One-loop corrections as the origin of spontaneous chiral symmetry breaking in the massless chiral sigma model

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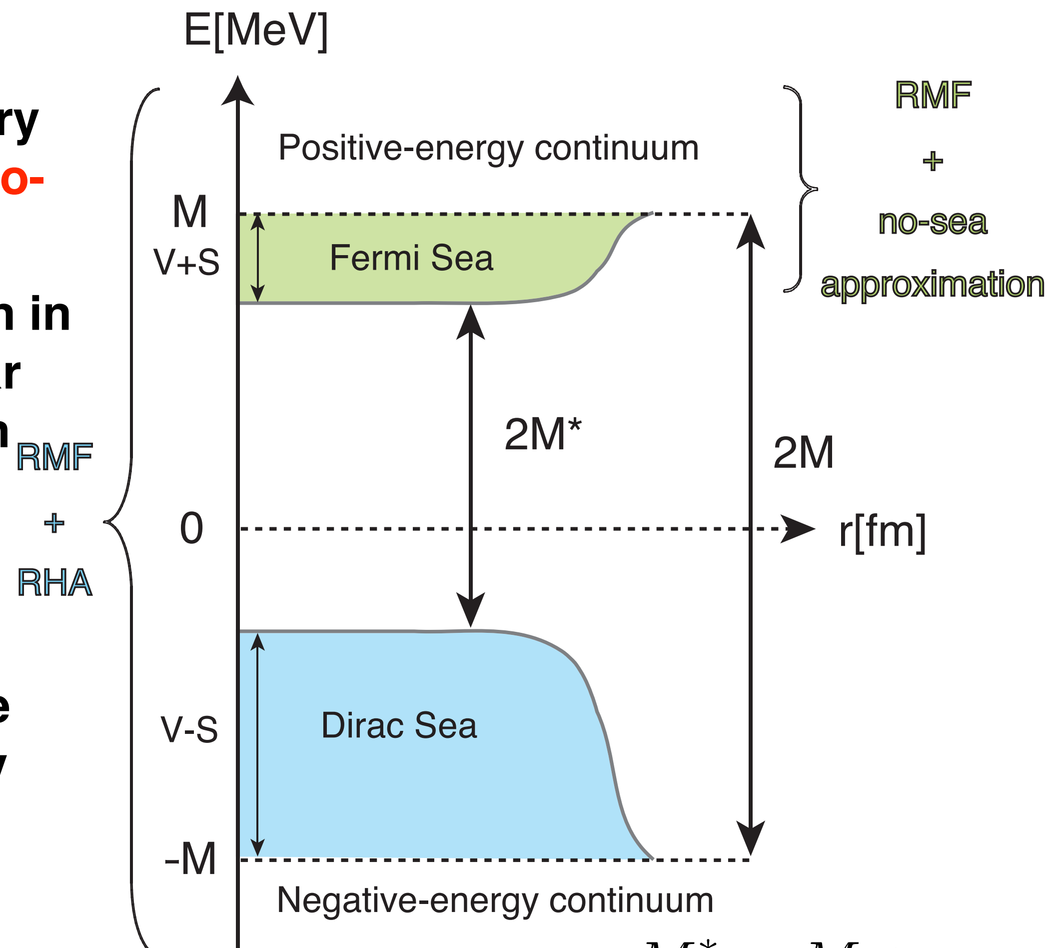
Contents

- Introduction & motivation
  - Pion and vacuum in the RMF theory
  - Chiral sigma model and the problem of instability
- Radiative corrections of boson as the origin of SSB (Coleman- Weinberg renormalization scheme)
- One-loop corrections as the origin of SCSB
  - The dependence on renormalization scale and the relationship between renormalization and symmetry
  - Naturalness and the naive dimensional analysis (NDA)
- Density distribution of ^{16}O
- Summary

Introduction

- **Since Walecka model despite the renormalizable model, RMF theory has been improved almost with no-sea approximation.**
- **Pion is the most important meson in realistic nuclear force and nuclear reaction, but pion had never been introduced to RMF theory due to parity conservation.**
- **Recently our group can introduce pionic correlations to RMF theory using CPPRMF method.**
- **We would like to discuss the importance of pion and Dirac sea in nuclear structure.**



$$M^* = M + g_\sigma \sigma$$

scalar potential

$$S \equiv g_\sigma \sigma$$

vector potential

$$V \equiv g_\omega \omega$$

Chiral sigma model

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left[i\gamma_\mu\partial^\mu - g_\sigma(\phi + i\gamma_5\tau\cdot\pi) - g_\omega\gamma_\mu\omega^\mu\right]\psi \\
&+ \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + \partial_\mu\pi\cdot\partial^\mu\pi\right) - \frac{\mu^2}{2}\left(\phi^2+\pi^2\right) - \frac{\lambda}{4}\left(\phi^2+\pi^2\right)^2 \\
&- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}\widetilde{g_\omega}^2\left(\phi^2+\pi^2\right)\omega_\mu\omega^\mu - \delta\mathcal{L}_{CTC}
\end{aligned}
$$

Y. Ogawa et al, PTP111, 75 (2004),

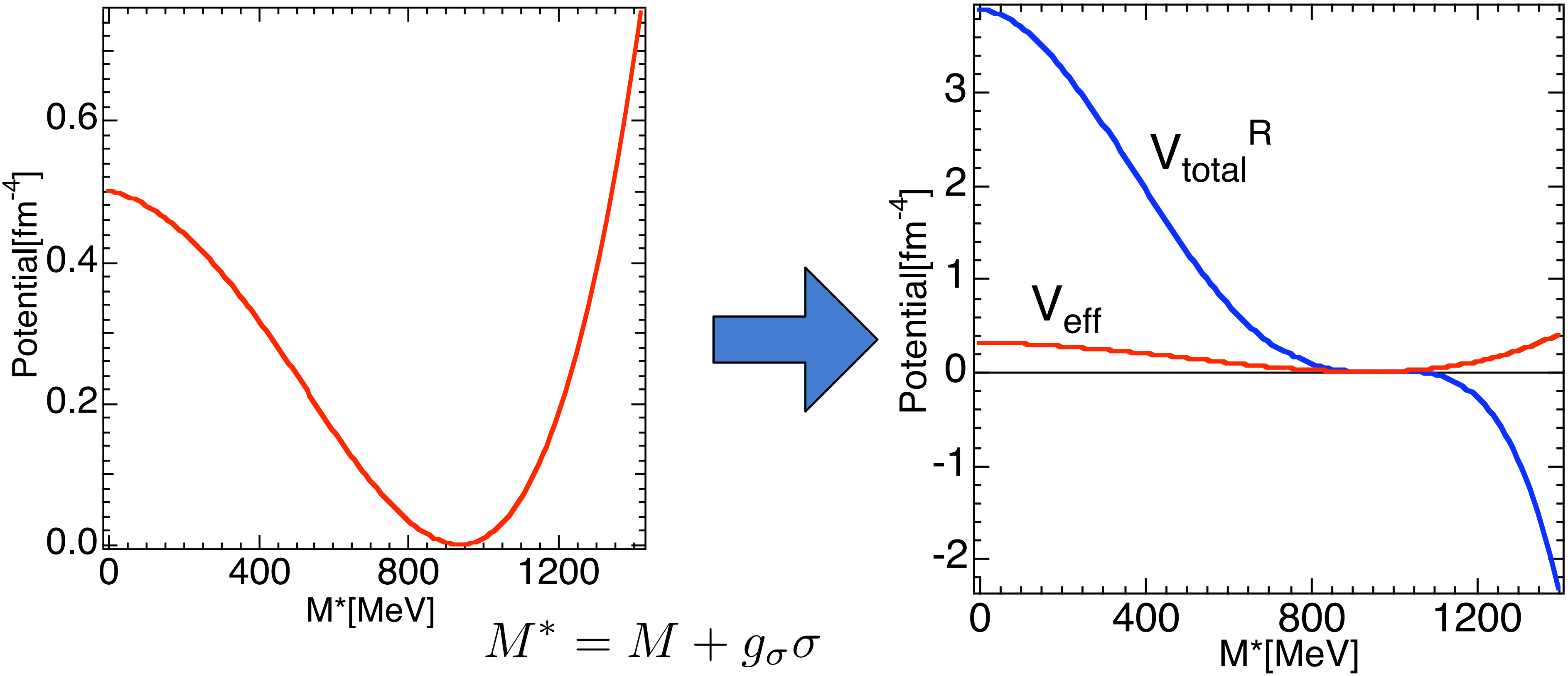
Phys. Rev. C73, 034301 (2006)

ϕ : **sigma field *before* the chiral symmetry breaking**

σ : **sigma field *after* the chiral symmetry breaking**

Chiral sigma model is a renormalizable one but ...

Problems of chirally symmetric renormalization



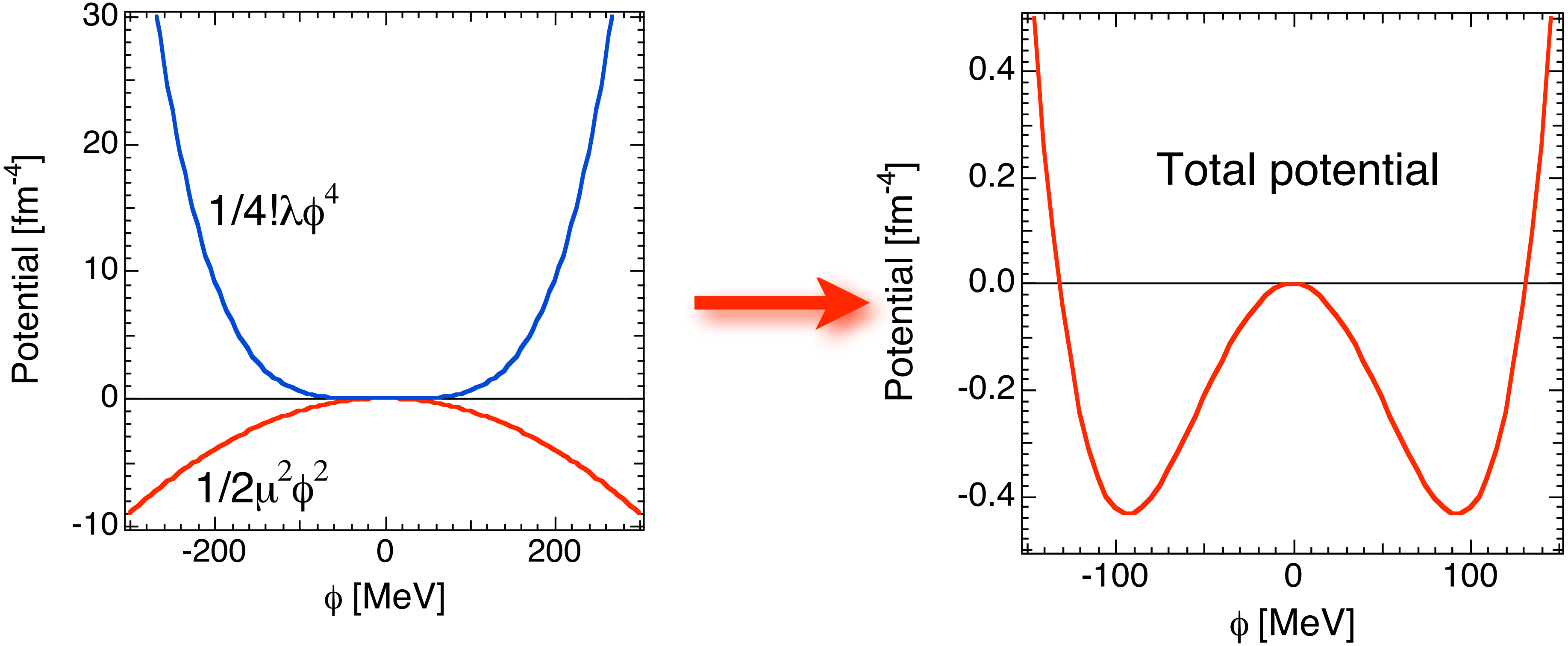
$M^* = M + g_\sigma \sigma$

Total effective potential from chirally symmetric renormalization becomes unstable, and vacuum fluctuation of nucleon loop has unnatural size??

J. Boguta, Nucl. Phys. A501, 637 (1989), R. J. Furnstahl, et al. NPA618, 446 (1997)

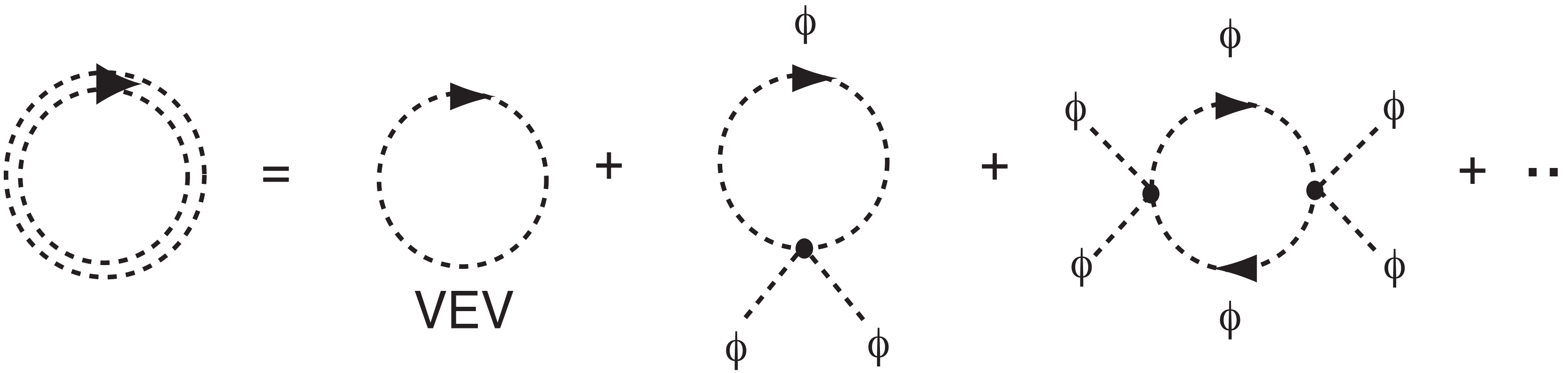
Spontaneous symmetry breaking in ϕ^4 theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$



Negative-mass term gives rise to spontaneous symmetry breaking.

Loop contributions in ϕ^4 theory



$$\frac{i}{2}\,\mathrm{Tr}\ln\left(p^2-\mu^2-\frac{\lambda}{2}\phi^2\right)-VEV-\delta\mathcal{L}_{CTC}$$

$$\Rightarrow\quad -V_B^R(\phi)+\frac{1}{2}Z(\phi)(\partial_\mu\phi)^2+\cdots$$

We define the renormalized potential of boson loop with counterterms and take two renormalization conditions for mass and coupling constant.

$$\boxed{\left.\frac{\partial^2 V_B^R}{\partial\phi^2}\right|_{\phi=0}=0}$$ **for mass**

$$\boxed{\left.\frac{\partial^4 V_B^R}{\partial\phi^4}\right|_{\phi=0}=0}$$ **for coupling constant**

$$V_B^R=\frac{\mu^4}{64\pi^2}\left[\left(1+\frac{\lambda\phi^2/2}{\mu^2}\right)^2\ln\left(1+\frac{\lambda\phi^2/2}{\mu^2}\right)-\frac{\lambda\phi^2/2}{\mu^2}-\frac{3}{2}\left(\frac{\lambda\phi^2/2}{\mu^2}\right)^2\right]$$

Radiative corrections as origin of SSB

Coleman & Weinberg redefine two renormalization conditions *before* the symmetry breaking in the massless ϕ^4 theory in order to avoid a logarithmical singularity.

S. R. Coleman and E. Weinberg, PRD 7, 1888 (1973)

$$\left.\frac{\partial^2 V_B^R}{\partial\phi^2}\right|_{\phi=0} = 0 \quad (\mu \to 0)$$

$$\left.\frac{\partial^4 V_B^R}{\partial\phi^4}\right|_{\phi=m} = 0 \quad (\mu \to 0)$$

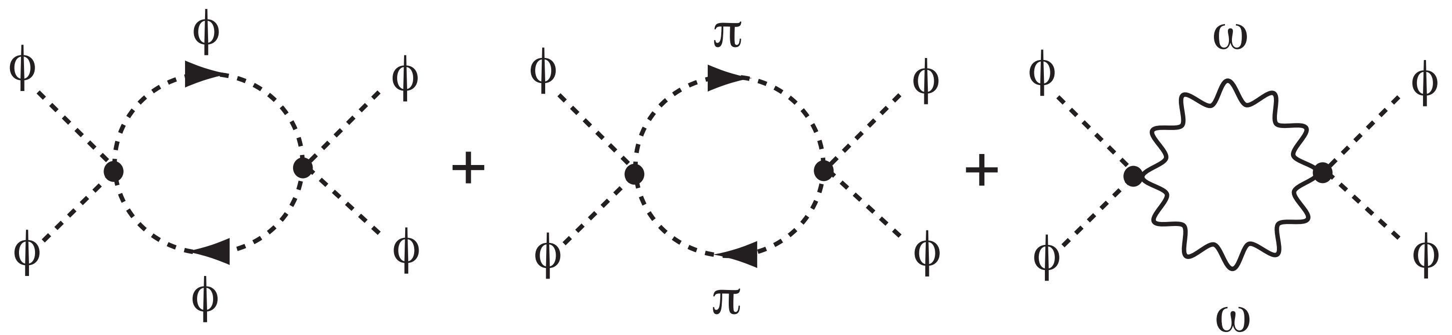
The renormalization scale

$$V_B^R = \frac{\lambda^2\phi^4}{256\pi^2}\left[\ln\left(\frac{\phi^2}{m^2}\right) - \frac{25}{6}\right]$$

$$V_B^{total} = \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2}\left[\ln\left(\frac{\phi^2}{m^2}\right) - \frac{25}{6}\right]$$

Since the second term of total potential is negative around the origin, it has an effect to make a new minimum at some point away from the origin. This mechanism plays the role of spontaneous symmetry breaking.

One-boson loop with chiral symmetry



We have to consider three diagrams especially for the four external lines of sigma meson in the massless chiral sigma model. Especially we need the internal line of omega meson. However we can neglect the external lines of omega meson due to current conservation.

$$V_B^R = \frac{(\phi^2+\pi^2)^2}{256\pi^2}\left[(6\lambda)^2\left\{1+3\left(\frac{1}{3}\right)^2\right\}+12\widetilde{g_\omega}^4\right]\left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right)-\frac{25}{6}\right]$$

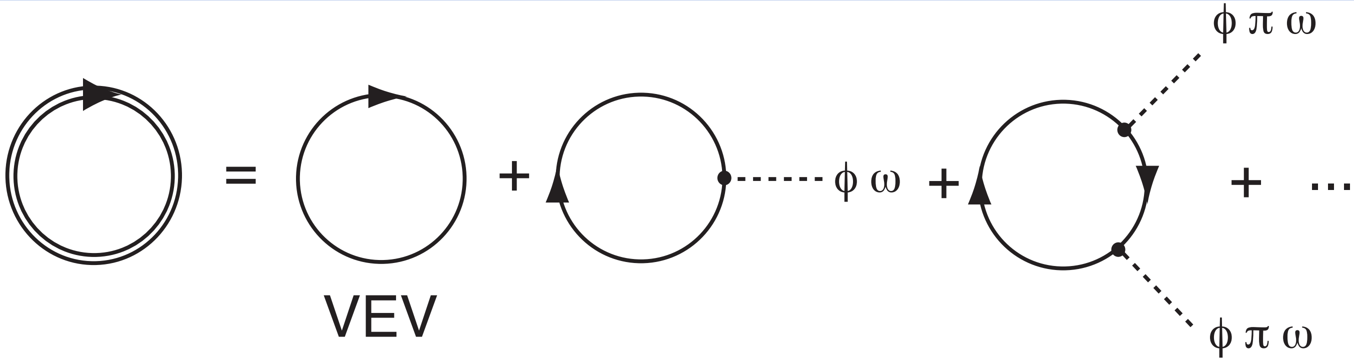
Three kinds of pion

The propagator of omega for Lorentz gauge

The ratio of coupling constant bewteen different particles

$$= \frac{3(4\lambda^2+\widetilde{g_\omega}^4)}{64\pi^2}(\phi^2+\pi^2)^2\left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right)-\frac{25}{6}\right]$$

New chirally symmetric renormalization



$$-i\,\mathrm{Tr}\ln\left[\not{k} - M - g_\sigma(\sigma + i\gamma_5\tau\cdot\pi) - g_\omega\gamma_\mu\omega^\mu\right] - VEV - \delta\mathcal{L}_{CTC}$$

$$= -V_F^R + \frac{1}{2}Z^F_{\sigma\pi}(\partial_\mu\phi\partial^\mu\phi + \partial_\mu\pi\partial^\mu\pi) + \frac{1}{4}Z^F_\omega\Omega_{\mu\nu}\Omega^{\mu\nu} + \cdots$$

We also apply the Coleman & Weinberg renormalization scheme to nucleon loop *before* the chiral symmetry breaking. In the same way of boson loop, we also introduce the same renormalization scale to avoid a logarithmical singularity.

$$\rho^2 = \phi^2 + \pi^2$$

$$\left.\frac{\partial^2 V_F^R}{\partial\rho^2}\right|_{\rho^2=0} = 0 \quad (M\to 0)$$

$$\left.\frac{\partial^4 V_F^R}{\partial\rho^4}\right|_{\rho^2=m^2} = 0 \quad (M\to 0)$$

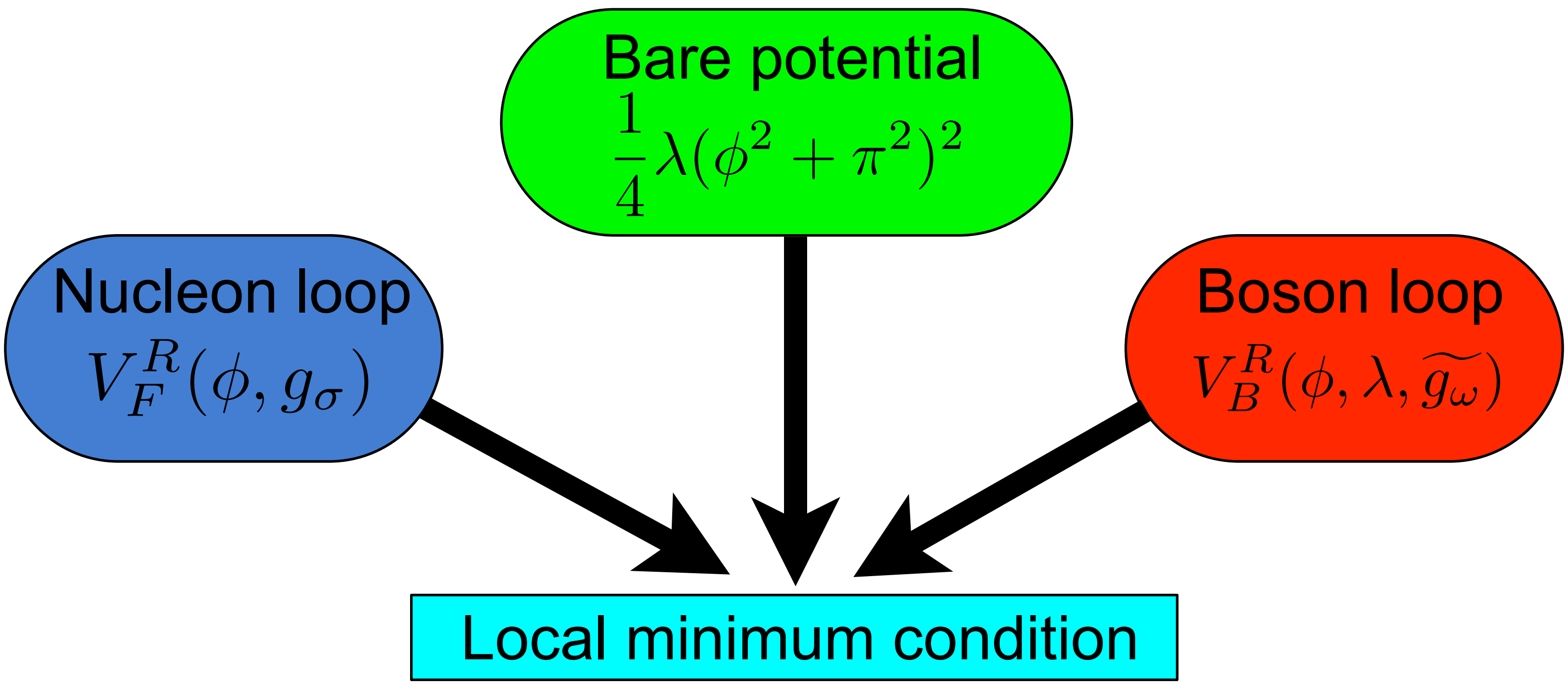
$$V_F^R = -\frac{g_\sigma^4}{8\pi^2}(\phi^2+\pi^2)^2\left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right) - \frac{25}{6}\right]$$

Massless nucleon and boson loops

$$V_F^R = -\frac{g_\sigma^4}{8\pi^2}(\phi^2+\pi^2)^2\left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right)-\frac{25}{6}\right]$$

$$V_B^R = \frac{3(4\lambda^2+\widetilde{g_\omega}^4)}{64\pi^2}(\phi^2+\pi^2)^2\left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right)-\frac{25}{6}\right]$$

The differences among boson and fermion loops are sign and coupling constants, but both of them have same function form!



One-loop corrections as origin of SCSB

$$U_{all} = V_{tree} + V_F^R + V_B^R - \epsilon\phi$$
$$= \frac{1}{4}\lambda(\phi^2+\pi^2)^2 + \frac{\gamma-1}{8\pi^2} g_\sigma^4 (\phi^2+\pi^2)^2 \left[\ln\left(\frac{\phi^2+\pi^2}{m^2}\right) - \frac{25}{6}\right] - \epsilon\phi$$

We can obtain the total potential *before* the symmetry breaking and take the condition for a non-trivial new local minimum away from the origin.

$$\boxed{\left.\frac{\partial U_{all}}{\partial \phi}\right|_{\phi=f_\pi} = 0}$$

$$\gamma = \left|\frac{V_B^R}{V_F^R}\right| = \frac{3(4\lambda^2 + \widetilde{g_\omega}^4)}{8 g_\sigma^4}$$

$$\Rightarrow \quad \frac{3}{2\pi^2}\left[\ln\left(\frac{f_\pi}{m}\right) - \frac{11}{6}\right]\lambda^2 + \lambda - \frac{g_\sigma^4 - \frac{3}{8}\widetilde{g_\omega}^4}{\pi^2}\left[\ln\left(\frac{f_\pi}{m}\right) - \frac{11}{6}\right] - \frac{\epsilon}{f_\pi^3} = 0$$

Input

$$f_\pi = 93[\mathrm{MeV}] \qquad M = g_\sigma f_\pi = 939[\mathrm{MeV}]$$

$$m_\omega = \widetilde{g_\omega} f_\pi = 783[\mathrm{MeV}] \qquad m_\pi = \sqrt{\frac{\epsilon}{f_\pi}} = 139[\mathrm{MeV}]$$

Output

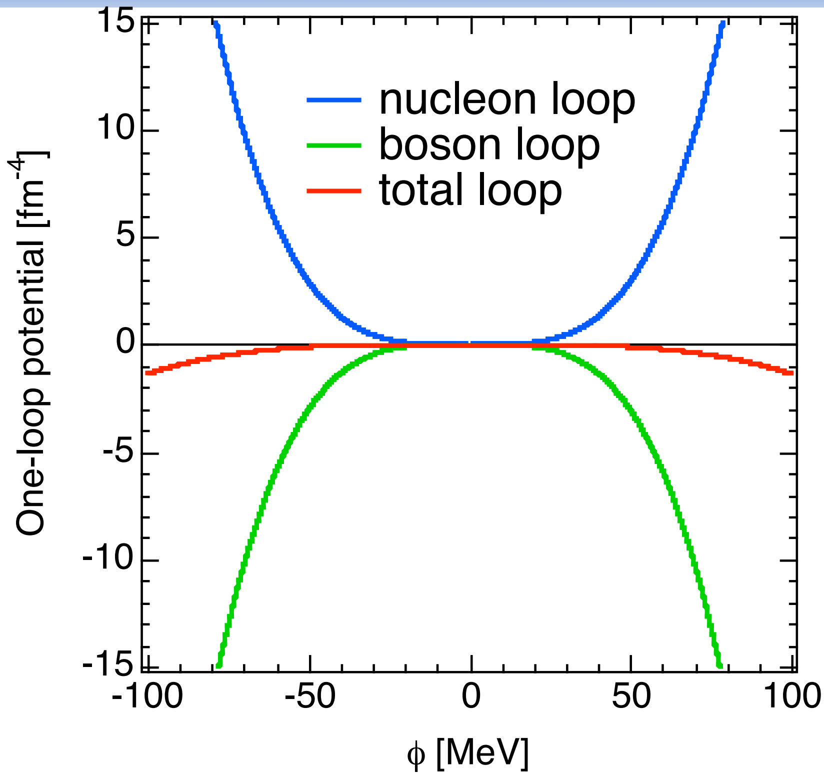
λ	γ	m_σ[MeV]	g_σ	$\widetilde{g_\omega}$
77.07	1.038	641	10.09	8.42

$$g_\omega = \widetilde{g_\omega}$$

Free parameter

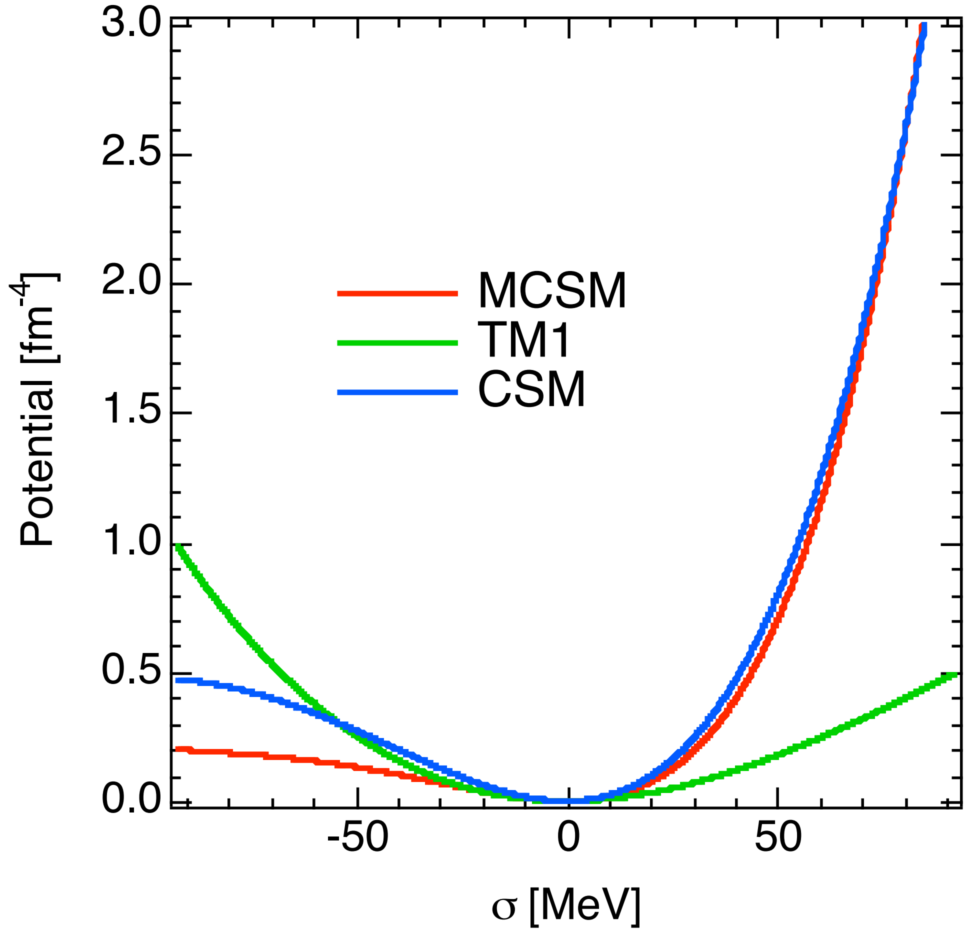
$$m = f_\pi$$

The stable effective potential



Before SCSB

After SCSB

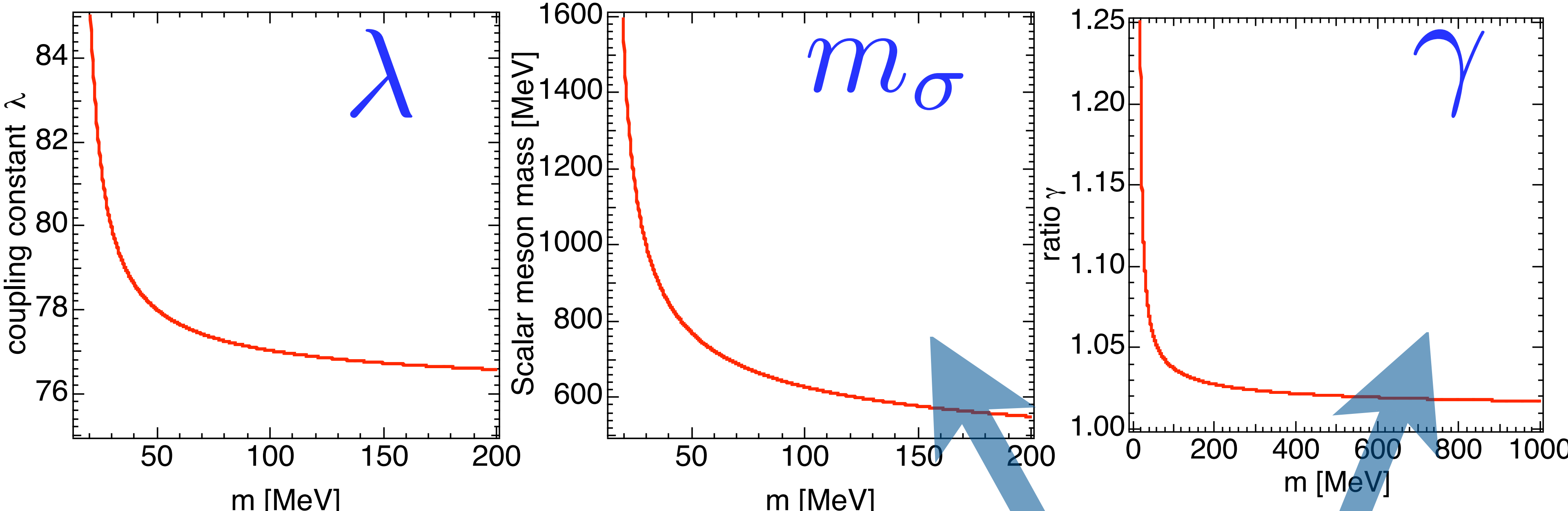


TM1; Y. Sugahara and H. Toki, NPA579, 557 (1994)

CSM; Y. Ogawa et al. PTP111, 75 (2004)

This stable effective potential is not made by us but appears naturally from the chiral symmetry!

Dependence on renormalization scale



$$m_\sigma^2 = \frac{\partial^2 U_{all}}{\partial \sigma^2} \quad \gamma = \left| \frac{V_B^R}{V_F^R} \right| = \frac{3(4\lambda^2 + g_\omega^4)}{8 g_\sigma^4}$$

We take the limit $m \to \infty$ especially for two cases ...

Broken symmetry between fermion and boson

$$\lim_{m\to\infty} m_\sigma = 0$$

$$\lim_{m\to\infty} \gamma = 1$$

$$\gamma = \left|\frac{V_B^R}{V_F^R}\right| = \frac{3(4\lambda^2 + \widetilde{g_\omega}^4)}{8g_\sigma^4}$$

1: The restoration of chiral symmetry

2: All of one-loop corrections are perfectly cancelled.

Symmetry

(Chiral symmetry and symmetry between fermion and boson)

Break

$m = \text{finite}$

Preserve

$m \to \infty$

Naturalness and naive dimensional analysis (NDA)

$$U_n(\sigma) = \kappa_n \frac{1}{n!} f_\pi^2 M^2 \left(\frac{\sigma}{f_\pi}\right)^n$$

H. Georgi, Adv. Nucl. Phys. 43, 209 (1993)

If naturalness holds, a dimensionless coefficient κ_n is O(1).
For example, we check the bare potential in massless ϕ^4 theory.

$$\frac{\lambda}{4}\sigma^4 = \kappa_4 \left(\frac{1}{4!}\frac{M^2}{f_\pi^2}\sigma^4\right) \longrightarrow \kappa_4 = \frac{3!\lambda f_\pi^2}{M^2} = 1.988$$

As next case, we check the vacuum fluctuation from nucleon loop in the Walecka model.

$$M^* = M + g_\sigma \sigma$$
$$\Phi = g_\sigma \sigma$$

$$\Delta\mathcal{E}_{VF}^{Walecka} = -\frac{1}{4\pi^2}\left[M^{*4}\ln\left(\frac{M^*}{M}\right) - M^3\Phi - \frac{7}{2}M^2\Phi^2 - \frac{13}{3}M\Phi^3 - \frac{25}{12}\Phi^4\right]$$
$$= -\frac{M^4}{4\pi^2}\left[\frac{1}{5}\left(\frac{\Phi}{M}\right)^5 - \frac{1}{30}\left(\frac{\Phi}{M}\right)^6 + \cdots + \frac{4!(n-5)!(-1)^{n-1}}{n!}\left(\frac{\Phi}{M}\right)^n\right]$$

$$-\frac{M^4}{4\pi^2}\frac{\Phi^4}{5M^5} = \kappa_5\left(\frac{1}{5!}\frac{M^2}{f_\pi^3}\sigma^5\right) \longrightarrow \kappa_5 = -\frac{3!g_\sigma}{\pi^2} = -61.97$$

We estimate the leading term in Dirac sea by NDA and it does not have natural coefficient.

Estimation of naturalness

In this estimation, we choose $m = f_\pi$ for simplicity.

$$\Delta\mathcal{E}_{VF}^{MCSM} = -\frac{\gamma-1}{4\pi^2}M^4\left[-9\left(\frac{\Phi}{M}\right)^2 - 4\left(\frac{\Phi}{M}\right)^3 + \frac{1}{5}\left(\frac{\Phi}{M}\right)^5 - \frac{1}{30}\left(\frac{\Phi}{M}\right)^6 + \cdots + \frac{4!(n-5)!(-1)^{n-1}}{n!}\left(\frac{\Phi}{M}\right)^n\right]$$

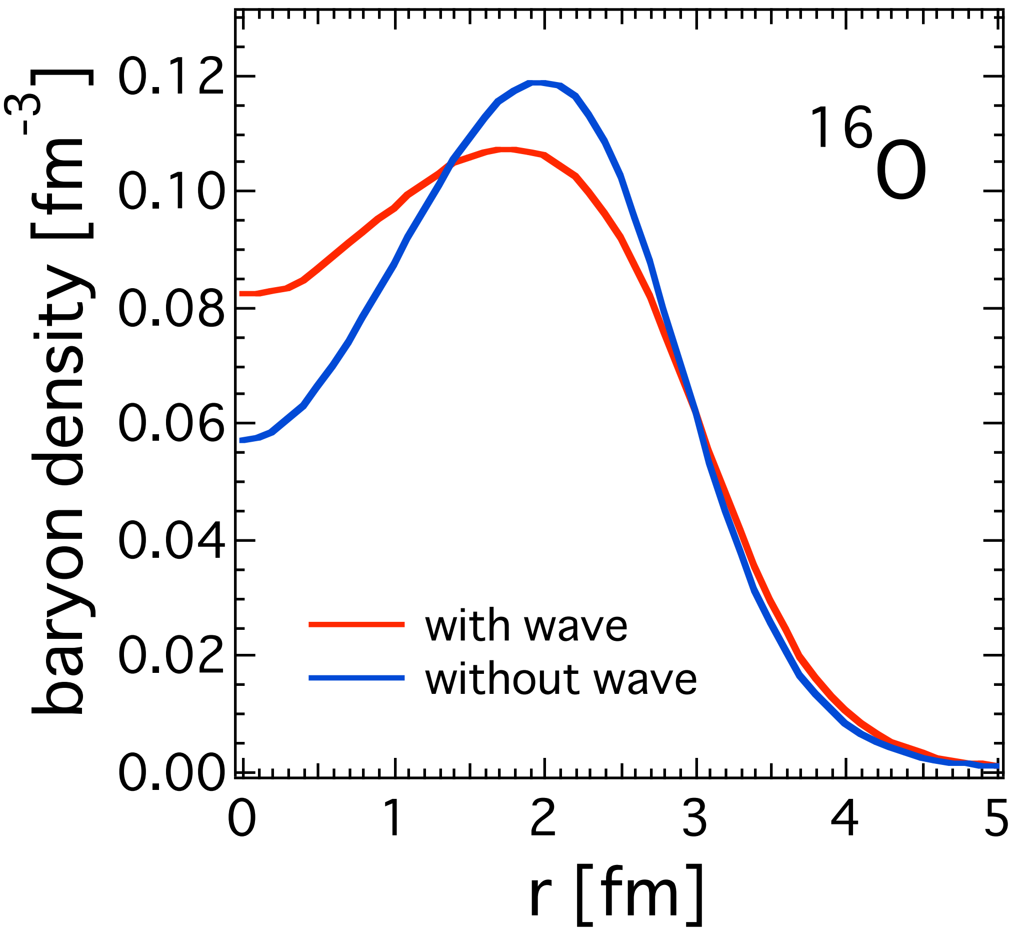
$$\gamma = 1.038 \quad (m \to f_\pi)$$

$$-\frac{\gamma-1}{4\pi^2}\frac{9M^4\Phi^2}{M^2} = \kappa_2\left(\frac{M^2}{2!}\sigma^2\right) \longrightarrow \kappa_2 = -\frac{9g_\sigma^2}{2\pi^2}\underline{(\gamma-1)} = -1.76$$

$$\frac{\gamma-1}{4\pi^2}\frac{M^4\Phi^5}{5M^5} = \kappa_5\left(\frac{M^2}{5!f_\pi^3}\sigma^5\right) \longrightarrow \kappa_5 = \frac{3!g_\sigma^2}{\pi^2}\underline{(\gamma-1)} = 2.35$$

When we consider the effect of only nucleon loop in any model and any renormalization scheme, it has too large non-linear potential and unnatural coefficient. By introducing both nucleon and boson loops *before* the symmetry breaking, two contributions from vacuum fluctuation are almost canceled. Thus we obtain the natural and stable potential with the effect of Dirac sea. In the RMF, *naturalness restores by introducing one-loop corrections of nucleon and bosons.*

Power of renormalization for wave function



preliminary result without pion mean field

Summary

- We can construct the massless chiral model with one-nucleon and one-boson loops in the Coleman-Weinberg scheme.
- In addition, we naturally obtain the stable effective potential with Dirac sea in the chiral model for the first time.
- SCSB is derived from broken balance among nucleon and bosons, and both nucleon and bosons become massive at the same time.
- By introducing nucleon and boson loop to RMF theory, naturalness restores in the massless chiral sigma model.
- It is possible to reveal the relationship between symmetry breaking and vacuum structure.

- As future works, we would like to study the properties in the finite nuclei, at finite temperature, at high density, and the gauge theory into this model; γ, ω meson, ρ meson, and a_1 meson.