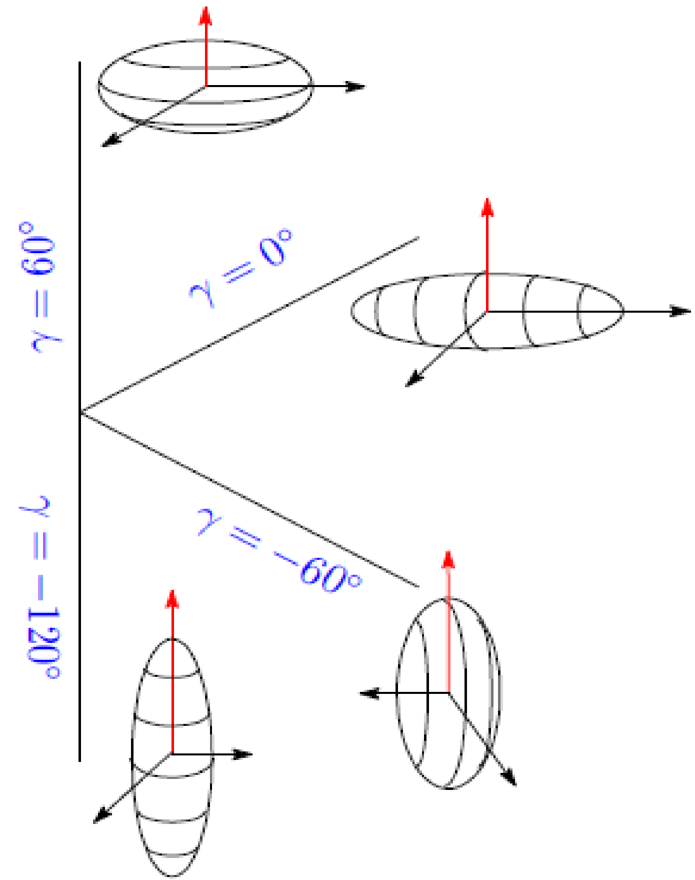
Shape of $^{163}_{71}\mathrm{Lu}$ nuclei at I=53/2

Cranked Nilsson Strutinsky

proton $i_{13/2}\ (\pi, \alpha) = (+, +1/2)$



Lund convention



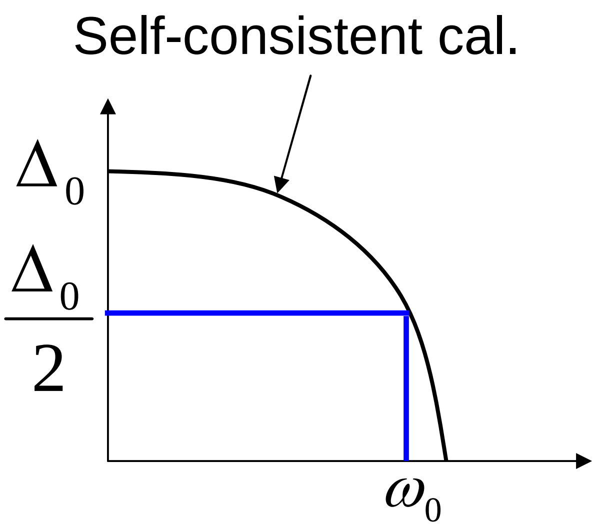
$\beta_2 = 0.42 \quad \gamma = 18^o$

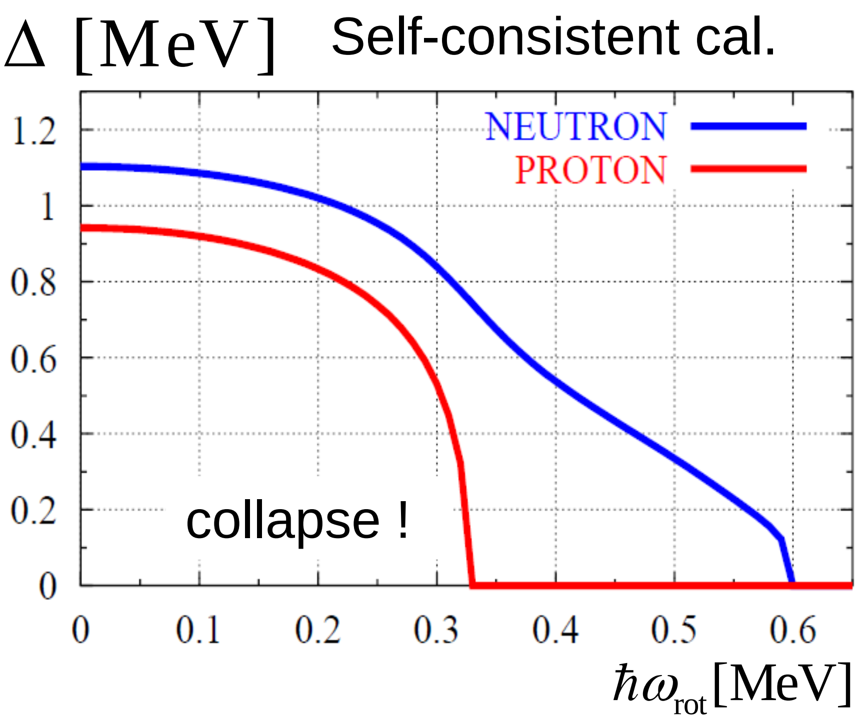
$\beta_4 = 0.034$

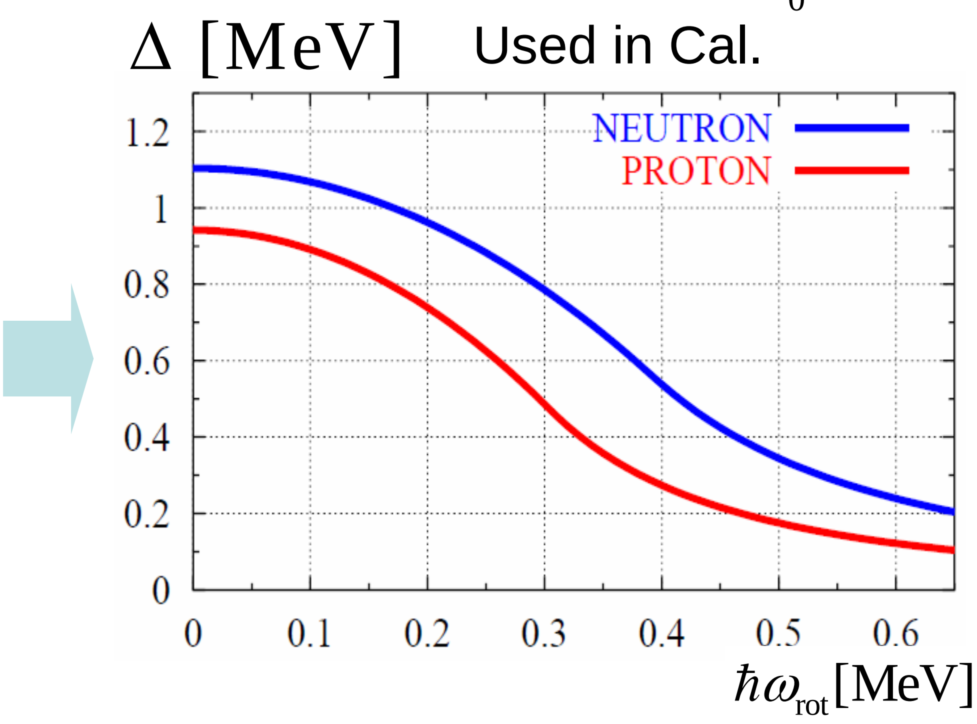
Paring gaps (monopole pairing)

$$\Delta(\omega_{rot}) = \begin{cases} \Delta_0 \left[1 - \frac{1}{2} \left(\frac{\omega_{rot}}{\omega_0} \right)^2 \right] & \text{for } \omega_{rot} < \omega_0 \\ \frac{\Delta_0}{2} \left(\frac{\omega_0}{\omega_{rot}} \right)^2 & \text{for } \omega_{rot} \geq \omega_0 \end{cases}$$

$$\Delta(\omega_0) = \frac{\Delta_0}{2}$$







RPA interaction ("minimal")

Separable force

$$V_{\mathrm{res}} = \frac{1}{2}\sum_{k=y,z} \kappa_k F_k^2$$

$$\kappa_k = \langle [[h_{\mathrm{def}}, iJ_k], iJ_k] \rangle$$

$$F_k = [h_{\mathrm{def}}, iJ_k]$$

Broken rotational symmetry

$$[h_{\mathrm{def}}, J_k] \neq 0$$

Restore broken symmetry in RPA order

$$[h_{\mathrm{def}} + V_{\mathrm{res}}, J_k]_{\mathrm{RPA}} = 0$$

No adjustable parameters

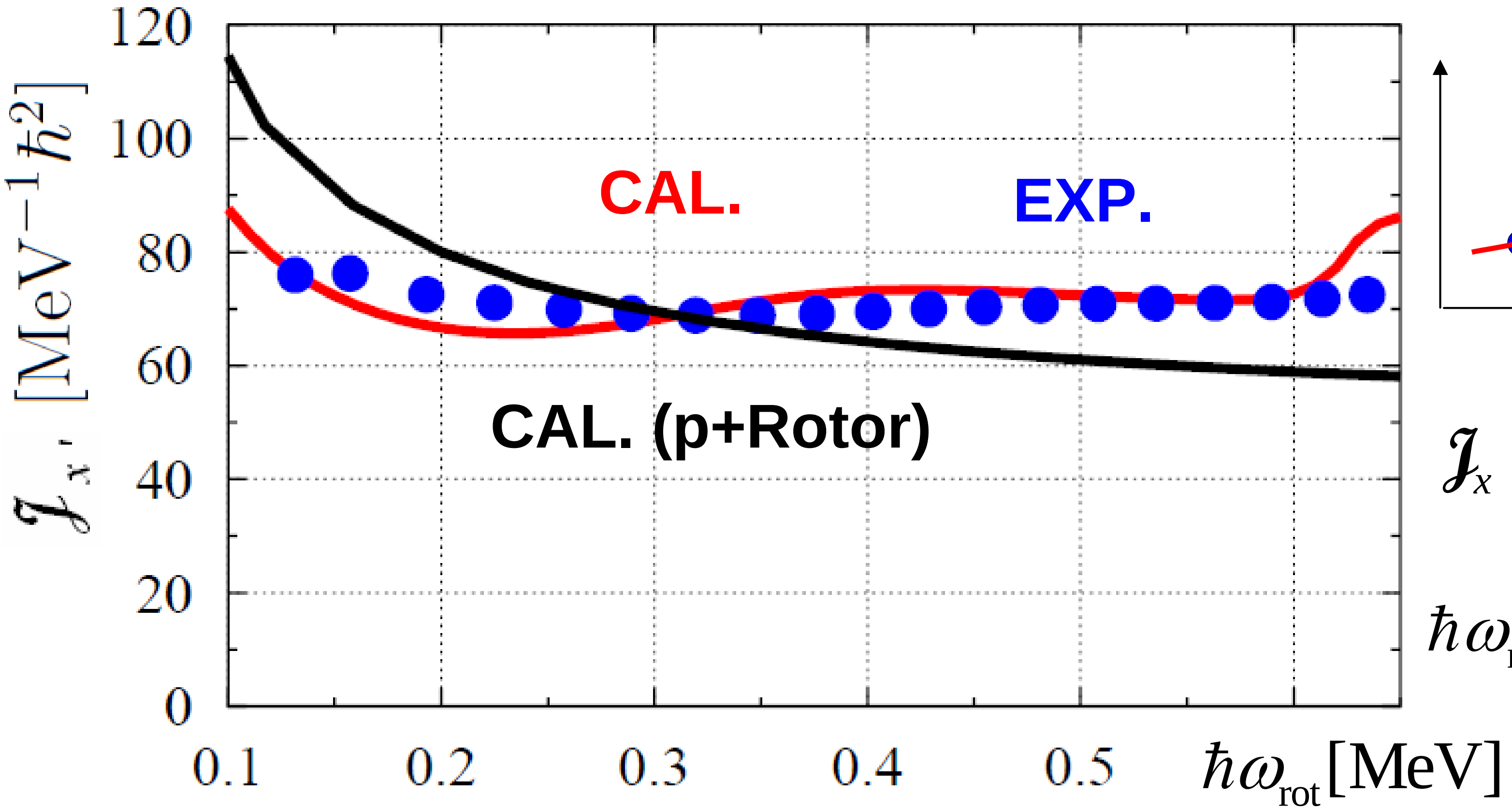
Bohr-Mottelson vol.II

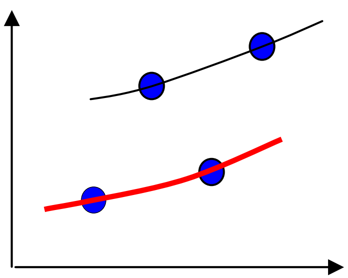
C. G. Andersson *et al.*, Nucl. Phys. A361 (1981) 147

RPA equation of motion is determined by a given mean field

$^{163}_{71}\mathrm{Lu}$

Moment of inertia





$$\mathcal{J}_x = \frac{\langle J_x \rangle}{\omega_{\mathrm{rot}}}$$

$$\hbar\omega_{\mathrm{rot}} = \frac{\partial E}{\partial I}$$

nice agreement with experimental data

$\gamma = 20^o$

$\mathcal{J}_x^{(R)} = 48$

$\mathcal{J}_y^{(R)} = 45$

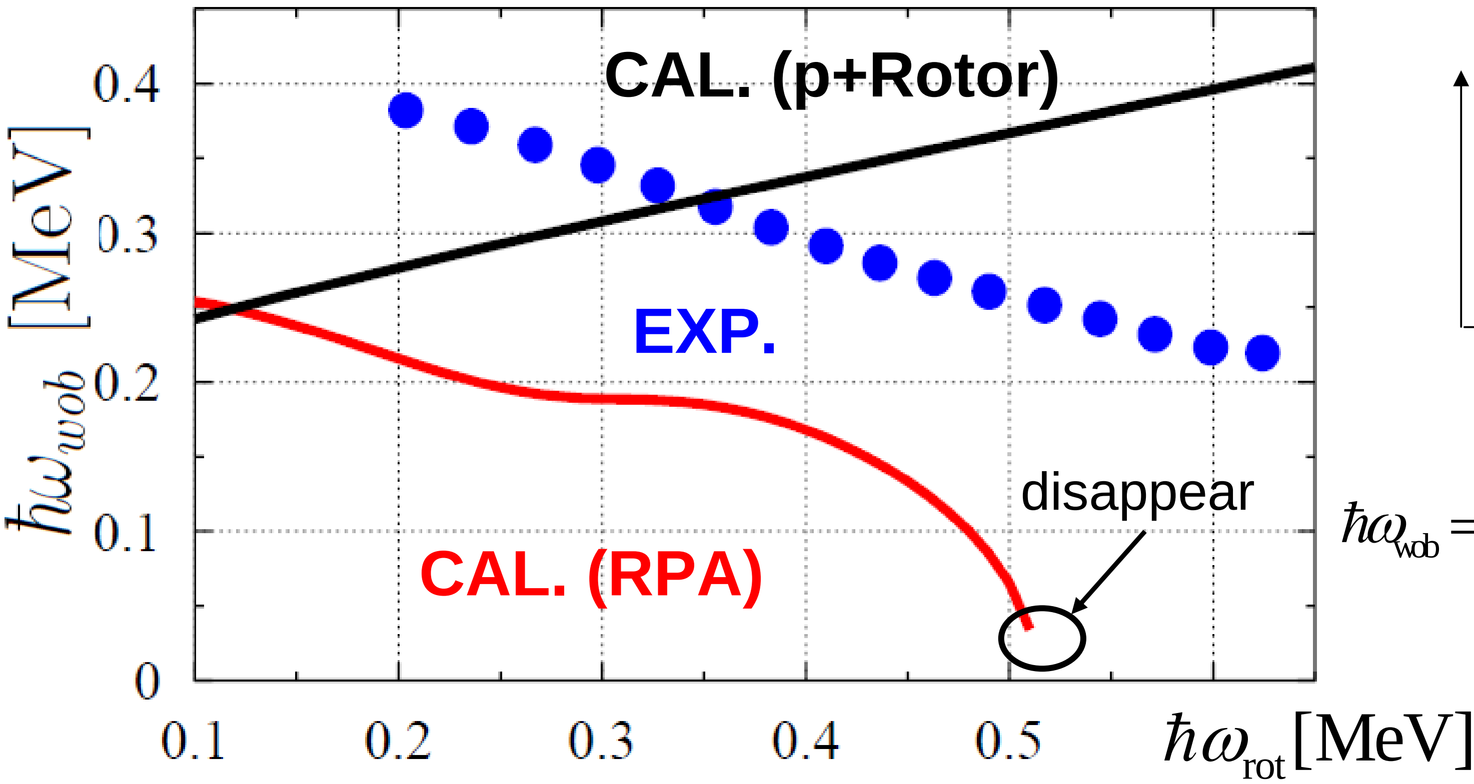
$\mathcal{J}_z^{(R)} = 17$

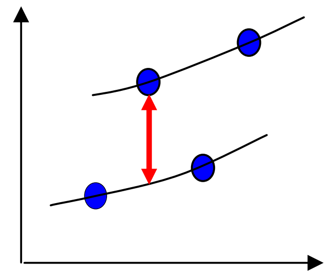
CAL. (p+Rotor)

I. Hamamoto,G. B. Hagemann, Phys. Rev. C **67**, 014319 (2003)

EXP. D. R. Jensen *et al*., Nucl. Phys. A **703** (2002) 3

$^{163}_{71}\mathrm{Lu}$ Excitation energy of RPA-phonon





$$\hbar\omega_{wob} = \hbar\omega_{rot}\sqrt{\left(\frac{\mathcal{J}_x}{\mathcal{J}_y}-1\right)\left(\frac{\mathcal{J}_x}{\mathcal{J}_z}-1\right)}$$

Wobbling-like RPA solution exists,

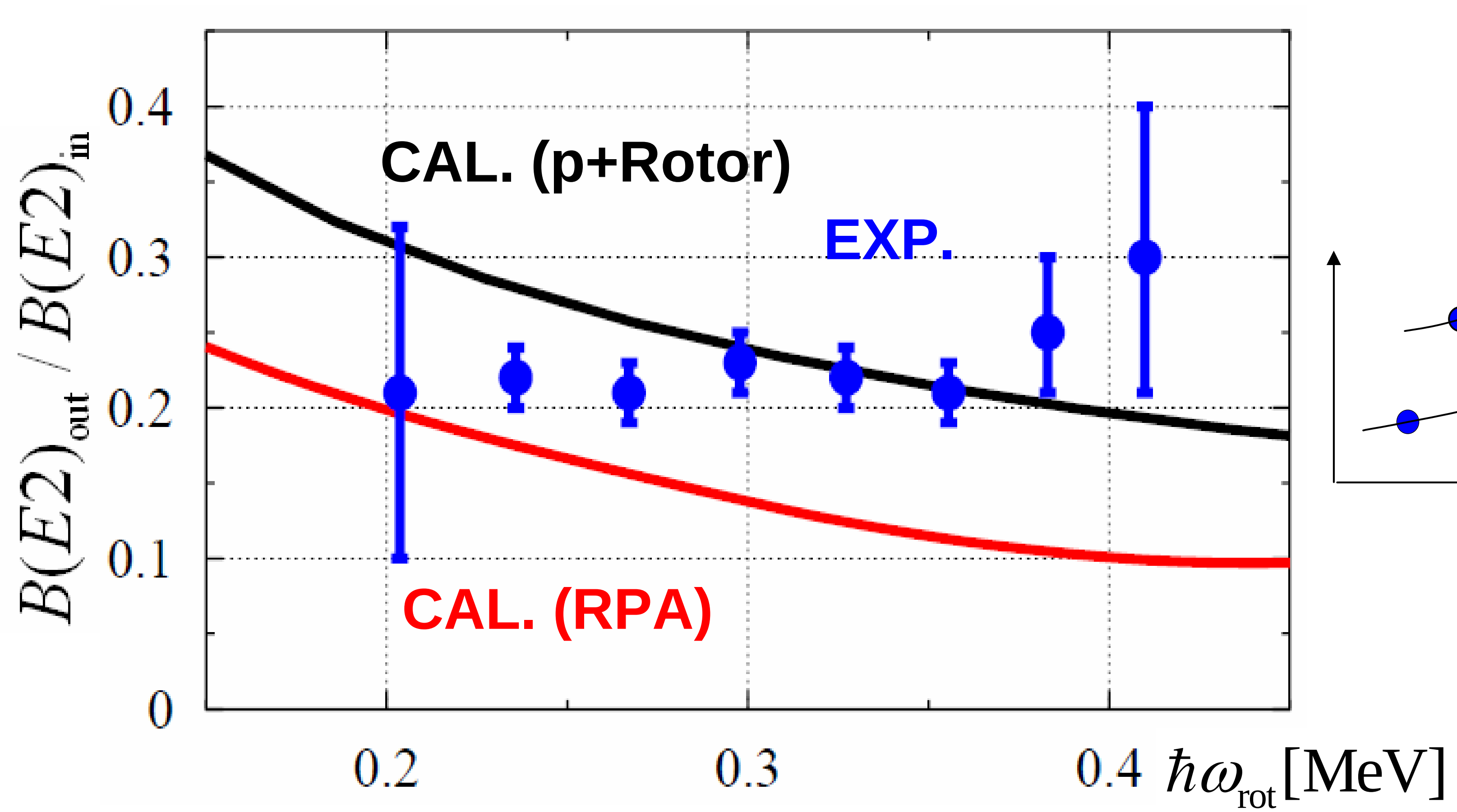
even if parameters are changed in a reasonable range.

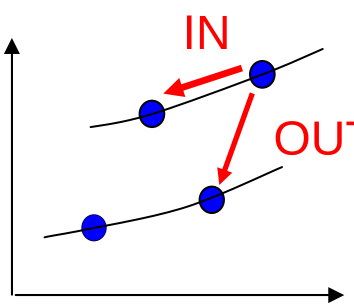
CAL. (p+Rotor) I. Hamamoto, G. B. Hagemann, Phys. Rev. C **67**, 014319 (2003)

EXP. D. R. Jensen *et al*., Nucl. Phys. A **703** (2002) 3

$^{163}_{71}Lu$

B(E2) ratio





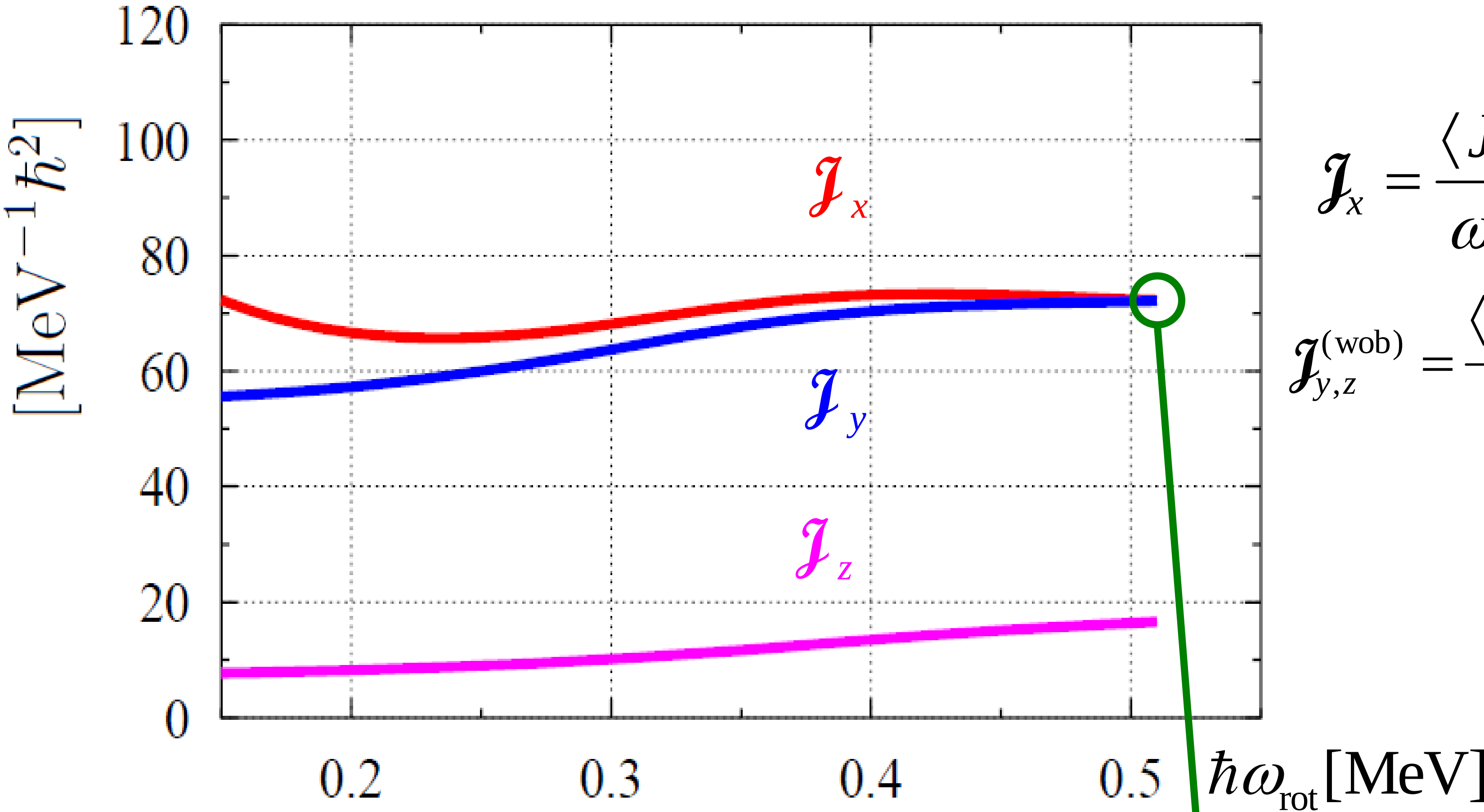
Extremely collective transitions ~ 50 Weisskopf units

CAL. (p+Rotor) I. Hamamoto, G. B. Hagemann, Phys. Rev. C **67**, 014319 (2003)

EXP. A. Görgen *et al.*, Phys. Rev. C **69**, 031301(R) (2004)

$^{163}_{71}\mathrm{Lu}$

Three moments of inertia



$$\mathcal{J}_x = \frac{\langle J_x \rangle}{\omega_{\mathrm{rot}}}$$

$$\mathcal{J}_{y,z}^{(\mathrm{wob})} = \frac{\langle J_{y,z} \rangle^{(\mathrm{wob})}}{\omega_{y,z}^{(\mathrm{wob})}}$$

$\mathcal{J}_{\mathrm{x}}$ is the largest

$$\hbar\omega_{\mathrm{wob}} \propto \sqrt{\left(\mathcal{J}_x - \mathcal{J}_y\right)\left(\mathcal{J}_x - \mathcal{J}_z\right)}$$

Conclusion

- We have studied nuclear wobbling motions by using Cranked mean field plus RPA on microscopic view point.
- Woods-Saxon mean field and symmetry restoring RPA interaction

Wobbling-like RPA solutions exist.

- The excitation energy is small compared with the experimental data (Effects of particle-rotation coupling ?)
- The B(E2) ratio of the in-band and out-of-band is small compared with the experimental data (Definitions of γ ?)

Microscopic justification of the rotor model

Effects of particle-rotation coupling

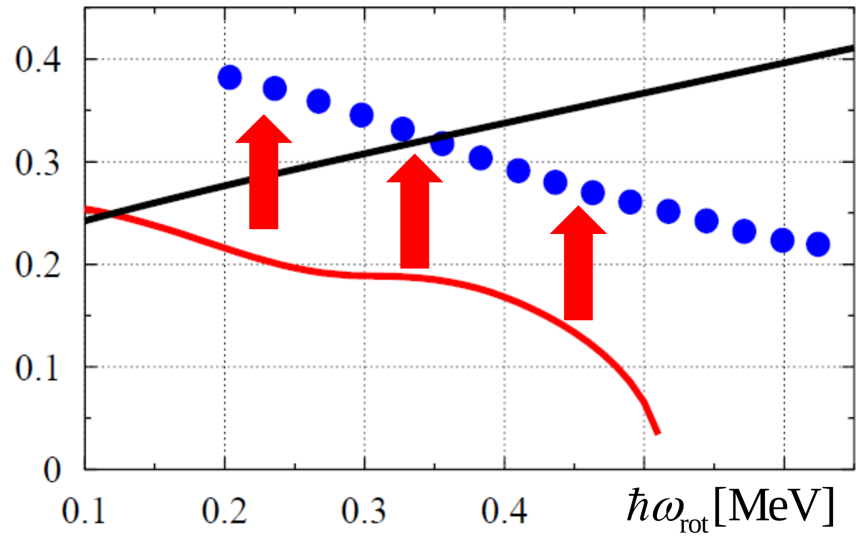
Particle plus Rotor Model   Hamamoto-Hagemann, Phys. Rev. C **67**,014319

$$H_{\rm rot} = \frac{(I_x - j)^2}{2\mathcal{J}_x} + \frac{I_y^2}{2\mathcal{J}_y} + \frac{I_z^2}{2\mathcal{J}_z} = \frac{I_x^2}{2\mathcal{J}_x} + \frac{I_y^2}{2\mathcal{J}_y} + \frac{I_z^2}{2\mathcal{J}_z} - \frac{I_x j}{\mathcal{J}_x} + \frac{j^2}{\mathcal{J}_x}$$

$$E(I+1, n=1) - E(I, n=0) = \frac{\hbar^2 I}{\mathcal{J}_x}\sqrt{\left(\frac{\mathcal{J}_x}{\mathcal{J}_y} - 1\right)\left(\frac{\mathcal{J}_x}{\mathcal{J}_z} - 1\right)} + \frac{j}{\mathcal{J}_x}$$

$$j \sim \frac{13}{2} \quad \mathcal{J}_x \sim 70\ [\mathrm{MeV}^{-1}\hbar^2]$$

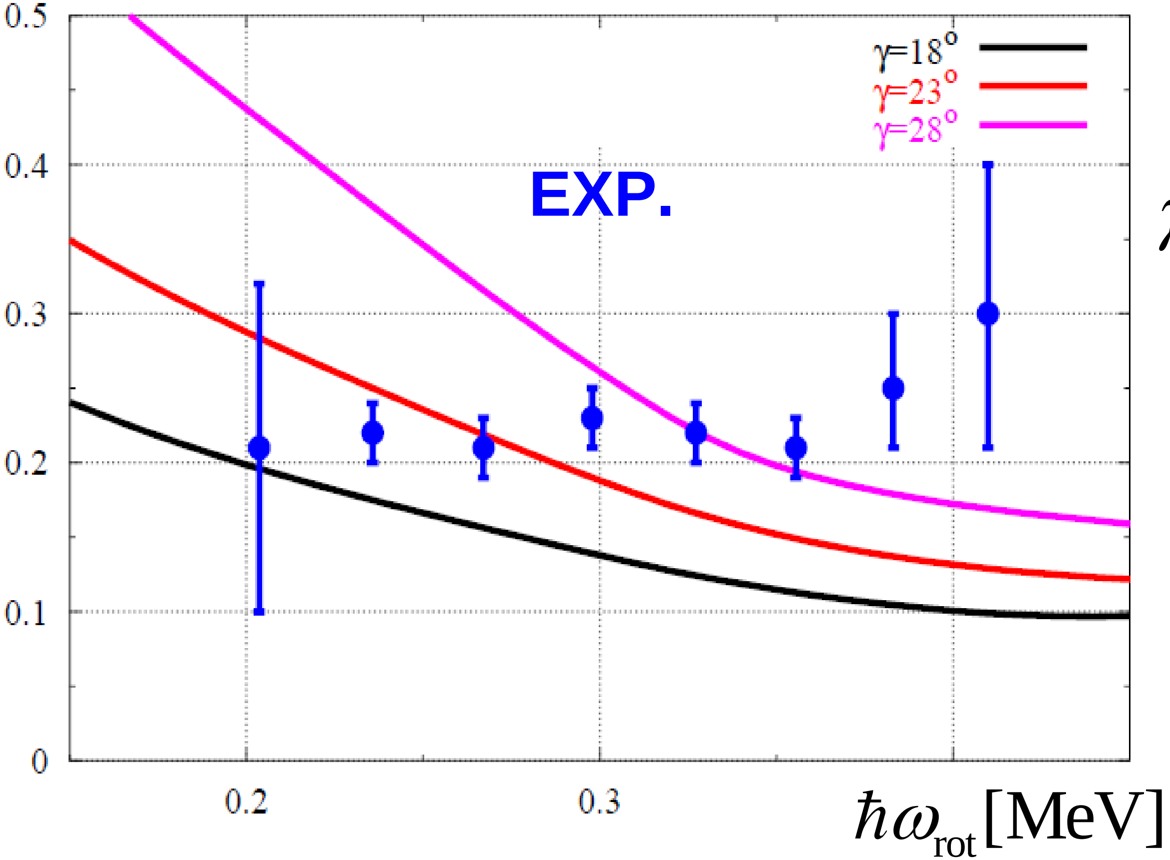
$$\frac{j}{\mathcal{J}_x} \sim 93\ [\mathrm{keV}]$$

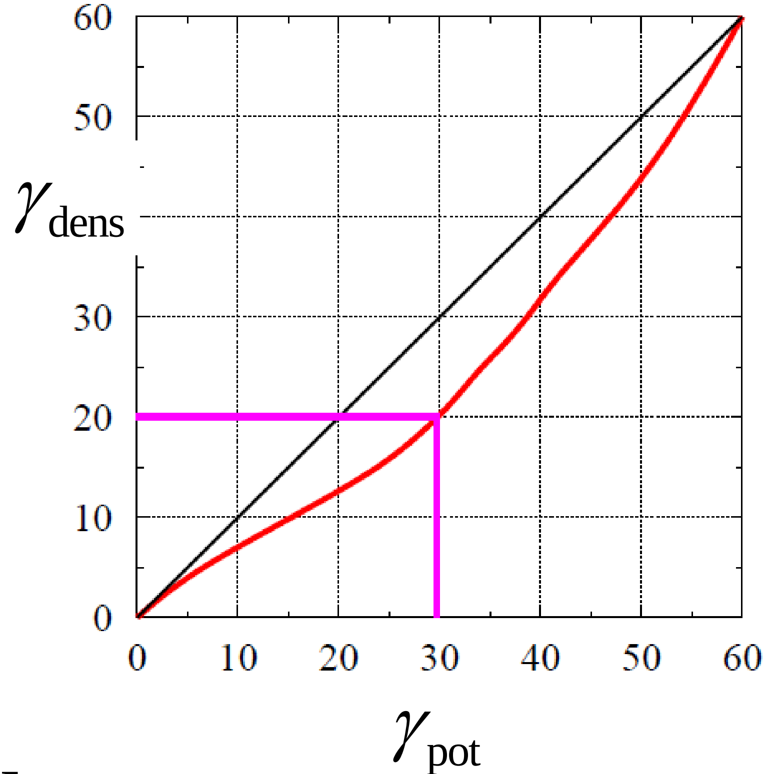


$^{163}_{71}\mathrm{Lu}$

Definitions of γ

$B(E2)_{\mathrm{out}} / B(E2)_{\mathrm{in}}$





in the rotor model

$$\gamma_{\mathrm{dens}} = -\frac{\sqrt{2}Q_{22}}{Q_{20}} = \tan^{-1}\frac{\sqrt{3}\langle y^2 - x^2\rangle}{\langle 2z^2 - x^2 - y^2\rangle}$$

if $\mathcal{J}_y = \mathcal{J}_z$,

$$B(E2)_{out}/B(E2)_{in} \propto \tan^2(\gamma + 30^o)$$

Conclusion

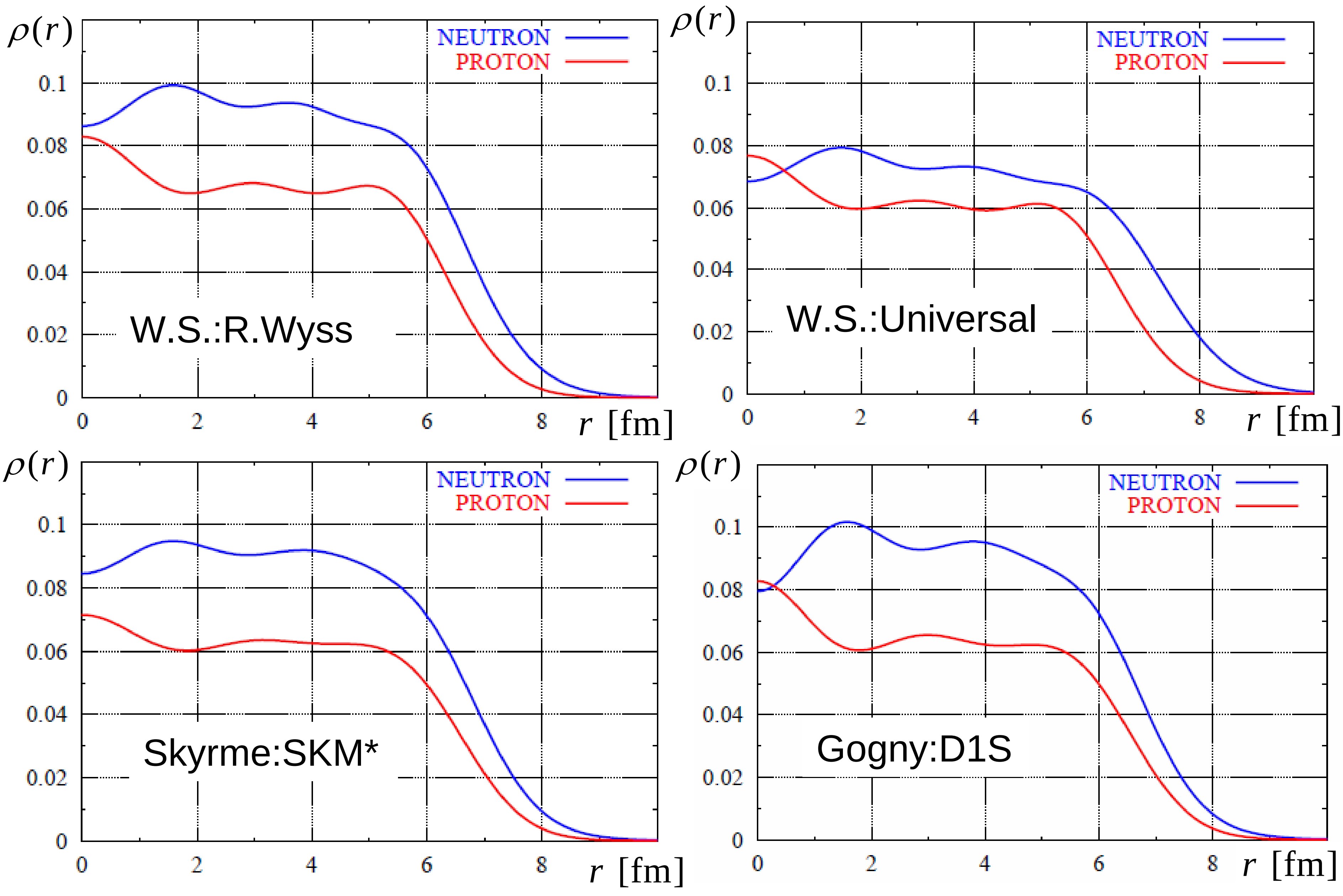
- We studied wobbling motion by using Cranked mean field plus RPA on microscopic view point.
- Woods-Saxon mean field and symmetry restoring RPA interaction

Wobbling-like RPA solutions exist.

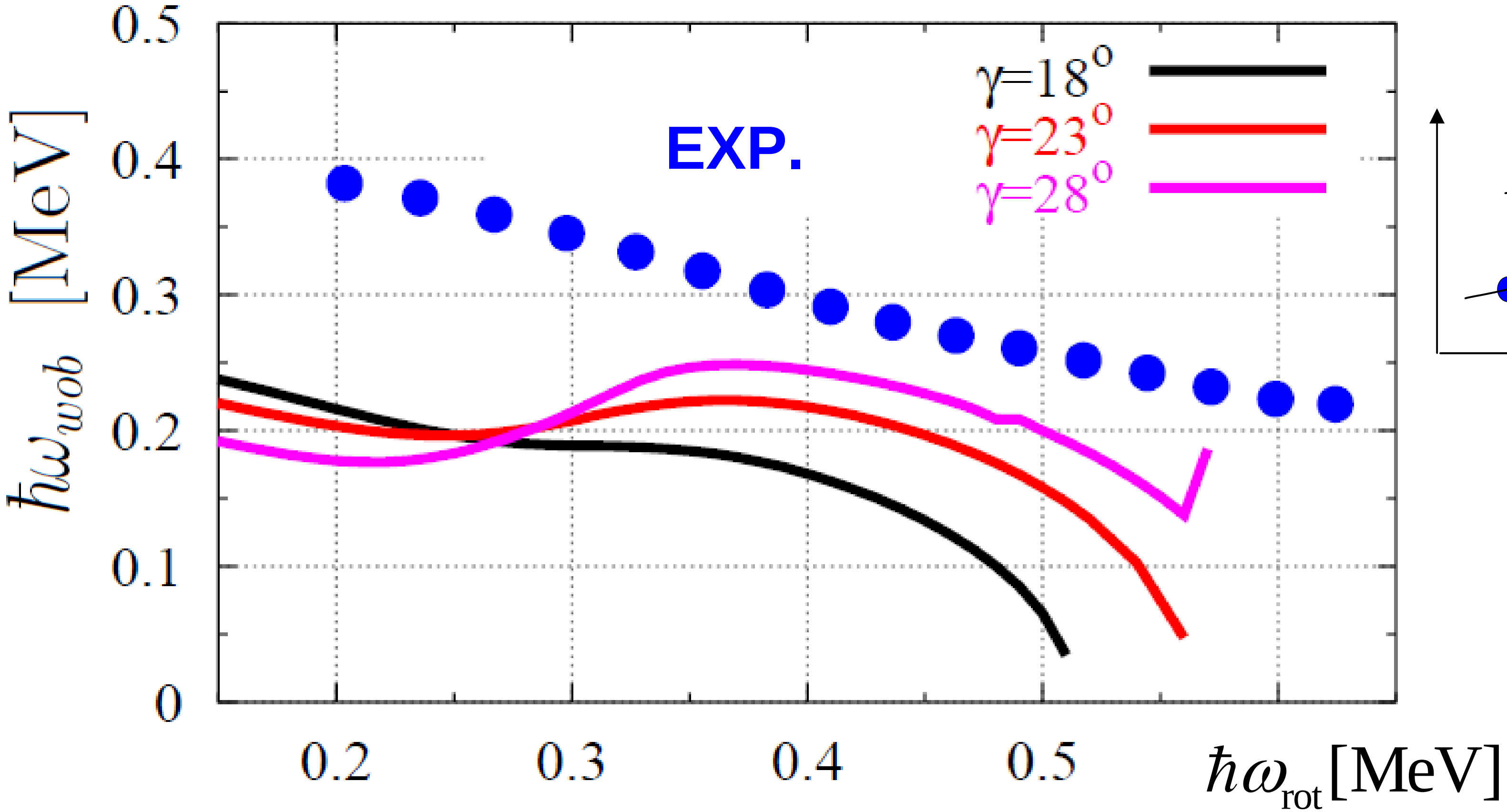
- The excitation energy is small compared with the experimental data (Effects of particle-rotation coupling ?)
- The B(E2) ratio of the in-band and out-band is small compared with experiments (Definitions of γ ?)

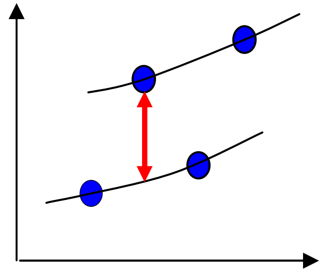
Microscopic justification of the rotor model

Density distribution $\rho(r)$ of $^{208}_{82}\mathrm{Pb}$



$^{163}_{71}$Lu Excitation energy of RPA-phonon



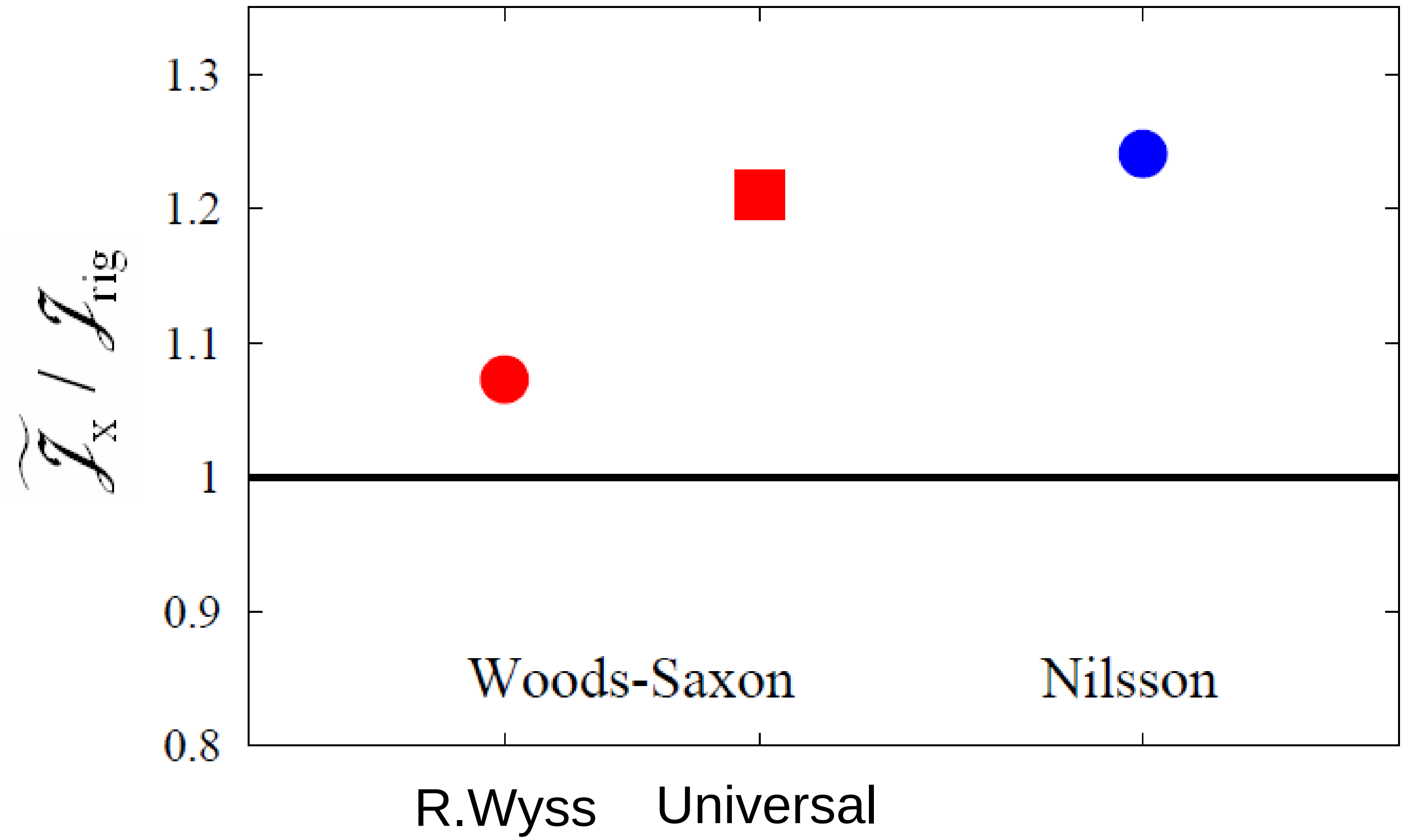


CAL. (RPA)

EXP. D. R. Jensen *et al*., Nucl. Phys. A **703** (2002) 3

$^{152}_{66}$Dy

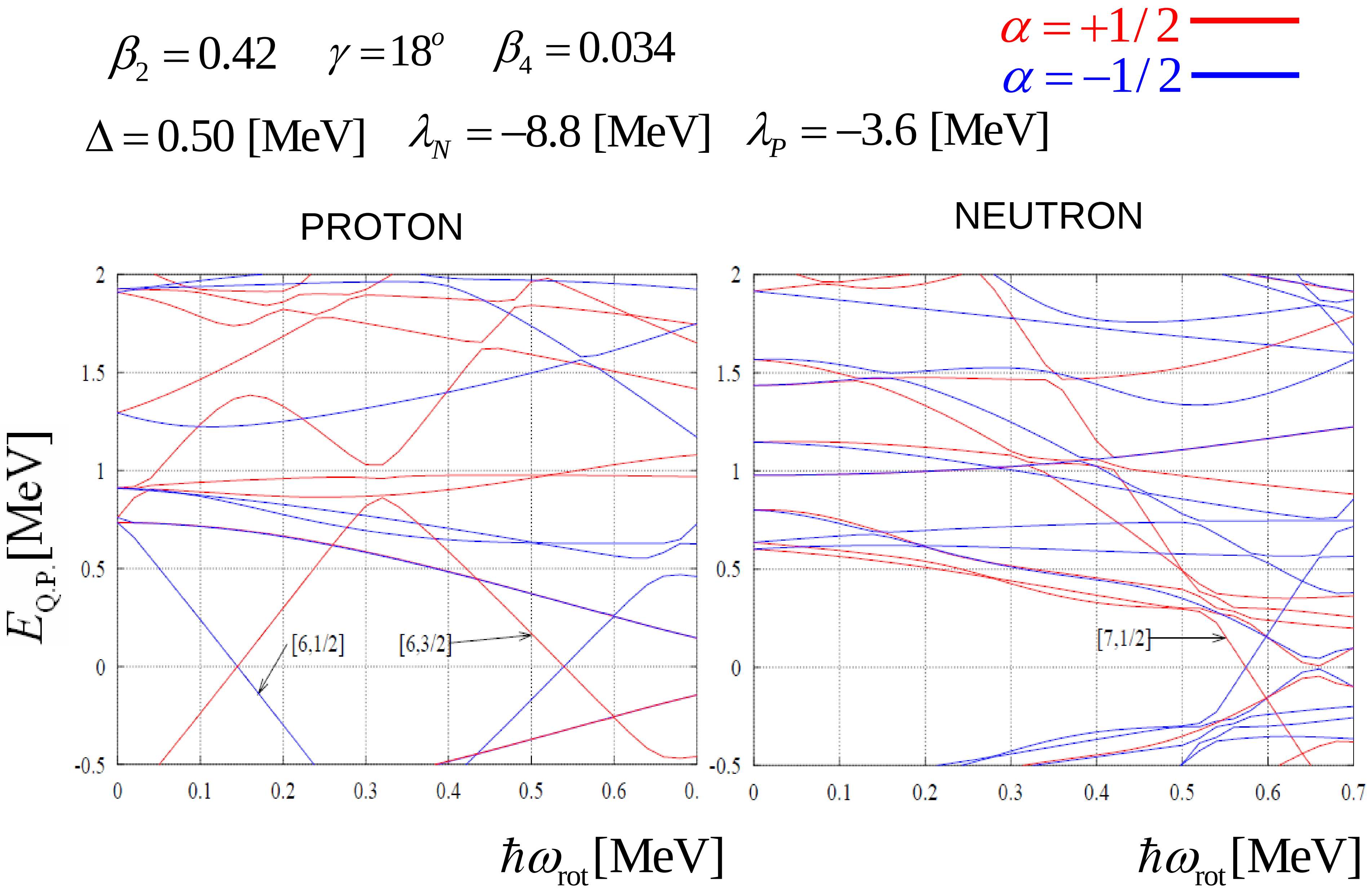
Woods-Saxon parameter set



${}^{162}_{70}$Yb Quasi-particle energies

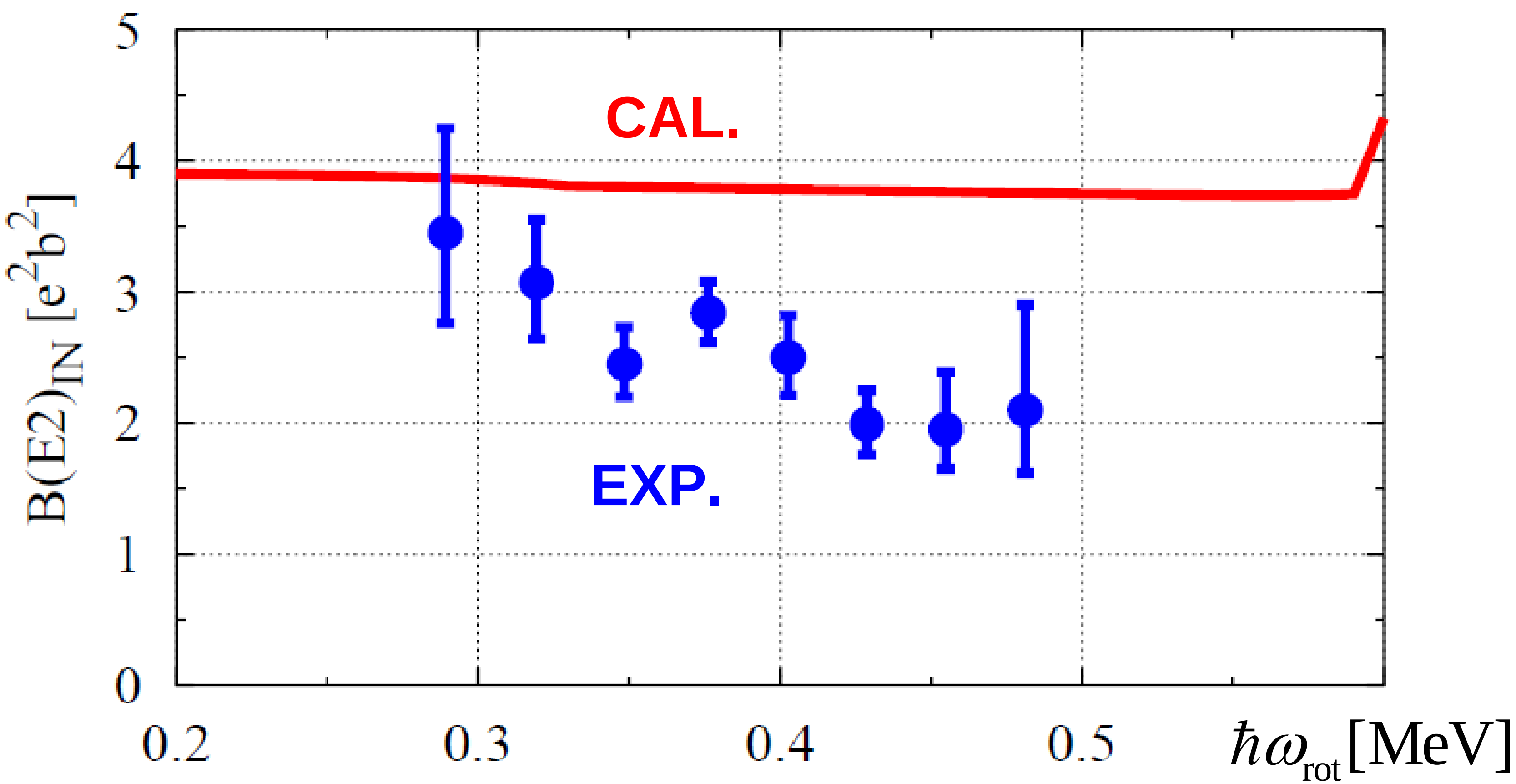
$\beta_2 = 0.42 \quad \gamma = 18^o \quad \beta_4 = 0.034$

$\Delta = 0.50$ [MeV] $\quad \lambda_N = -8.8$ [MeV] $\quad \lambda_P = -3.6$ [MeV]



$^{163}_{71}Lu$

In-band transitions

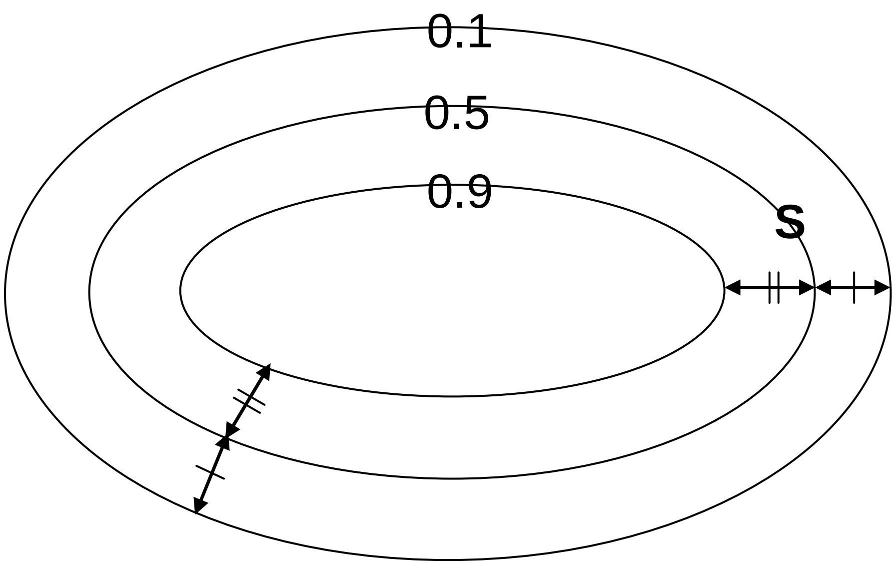


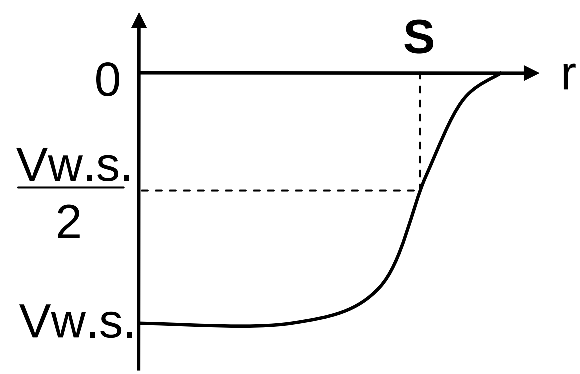
EXP. A. Görgen *et al.*, Phys. Rev. C **69**, 031301(R) (2004)

Woods-Saxon potential

$$h_{W.S.}(\mathbf{r}) = \frac{V_{W.S.}}{1 + \exp\left[dist_S(\mathbf{r})/a\right]}$$

$$dist_S(\mathbf{r}) = \mp \min_{\mathbf{r}' \in S} |\mathbf{r} - \mathbf{r}'| \quad \begin{cases} - & \mathbf{r}' \in V \\ + & \mathbf{r}' \notin V \end{cases}$$





- **LS potential**

$$h_{LS}(\mathbf{r}) = V_{LS}\frac{1}{i}\left[\sigma_{Pauli} \cdot \{(\nabla h_{W.S.}(\mathbf{r})) \times \nabla\}\right]$$

- **Coulomb potential**

$$h_{Coul.}(\mathbf{r}) = (Z-1)\, e^2 \rho_0 \iiint_V \frac{1}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

- **Pairing potential**

$$\hat{h}_{Pair.} = \Delta\left(\hat{P}^\dagger + \hat{P}\right) - \sum_\tau \lambda_\tau \hat{N}_\tau$$

$$\hat{P}^\dagger = \sum_i \hat{a}_i^\dagger \hat{a}_{\bar{i}}^\dagger$$

$$\hat{N}_\tau = \sum_{i\in\tau} \hat{a}_i^\dagger \hat{a}_i$$