Study of the tensor correlation using a mean-field-type model

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Introduction

- The correlations induced by the tensor force (tensor correlation) are important for structure of nuclei
- But the tensor force is not usually included in the mean-field-type calculation.
- We want to construct a mean-field-type model which can treat the tensor correlation and study the effect of the tensor correlation on structure of nuclei, which is probably different from those of the correlations by the central and LS forces.

$$S_{12} \propto \left[[\sigma_1 \times \sigma_2]^{(2)} \times Y_2(\hat{r}) \right]^{(2)}$$

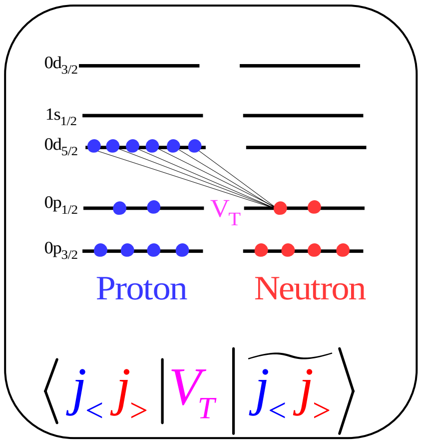
$$\Delta L = 2; \ \Delta S = 2$$

The correlation to be included

Hartree-Fock cal. (MV1(VC)+G3RS(VT, VLS))

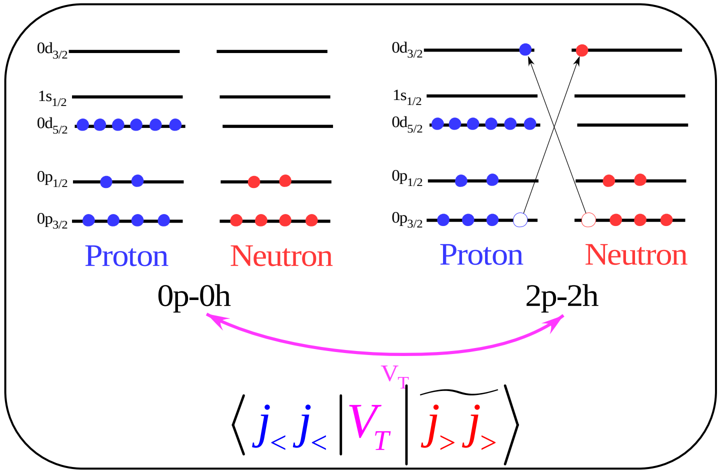
	E	K	V	VC	VT	VLS
14O	−89.1	199.0	−288.1	−333.2	1.1	−8.9
16O	−124.1	230.0	−354.1	−418.1	0.0	−0.9
22O	−156.2	354.2	−510.3	−579.6	1.8	−21.5
24O	−163.2	375.0	−538.3	−612.2	1.7	−20.5
28O	−176.4	424.4	−600.8	−691.0	0.1	−2.2

Single-particle (H-F) correlation



- In the simple HF calculation, the tensor correlation cannot be exploited.
- We need to include at least 2p-2h correlation to exploit the tensor correlation.
  ⇒**beyond mean field model**

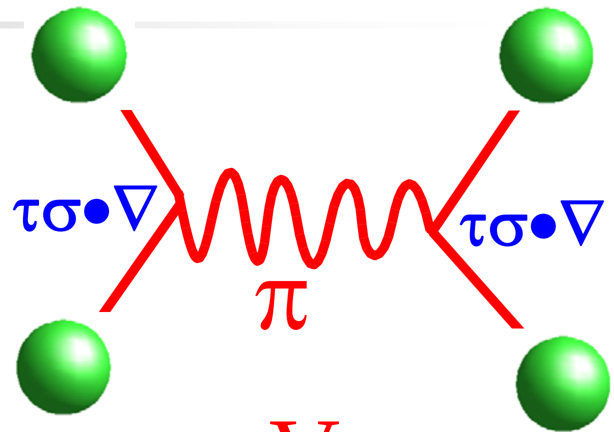
2p-2h correlation

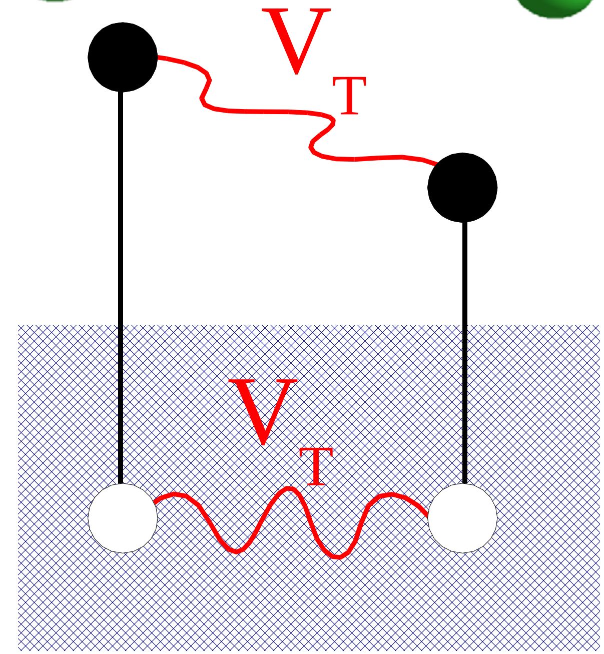


Charge- and parity-projected Hartree-Fock method

Sugimoto *et al.*, Nucl. Phys. A **740** (2004) 77
Ogawa *et al.*, PRC **73** (2006) 034301

Charge- and parity-symmetry breaking mean field method





- Tensor force is mediated by the pion.
- Pseudo scalar ($\sigma\cdot\nabla$)
  - To exploit the pseudo scalar character of the pion, we introduce parity-mixed single particle state. (over-shell correlation)
- Isovector (τ)
  - To exploit the isovector character of the pion, we introduce charge-mixed single particle state.
- Projection
  - Because the total wave function made from such parity- and charge-mixed single particle states does not have good parity and a definite charge number. We need to perform the parity and charge projections.

Toki *et al.*, Prog. Theor. Phys. **108** (2002) 903.
Sugimoto *et al.*, Nucl. Phys. A **740** (2004) 77.
Ogawa *et al.*, Prog. Thoer. Phys. **111** (2004) 75.
cf. Bleuler, *Proceeding of the international school of physics "Enrico Fermi"* **36** (1966) 464.

Schematic example (^{4}He;A=4,Z=2)

$$(\pi 0s)^2(\nu 0s)^2 \Rightarrow \left(\alpha(\pi 0s)+\beta(\nu 0s)+\gamma(\pi\widetilde{0p})+\delta(\nu\widetilde{0p})\right)^4$$

simple $(0s)^4$ — mixed wave function

$$=\underbrace{6\alpha^2\beta^2(\pi 0s)^2(\nu 0s)^2}_{\text{0p-0h}}$$

$$+\underbrace{6\alpha^2\delta^2(\pi 0s)^2(\nu\widetilde{0p})^2+6\beta^2\gamma^2(\nu 0s)^2(\pi\widetilde{0p})^2+24\alpha\beta\gamma\delta(\pi 0s)(\nu 0s)(\pi\widetilde{0p})(\nu\widetilde{0p})}_{\text{2p-2h}}$$

$$+\underbrace{6\gamma^2\delta^2(\pi\widetilde{0p})^2(\nu\widetilde{0p})^2}_{\text{4p-4h}}$$

^{4}He, 0^+

$$+\underbrace{12\alpha^2\beta\delta(\pi 0s)^2(\nu 0s)(\nu\widetilde{0p})+12\alpha\beta^2\gamma(\pi 0s)(\nu 0s)^2(\pi\widetilde{0p})}_{\text{1p-1h}}$$

$$+\underbrace{12\beta\gamma^2\delta(\nu 0s)(\pi\widetilde{0p})(\nu\widetilde{0p})+12\alpha\gamma\delta^2(\pi 0s)(\pi\widetilde{0p})(\nu\widetilde{0p})^2}_{\text{3p-3h}}$$

^{4}He, 0^-

+(other 160 terms (4n, ^{4}H, ^{4}Li, ^{4}Be))

By combining <u>the charge and parity mixings and projections</u>, we can obtain a wave function which includes <u>the correlations induced by the tensor force.</u>

Symmetry projected Hartree-Fock method

1. We take a Slater determinant made from single particle wave functions with parity and charge mixing as an intrinsic wave function.
2. We perform the parity and charge projections on the mixed parity and charge number wave function.
3. Taking a variation with the projected wave function, the charge-and parity-projected Hartree-Fock equation is obtained.

1. Intrinsic wave function:

$$\Phi^{\mathrm{intr}} = \frac{1}{\sqrt{A!}} \mathcal{A} \prod_{\alpha jm} \psi^{\mathrm{mix}}_{\alpha jm}$$

2. Projected wave function:

$$\Psi^{(\mathrm{Z};\pm)} = \mathcal{P}^{c}(Z)\mathcal{P}^{p}(\pm)\Phi^{\mathrm{intr}}$$

$$= \underbrace{\frac{1}{2\pi}\int_0^{2\pi} d\theta e^{i((1+\tau_3)/2-Z)\theta}}_{\text{charge projection}} \underbrace{\frac{1\pm\hat{P}}{2}}_{\text{parity projection}} \frac{1}{\sqrt{A!}} \mathcal{A} \prod_{\alpha jm} \psi^{\mathrm{mix}}_{\alpha jm}$$

3. Variation for single particle wave functions:

$$\frac{\delta}{\delta\psi^{\mathrm{mix}\,\dagger}_{\alpha jm}} \frac{\langle \Psi^{\mathrm{intr}} | H | \Psi^{(Z;\pm)} \rangle}{\langle \Psi^{\mathrm{intr}} | \Psi^{(Z;\pm)} \rangle} = 0 \Rightarrow$$ charge- and parity-projected HF equation

The method beyond mean filed model !

Charge- and parity- projected Hartree- Fock equation

$$
\begin{aligned}
\frac{1}{4\pi}\int_0^{2\pi} d\theta e^{-iZ\theta}\Big[n^{(0)} \Big\{ & \hat{t}(x_a)\tilde{\psi}_{\alpha_a}(x_a,\theta) + \sum_{b=1}^{A}\left\langle \psi_{\alpha_b} \middle| \hat{v}(x_{a1}) \middle| \tilde{\psi}_{\alpha_b}(\theta)\right\rangle_1 \tilde{\psi}_{\alpha_a}(x_a,\theta) - \sum_{b=1}^{A}\left\langle \psi_{\alpha_b} \middle| \hat{v}(x_{a1}) \middle| \tilde{\psi}_{\alpha_a}(\theta)\right\rangle_1 \tilde{\psi}_{\alpha_b}(x_a,\theta) \\
& -\left(E^{(\pm;Z)} - E^{(0)}(\theta)\right)\tilde{\psi}_{\alpha_a}(x_a,\theta) - \sum_{b=1}^{A}\eta_{ba}^{(0)}(\theta)\tilde{\psi}_{\alpha_b}(x_a,\theta) \Big\} \\
\pm n^{(P)} \Big\{ & \hat{t}(x_a)\tilde{\psi}^{(P)}_{\alpha_a}(x_a,\theta) + \sum_{b=1}^{A}\left\langle \psi_{\alpha_b} \middle| \hat{v}(x_{a1}) \middle| \tilde{\psi}^{(P)}_{\alpha_b}(\theta)\right\rangle_1 \tilde{\psi}^{(P)}_{\alpha_a}(x_a,\theta) - \sum_{b=1}^{A}\left\langle \psi_{\alpha_b} \middle| \hat{v}(x_{a1}) \middle| \tilde{\psi}^{(P)}_{\alpha_a}(\theta)\right\rangle_1 \tilde{\psi}^{(P)}_{\alpha_b}(x_a,\theta) \\
& -\left(E^{(\pm;Z)} - E^{(P)}(\theta)\right)\tilde{\psi}^{(P)}_{\alpha_a}(x_a,\theta) - \sum_{b=1}^{A}\eta_{ba}^{(P)}(\theta)\tilde{\psi}^{(P)}_{\alpha_b}(x_a,\theta) \Big\}\Big] = n^{(\pm;Z)}\sum_{b=1}^{A}\varepsilon_{ab}\psi_{\alpha_b}(x_a)
\end{aligned}
$$

H-F part

$$\hat{P} \equiv \prod_{a=1}^{A}\hat{p}_a,\ \hat{C}(\theta) \equiv e^{i\hat{Z}\theta},\ B^{(0)}(\theta)_{ab} \equiv \left\langle \psi_{\alpha_a} \middle| e^{i\theta(1+\tau^3)/2}\psi_{\alpha_b}\right\rangle,\ B^{(P)}(\theta)_{ab} \equiv \left\langle \psi_{\alpha_a} \middle| \hat{p}e^{i\theta(1+\tau^3)/2}\psi_{\alpha_b}(\theta)\right\rangle$$

$$n^{(0)}(\theta) \equiv \det B^{(0)}(\theta),\ n^{(P)}(\theta) \equiv \det B^{(P)}(\theta),\ E^{(0)}(\theta) \equiv \left\langle \Phi^{\mathrm{intr}} \middle| \hat{H}\hat{C}(\theta) \middle| \Phi^{\mathrm{intr}}\right\rangle,\ E^{(P)}(\theta) \equiv \left\langle \Phi^{\mathrm{intr}} \middle| \hat{H}\hat{P}\hat{C}(\theta) \middle| \Phi^{\mathrm{intr}}\right\rangle$$

$$\tilde{\psi}_{\alpha_a}(x;\theta) \equiv \sum_{b=1}^{A} e^{i\theta(1+\tau^3)/2}\psi_{\alpha_b}(x)\left(B^{(0)}(\theta)^{-1}\right)_{ba},\quad \eta_{ab}^{(0)} \equiv \left\langle \psi_{\alpha_a} \middle| \hat{t} \middle| \tilde{\psi}_{\alpha_b}(\theta)\right\rangle + \sum_{c=1}^{A}\left\langle \psi_{\alpha_a}\psi_{\alpha_c} \middle| \hat{v} \middle| \tilde{\psi}_{\alpha_b}(\theta)\tilde{\psi}_{\alpha_c}(\theta) - \tilde{\psi}_{\alpha_c}(\theta)\tilde{\psi}_{\alpha_b}(\theta)\right\rangle$$

$$\tilde{\psi}^{(P)}_{\alpha_a}(x;\theta) \equiv \sum_{b=1}^{A} \hat{p}e^{i\theta(1+\tau^3)/2}\psi_{\alpha_b}(x)\left(B^{(P)}(\theta)^{-1}\right)_{ba},\quad \eta_{ab}^{(P)} \equiv \left\langle \psi_{\alpha_a} \middle| \hat{t} \middle| \tilde{\psi}^{(P)}_{\alpha_a}(\theta)\right\rangle + \sum_{c=1}^{A}\left\langle \psi_{\alpha_a}\psi_{\alpha_c} \middle| \hat{v} \middle| \tilde{\psi}^{(P)}_{\alpha_b}(\theta)\tilde{\psi}^{(P)}_{\alpha_c}(\theta) - \tilde{\psi}^{(P)}_{\alpha_c}(\theta)\tilde{\psi}^{(P)}_{\alpha_b}(\theta)\right\rangle$$

Sugimoto *et al.*, Nucl. Phys. A **740** (2004) 77

cf. PPHF equation (Takami *et al.*, Prog. Theor. Phys. **96** (1996) 407)

We solve the CPPHF equation selfconsistently with the gradient method.

The effect of the mixings and the projections (^{4}He)

m=0.6,b=h=0.0

unit: MeV	Simple H-F				PPHF		CPPHF	
X_T	original Volkov No.1		1.5		1.5		1.5	
X_{TE}			0.81		0.81		0.81	
$\langle V_C^0 \rangle$	-19.17	-76.67	-12.94	-56.85	-14.36	-61.31	-15.92	-64.75
$\langle \sigma\cdot\sigma V_C^S \rangle$	-11.50		-11.33		-12.08		-11.67	
$\langle \tau\cdot\tau V_C^T \rangle$	-11.50		-6.24		-6.59		-7.43	
$\langle \sigma\cdot\sigma\tau\cdot\tau V_C^{ST} \rangle$	-34.50		-26.35		-28.28		-29.73	
$\langle V_T^0 \rangle$	0.00	0.00	0.00	0.00	-0.19	-10.91	-0.43	-30.59
$\langle \tau\cdot\tau V_T^T \rangle$	0.00		0.00		-10.72		-30.16	
$\langle V_{LS}^0 \rangle$	0.00	0.00	0.00	0.00	0.56	0.67	1.78	1.91
$\langle \tau\cdot\tau V_{LS}^T \rangle$	0.00		0.00		0.11		0.13	
$\langle V_{coul} \rangle$	0.83		0.76		0.78		0.85	
PE sum	-75.84		-56.10		-70.76		-92.58	
KE	48.54		39.98		49.67		64.39	
E_{total}	-27.30		-16.12		-21.09		-28.19	
rms R_m (fm)	1.48		1.63		1.50		1.37	
P(-)	0.00		0.00		0.08		0.16	

Sugimoto *et al.*, Nucl. Phys. A **740** (2004) 77.

$$\overbrace{\tau_1^0\tau_2^0}^{\text{PPHF}} \Rightarrow \overbrace{\underbrace{\tau_1^0\tau_2^0}_{\pi^0} + \underbrace{\tau_1^+\tau_2^- + \tau_1^-\tau_2^+}_{\pi^+,\pi^-}}^{\text{CPPHF}}$$

1. By performing the parity projection, we can obtain the correlation induced by the tensor force.
2. Performing the charge projection further, we get much more contribution from the tensor force. It becomes three times larger than the PPHF case.
3. Projection before variation is necessary to get the tensor correlation.

Application to O isotopes

$$H=\sum_{i=1}^{A}\left(-\frac{\hbar^2}{2M}\Delta_i\right)+\underbrace{V_{\mathrm{C}}+V_{\mathrm{3B}}}_{\mathrm{MV1}}+\underbrace{V_{\mathrm{T}}+V_{\mathrm{LS}}}_{\mathrm{G3RS}}+V_{\mathrm{Coul}}-E_{\mathrm{CM}}$$

- We calculated sub-shell closed oxygen isotopes, ^{14}O, ^{16}O, ^{22}O, ^{24}O and ^{28}O.
- The spherical symmetry is assumed.
  - Only the couplings between the same j states ($s_{1/2}$ and $p_{1/2}$, $p_{3/2}$ and $d_{3/2}$) are included.
- NN potential
  - MV1 (PTP **64** (1980) 1608) for the central part.
  - G3RS (PTP **39** (1968) 91) for the tensor and LS forces.
- The attraction part of the ^{3}E part of the central force and the 3-body force are adjusted to reproduce the biding energy and the charge radius of ^{16}O.
- The strength of the $\tau\cdot\tau$ part of the tensor force is changed by multiplying a numerical factor, x_T to take into account correlations like <s1/2 s1/2|V_T|s1/2 d3/2> which is not included in the present calculation, effectively. (only <j1 j2|V_T|j1 j2> type correlations are included in the CPPHF method.)
- The strength of the LS force is multiplied by 2.

Single particle wave function (spherical case)

Single particle wave function with charge and parity mixing

$$\psi^{\text{mix}}_{\alpha jm}(\mathbf{r}) = \underbrace{\phi_{\alpha lj\pi}(r)\mathcal{Y}_{ljm}(\Omega)\zeta(\pi) + \phi_{\alpha lj\nu}(r)\mathcal{Y}_{ljm}(\Omega)\zeta(\nu)}_{\text{positve parity}}$$

$$+\underbrace{\phi_{\alpha\bar{l}j\pi}(r)\mathcal{Y}_{\bar{l}jm}(\Omega)\zeta(\pi) + \phi_{\alpha\bar{l}j\nu}(r)\mathcal{Y}_{\bar{l}jm}(\Omega)\zeta(\nu)}_{\text{negative parity}}$$

$\mathcal{Y}_{ljm} = \left[Y_{lm} \times \chi_{1/2}\right]^{(j)}_{m}$: eigenfunction of the total spin, $\mathbf{l}+\mathbf{s}$

$\mathcal{Y}_{\bar{l}jm} = \underbrace{\boldsymbol{\sigma}\bullet\hat{\mathbf{r}}}_{0^-}\mathcal{Y}_{ljm} \Rightarrow \mathcal{Y}_{ljm}$ and $\mathcal{Y}_{\bar{l}jm}$ have the same j but different parities.

Gaussian basis expansion:

$$\phi_{\alpha lj}(r) = \sum_{i=1}^{n} C^{i}_{\alpha lj} N_l(a_i) r^l \exp\left(-\left(r/a_i\right)^2\right),\ (a_i = a_1\nu^{i-1}(i=1,\dots n))$$

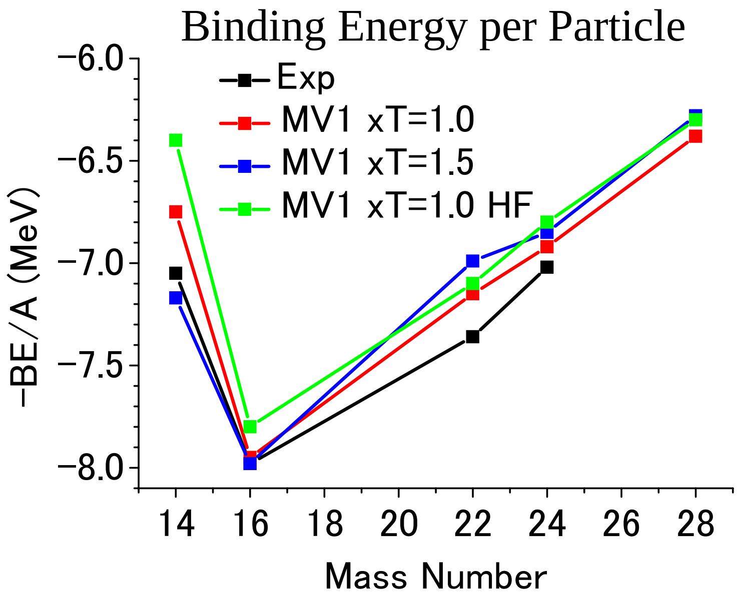
We expand each ϕ by the Gaussian basis with a geometric-series widths.

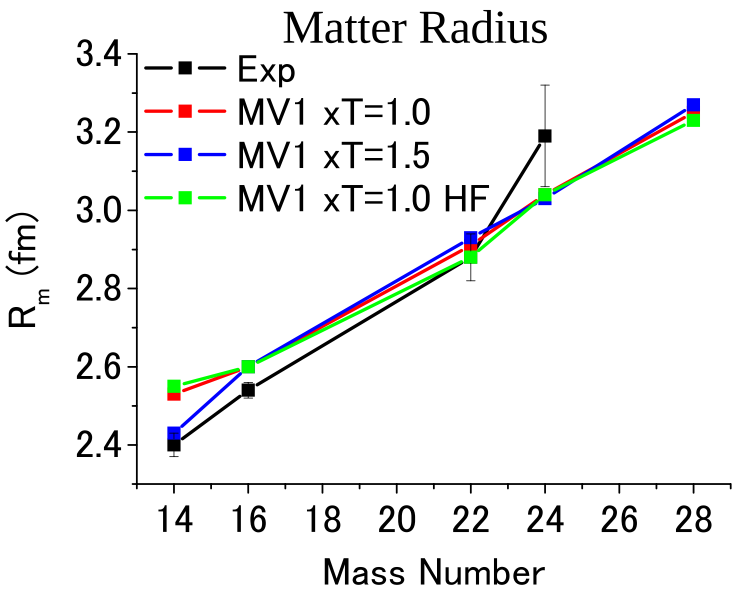
Results for ^{16}O

	x_T	E	K	V	V_T	V_{LS}
HF	1.0	-124.1	230.0	-354.1	0.0	-0.9
CPPHF	1.0	-127.1	237.1	-364.24	-11.7	-1.0
CPPHF	1.5	-127.6	253.9	-381.6	-38.3	-1.0

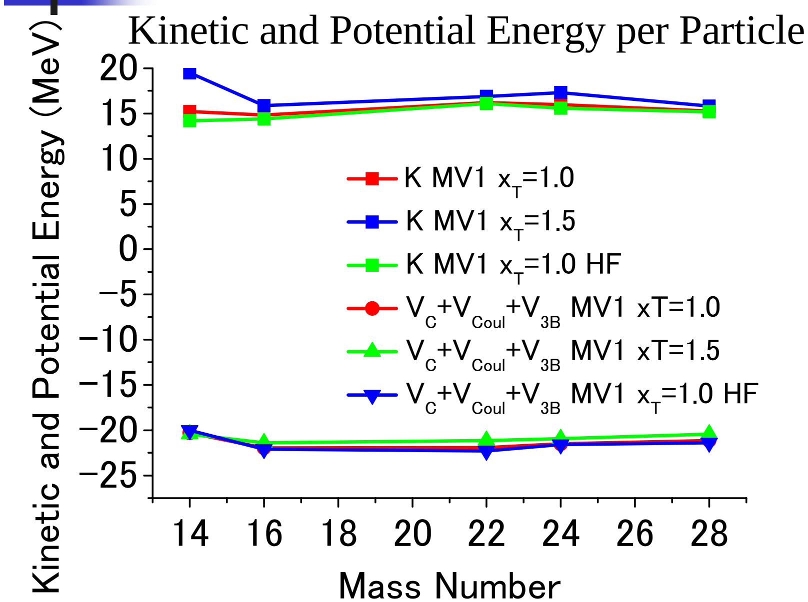
- By performing the parity and charge projection the potential energy from the tensor force becomes sizable value.

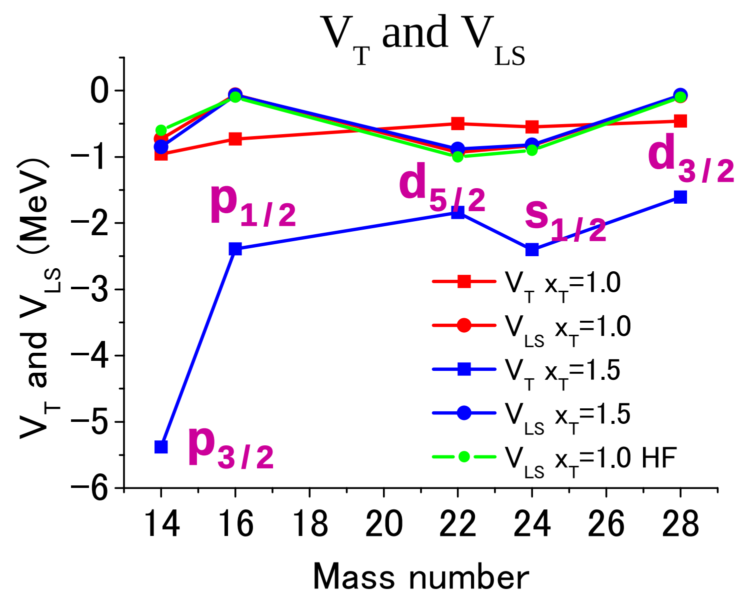
BE and R_m


Binding Energy per Particle
−6.0
−6.5
−7.0
−7.5
−8.0
−BE/A (MeV)
Exp
MV1 xT=1.0
MV1 xT=1.5
MV1 xT=1.0 HF
14 16 18 20 22 24 26 28
Mass Number


Matter Radius
3.4
3.2
3.0
2.8
2.6
2.4
R_m (fm)
Exp
MV1 xT=1.0
MV1 xT=1.5
MV1 xT=1.0 HF
14 16 18 20 22 24 26 28
Mass Number

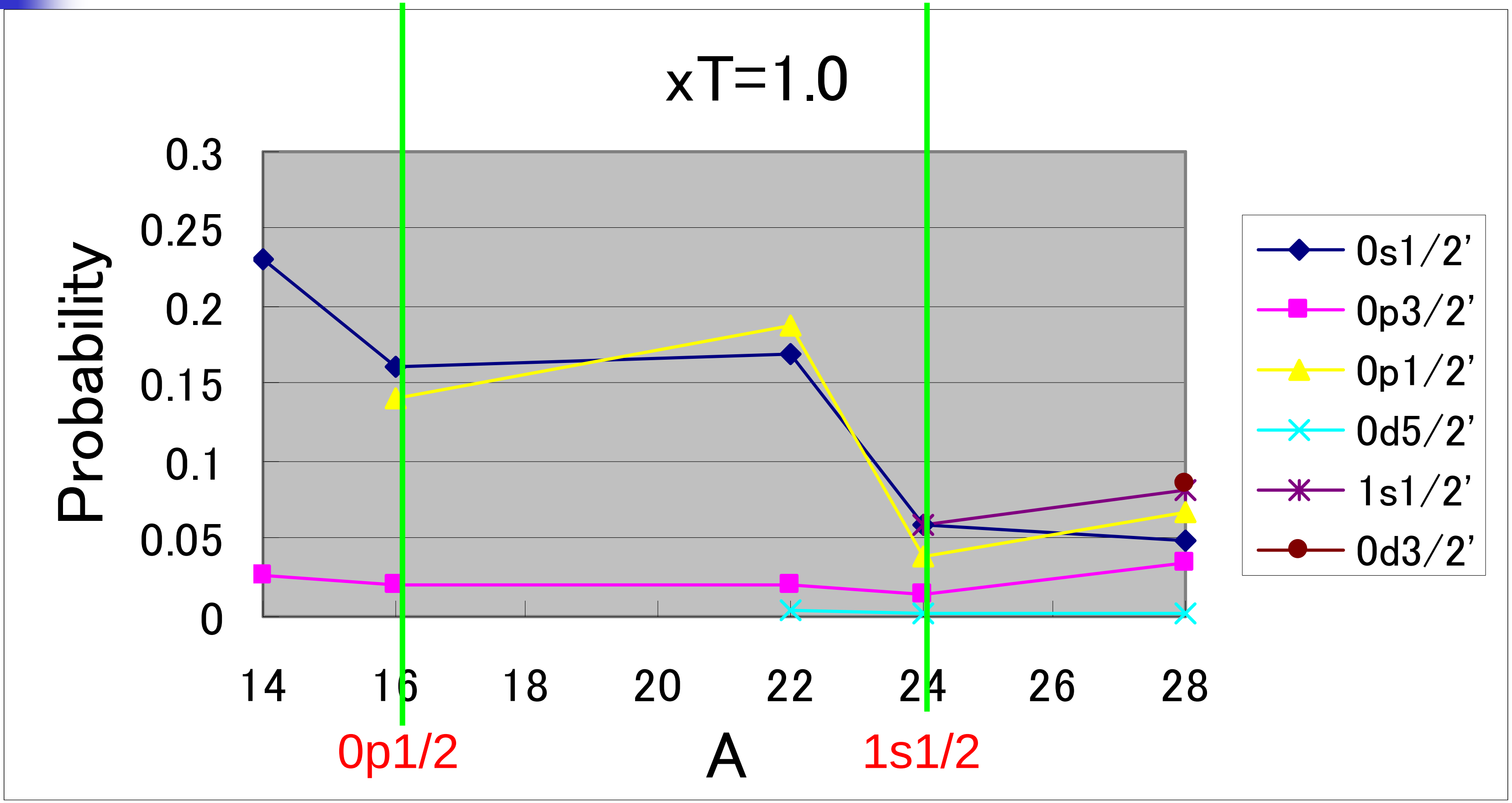
V and K per particle





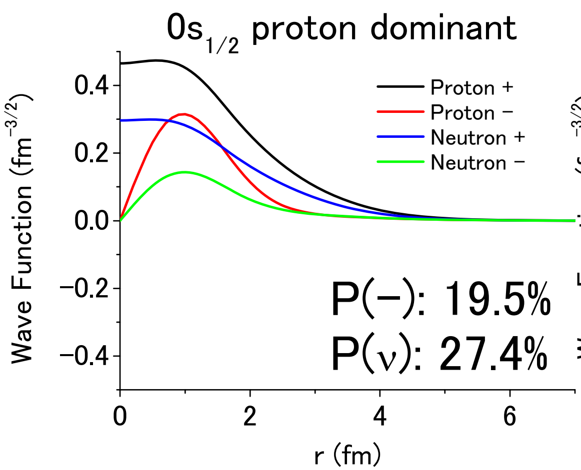
- The potential energy of the tensor force behaves differently from those of the central and LS forces.
- It indicates that the tensor force affects the shell structure differently from the central and LS forces.

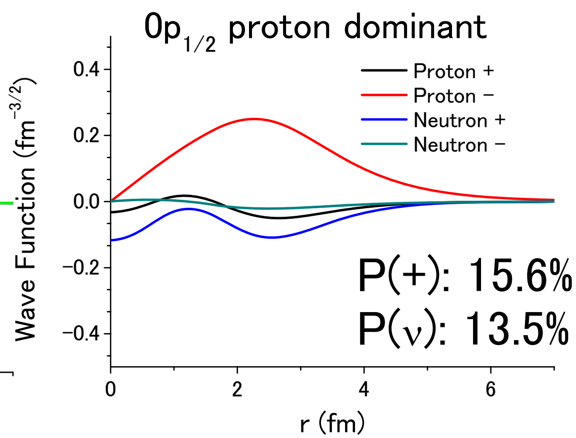
Mixing of the opposite parity components

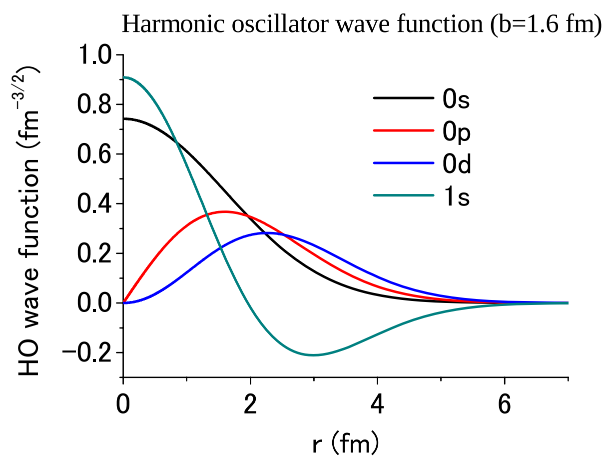


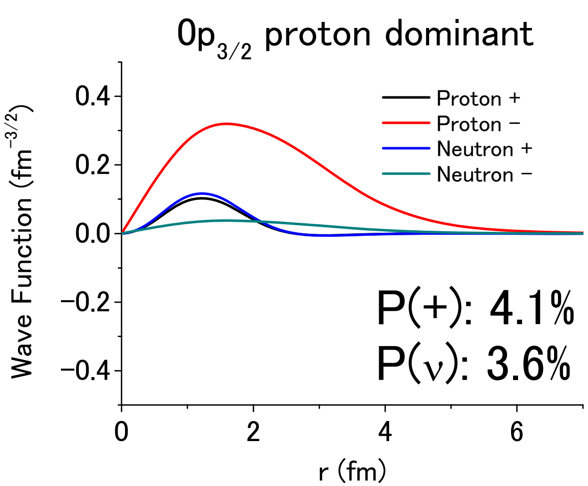
- j=1/2 states are important for the tensor correlation.

Wave function (^{16}O, x_T=1.5)

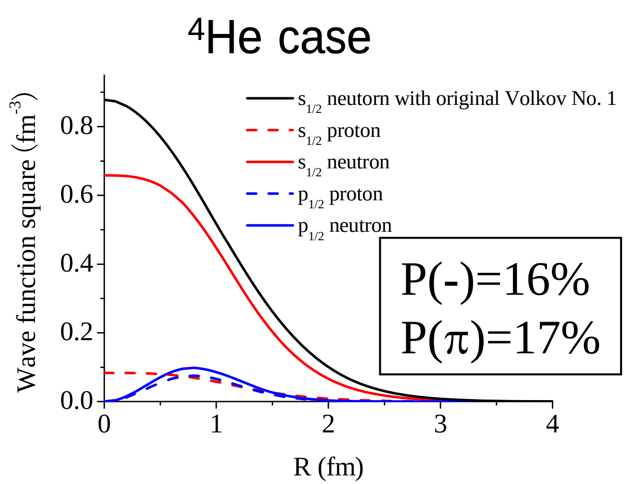




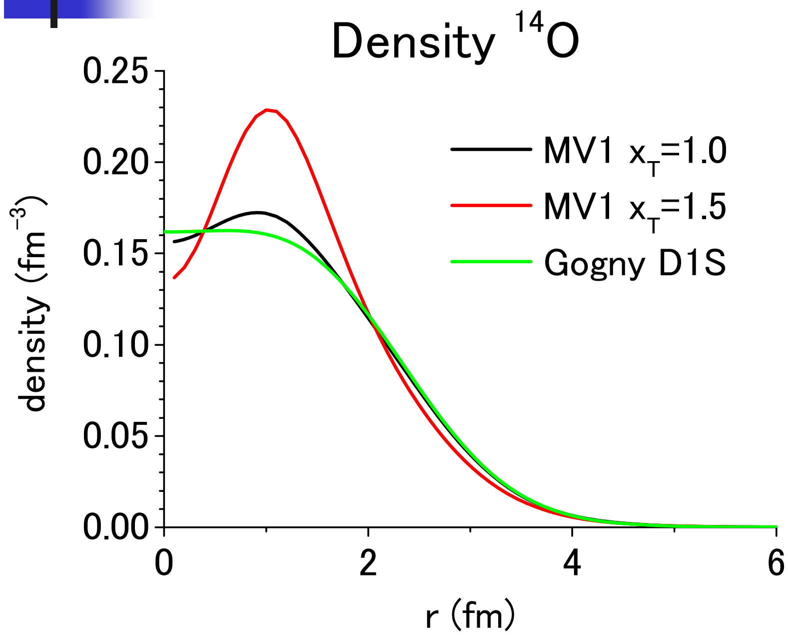




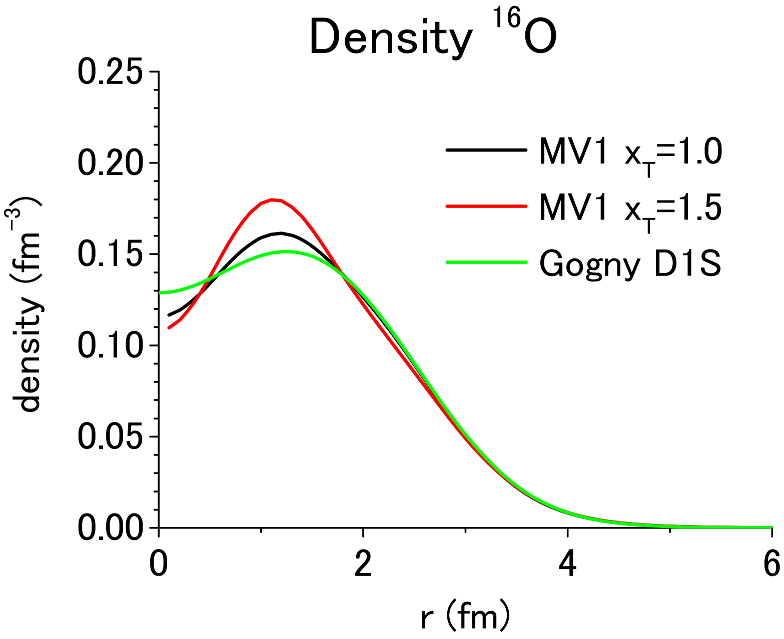
Opposite parity components mixed by the tensor force have narrow widths. It suggests that the tensor correlation needs high-momentum components.



Density

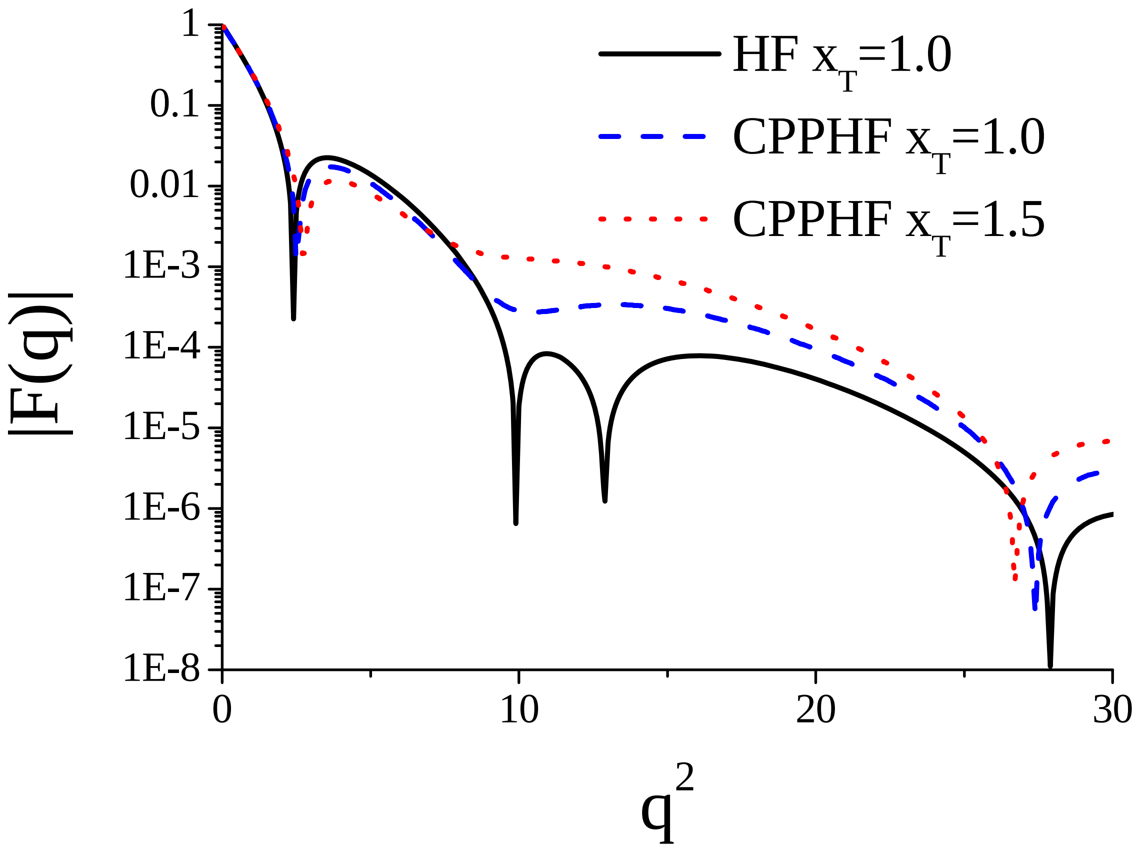


Density ^{14}O
0.25
0.20
0.15
0.10
0.05
0.00
density (fm^{-3})
0
2
4
6
r (fm)
MV1 x_T=1.0
MV1 x_T=1.5
Gogny D1S



Density ^{16}O
0.25
0.20
0.15
0.10
0.05
0.00
density (fm^{-3})
0
2
4
6
r (fm)
MV1 x_T=1.0
MV1 x_T=1.5
Gogny D1S

Charge Formfactor ^{16}O



- The tensor correlation induce higher momentum component.

Summary

- We make a mean-field model which can treat the tensor correlation by mixing parities and charges in single-particle states. (the CPPHF method)
- The opposite parity components induced by the tensor force is compact in size. (high-momentum component)
- The CPPHF calculation with the spherical symmetry shows that in the oxygen isotopes j=1/2 states are important for the tensor correlation.