Nuclear pairing: new methods, new perspectives and new questions

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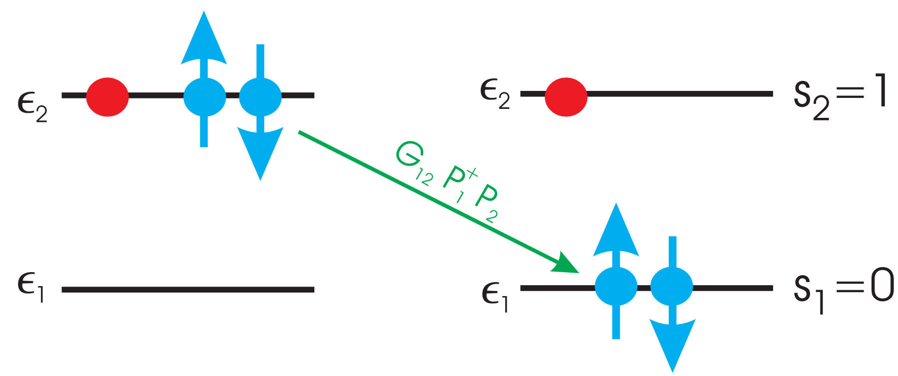
Topics of Discussion

- Exact solution of nuclear pairing
- Properties of the ground state
  - Pairing correlation energy
  - Occupation numbers
  - Spectroscopic factors
- Excited states
- Beyond pairing
  - Perturbation theory starting from pairing
  - Pairing and deformation
  - Hartree-Fock + pairing

Pairing Hamiltonian

- Pairing on degenerate time-conjugate orbitals
  $|1\rangle \leftrightarrow |\tilde{1}\rangle \qquad |j\tilde{m}\rangle = (-1)^{j-m}|j-m\rangle$
- Pair operators P=$(a_1 a_1)_{J=0}$ (J=0, T=1)
- Number of unpaired fermions is seniority s
- Unpaired fermions are untouched by H

$$H = \sum_1 \epsilon_1 N_1 - \sum_{12} G_{12} P_1^\dagger P_2$$



Approaching the solution of pairing problem

- Approximate
  - BCS theory
    - HFB+correlations+RPA
  - Iterative techniques
- Exact solution
  - Richardson solution
  - Algebraic methods
  - Direct diagonalization + quasispin symmetry

Shortcomings of BCS

- Particle number non-conservation

$$|\mathrm{BCS}\rangle = \prod_{\nu(\mathrm{doublets})} \left\{ u_\nu - v_\nu P_\nu^\dagger \right\} |0\rangle$$

- Phase transition and weak pairing problem

Example $$G = G_{\nu\nu'}\,, \quad \text{gap eq. } 1 = G \sum_\nu \frac{1}{2E_\nu}$$

$$G < G_c \quad \Delta = 0\,, \quad \text{where } 1 = G_c \sum_\nu \frac{1}{2\epsilon'_\nu}$$

- Excited states, pair vibrations

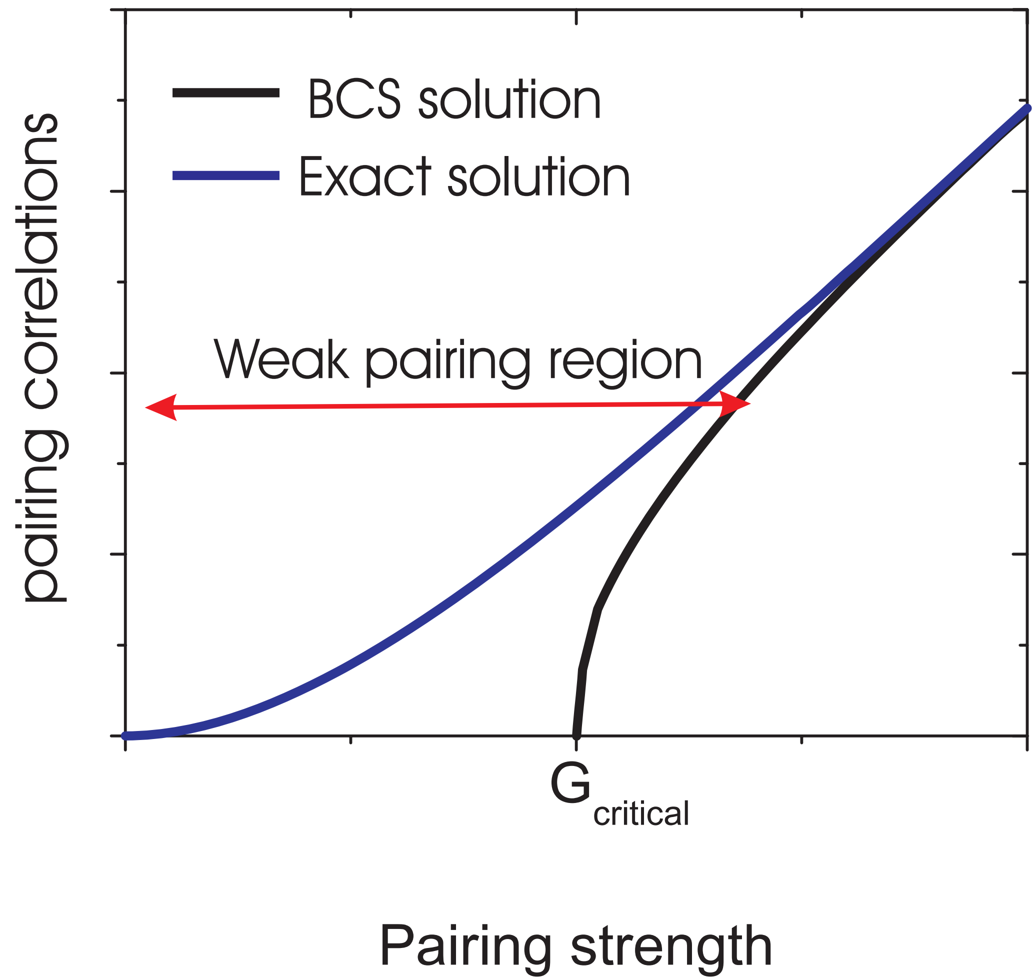
Quasispin and exact solution of pairing problem (EP)

On each single j-level

- Operators $P^\dagger_j$, P_j and N_j form a SU(2) group
  $P^\dagger_j \sim L^+_j$, $P_j \sim L_j$, and $N_j \sim L_j^z$
- Quasispin L^2_j is a constant of motion,
  seniority $s_j=(2j+1) -2L_j$
- States can be classified with set
  $(L_j\ L_j^z) \Leftrightarrow (s_j,\ N_j)$
- Each s_j is conserved but N_j is not
- Extra conserved quantity simplifies solution.
  Example: ^{116}Sn: 601,080,390 m-scheme states
  272,828 J=0 states
  110 s=0 states
  Linear algebra with sparse matrices is fast. Deformed basis Nmax~50-60
- Generalization to isovector pairing, R_5 group

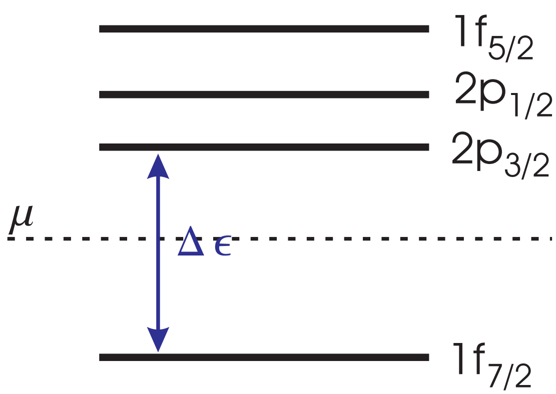
Pairing phase transition

- BCS has a sharp phase transition

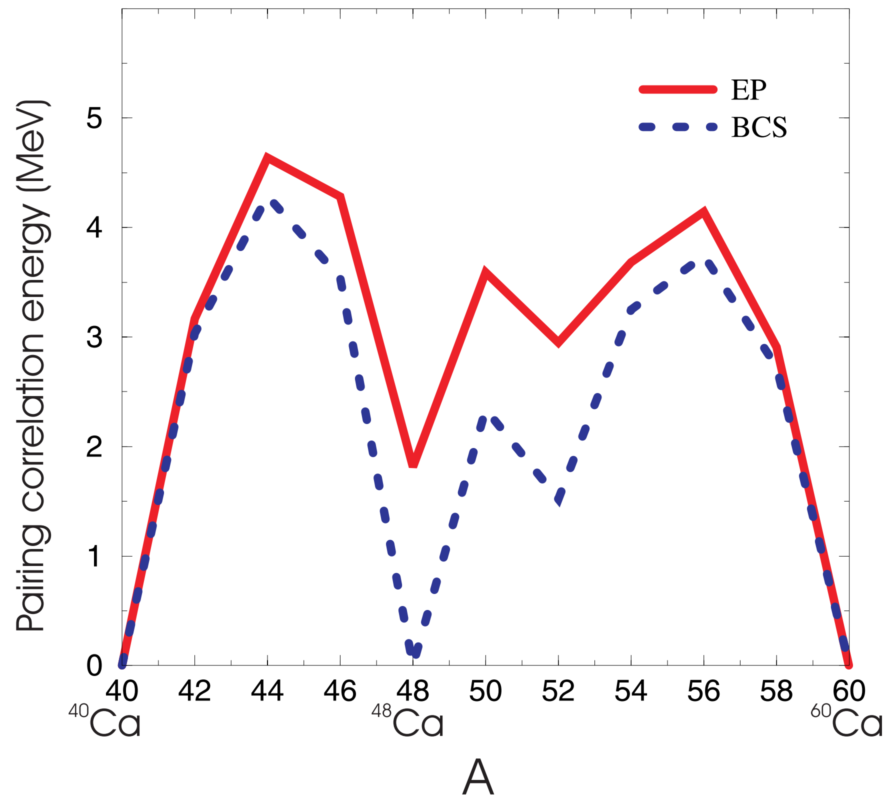


Pairing correlations in calcium

BCS fails to describe pairing correlations in ^{48}Ca



Pairing becomes weak if $G/\Delta\varepsilon \sim 1$ at Fermi surface

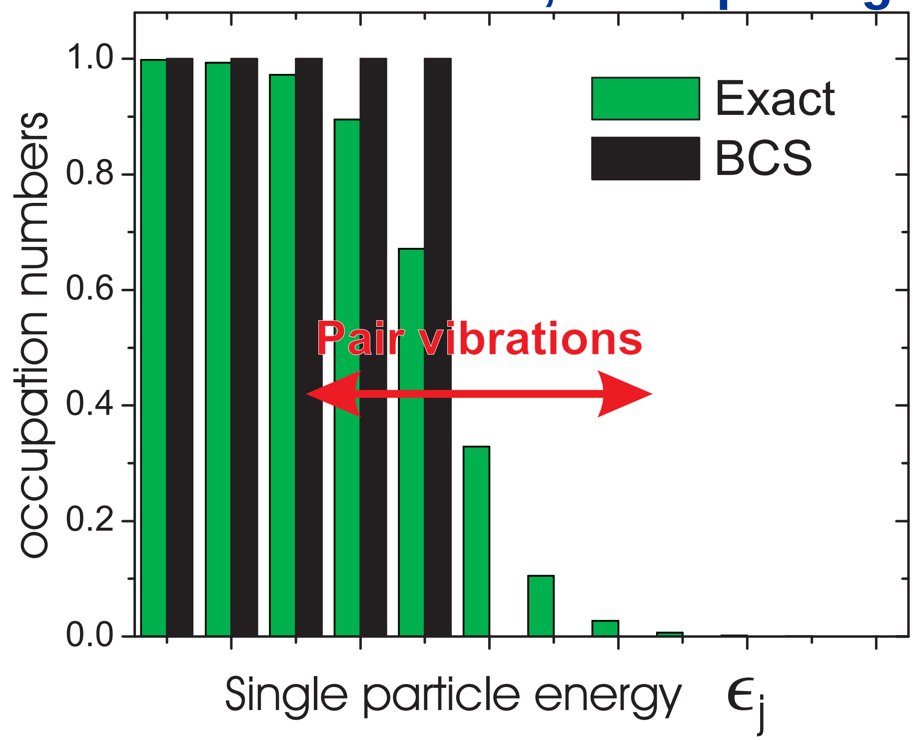


Occupation numbers and spectroscopic factors

- Occupation numbers
  $$n_j = \langle N | a_j^\dagger a_j | N \rangle$$
- Spectroscopic factors
  $$v_j = \langle N-1 | a_j | N \rangle$$
  $$u_j = \langle N+1 | a_j^\dagger | N \rangle$$
- Two-body spectroscopic factors

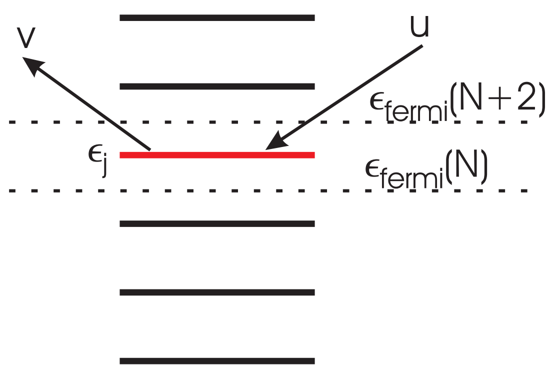
$$\mathcal{P}_j(N) = \langle N-2 | P_j | N \rangle$$
$$\mathcal{P}_j^\dagger(N) = \mathcal{P}_J(N+2)^* = \langle N+2 | P_j^\dagger | N \rangle$$

Below critical G, weak pairing



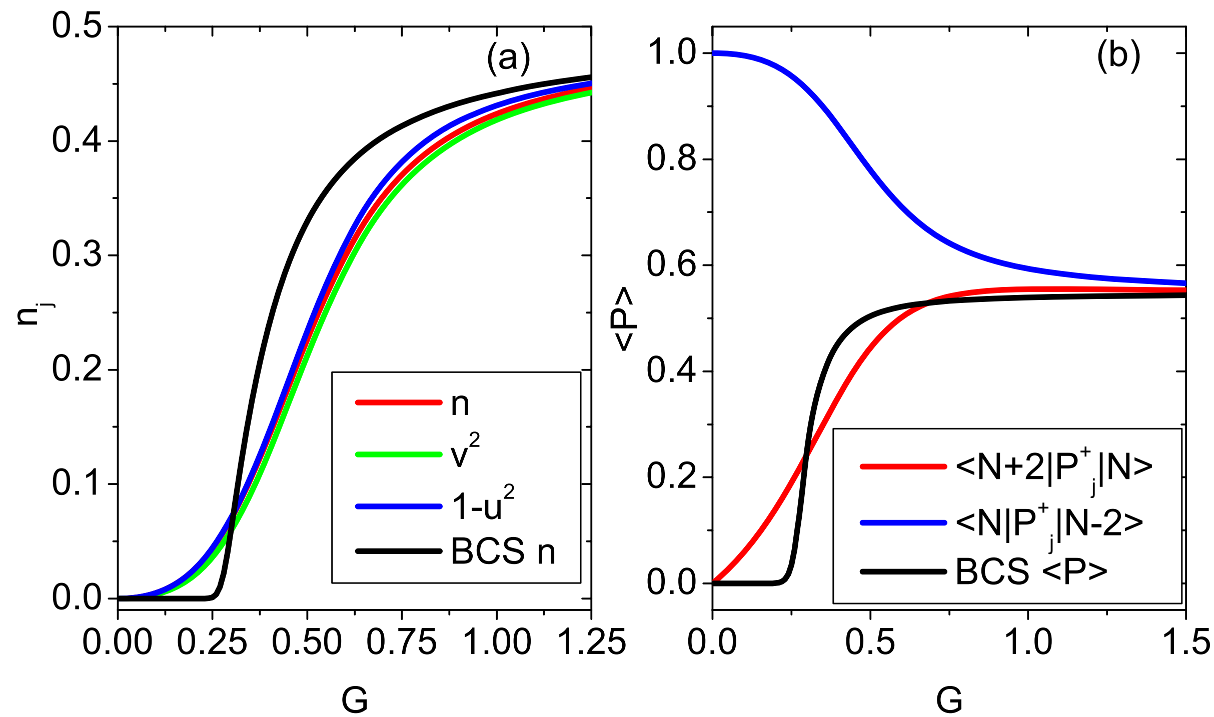
BCS: $$n_j = v_j^2 = 1 - u_j^2, \quad \mathcal{P}_j(\overline{N}) = \sqrt{n_j(1-n_j)} = u_j v_j$$

Ladder-system with constant G



occupation numbers single nucleon transitions

two nucleon emission



Ladder with 12 levels, N=12

Occupation numbers and spectroscopic factors ^{114}Sn

Exact calculation and BCS

Separation energy:

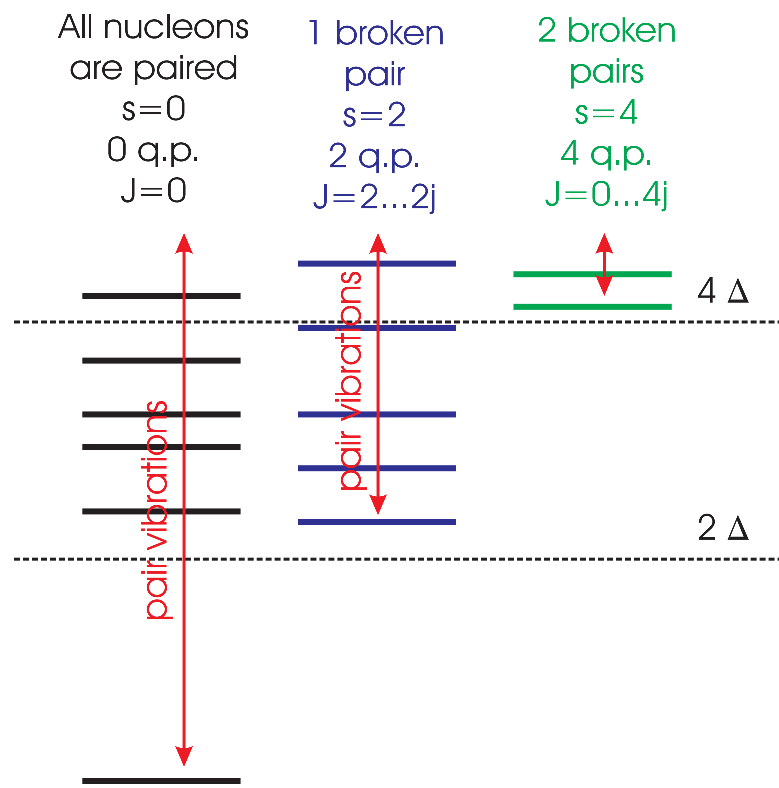
$S(N)=E(N-1)-E(N)$

j	$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$s_{1/2}$	$h_{11/2}$
N_j	6.96	4.46	0.627	0.356	1.60
N_j	6.71	4.14	0.726	0.507	1.91
n_j	0.870	0.744	0.157	0.178	0.133
$1-u^2_j$	0.872	0.748	0.162	0.183	0.137
v^2_j	0.865	0.736	0.155	0.177	0.131
n_j	0.839	0.690	0.181	0.254	0.159
S(N+1)	2.80	3.13	3.14	3.39	3.29
S(N+1)	2.89	3.21	3.11	3.21	3.26
S(N)	6.86	6.55	7.25	6.98	7.12
S(N)	6.89	6.64	7.20	7.03	7.06
P(N+2)	0.680	0.779	0.617	0.514	1.03
P(N)	0.810	0.930	0.524	0.396	0.845
P	0.734	0.801	0.545	0.435	0.896

Low-lying states in paired systems

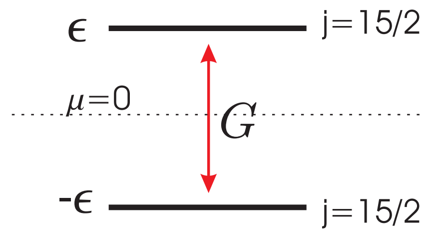
- Exact treatment
  - No phase transition and G_{critical}
  - Different seniorities do not mix
  - Diagonalize for pair vibrations
- BCS treatment

	$G<G_{\text{critical}}$	$G>G_{\text{critical}}$
Ground state	Hartree-Fock	BCS
Elementary excitations	single-particle excitations $E_{s=2}=2\,\varepsilon$	quasiparticle excitation $E_{s=2}=2\,e$
Collective excitations	HF+RPA	HFB+RPA



Low-lying states: Two-level pairing model

Two-level system N=16



BCS solution

$$V_{\text{critical}} = \frac{4\epsilon}{\Omega} = 0.25$$

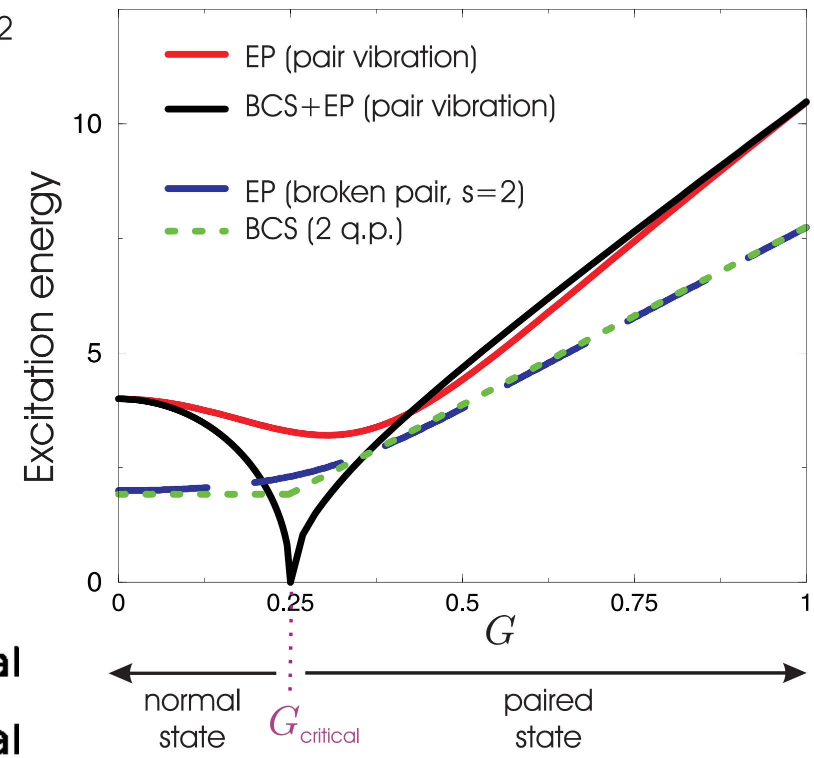
$$e_1 = e_2 = \frac{G\Omega}{4}, \quad \Delta^2 = \frac{G^2\Omega^2}{16} - \epsilon^2$$

Quasiparticle excitations

$$E_{s=2} = \begin{cases} 2e & G > G_{\text{critical}} \\ 2\mathcal{E} & G < G_{\text{critical}} \end{cases}$$

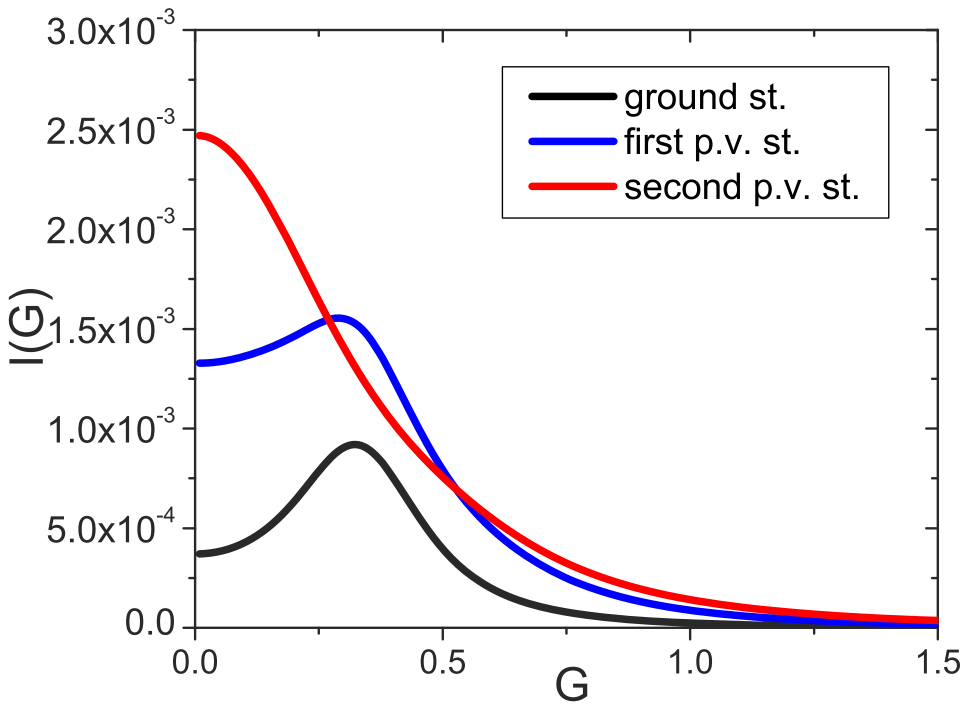
Pair vibrations:

$$E^2_{\text{ex}} = \begin{cases} \frac{G^2\Omega^2}{2} - 8\epsilon^2 = 8\Delta^2 & G > G_{\text{critical}} \\ 4\epsilon^2 - \frac{V^2\Omega^2}{4} & G < G_{\text{critical}} \end{cases}$$



Is there a pairing phase transition in mesoscopic system?

Invariant entropy



Mixing of states $|\alpha\rangle = \sum_k C_k^\alpha(G)|k\rangle$

$|k\rangle$- some reference basis states

Matrix ρ for each state α

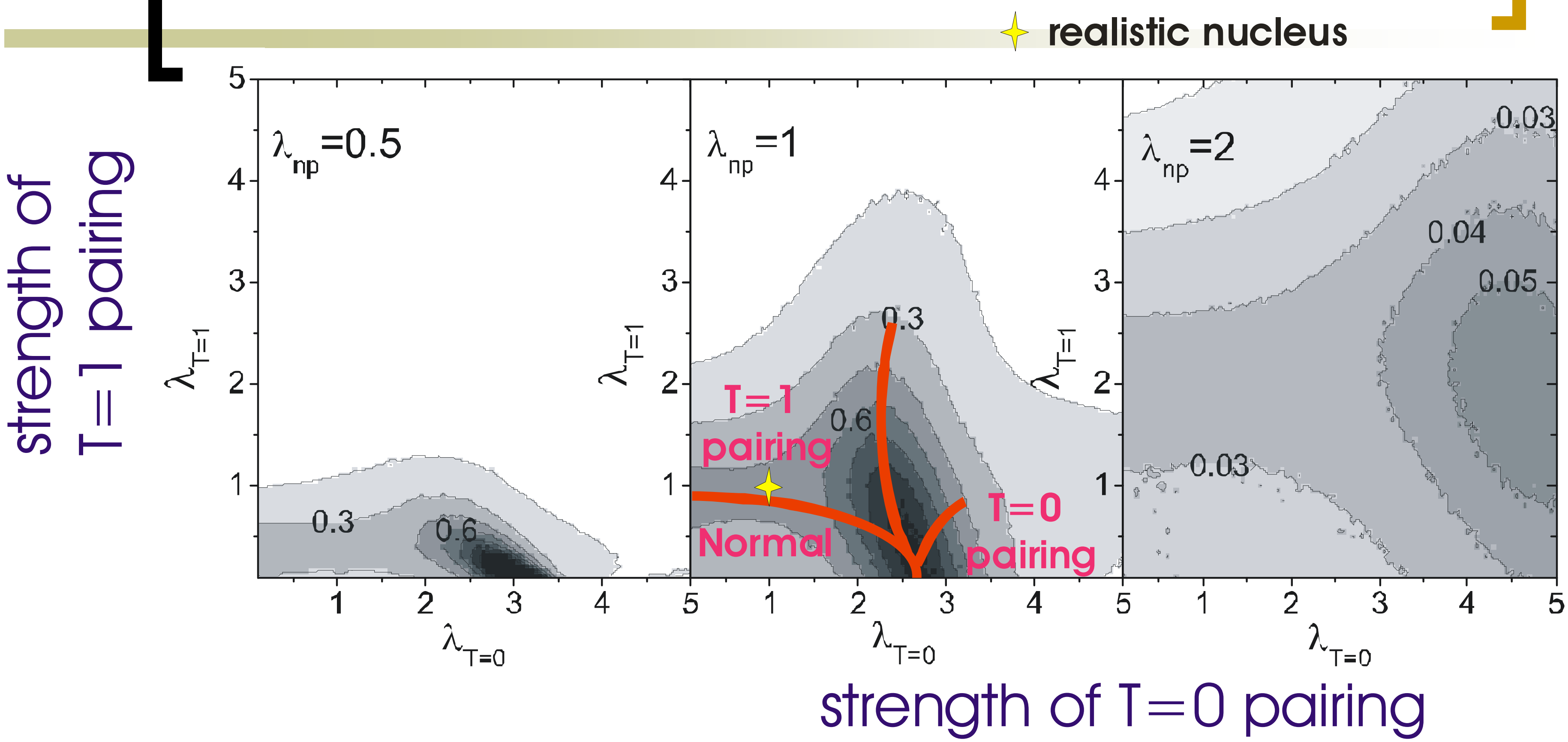
$$\rho^\alpha_{k\,k'}(G,\delta G) = \frac{1}{\delta G}\int_G^{G+\delta G} C^\alpha_{k'}(G)^* C^\alpha_k(G)dG$$

Invariant entropy

$$I^\alpha(G,\delta G) = -\mathrm{Tr}\,(\rho^\alpha \ln(\rho^\alpha))$$

- **Invariant entropy is basis independent**
- **Indicates the sensitivity of eigenstate α to parameter G in interval [G,G+δ G]**

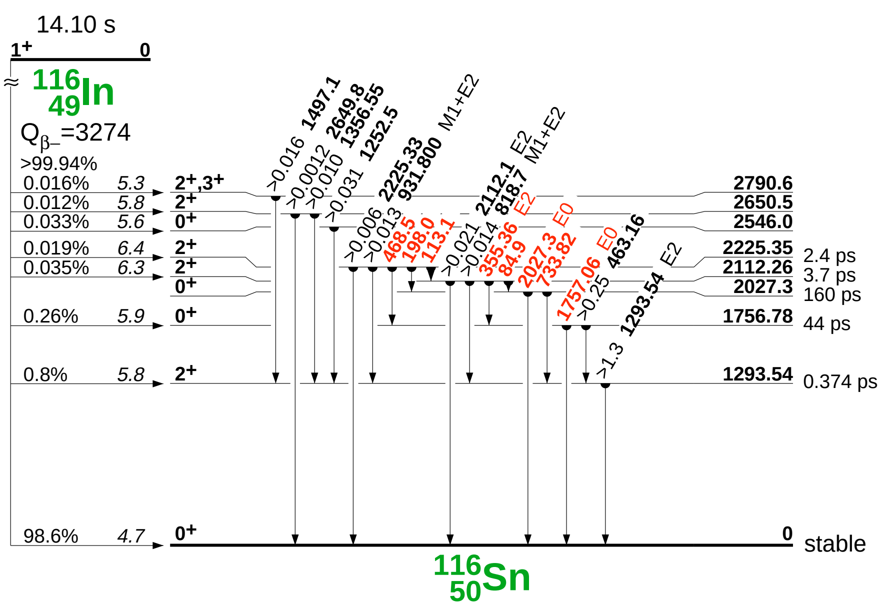
^{24}Mg phase diagram



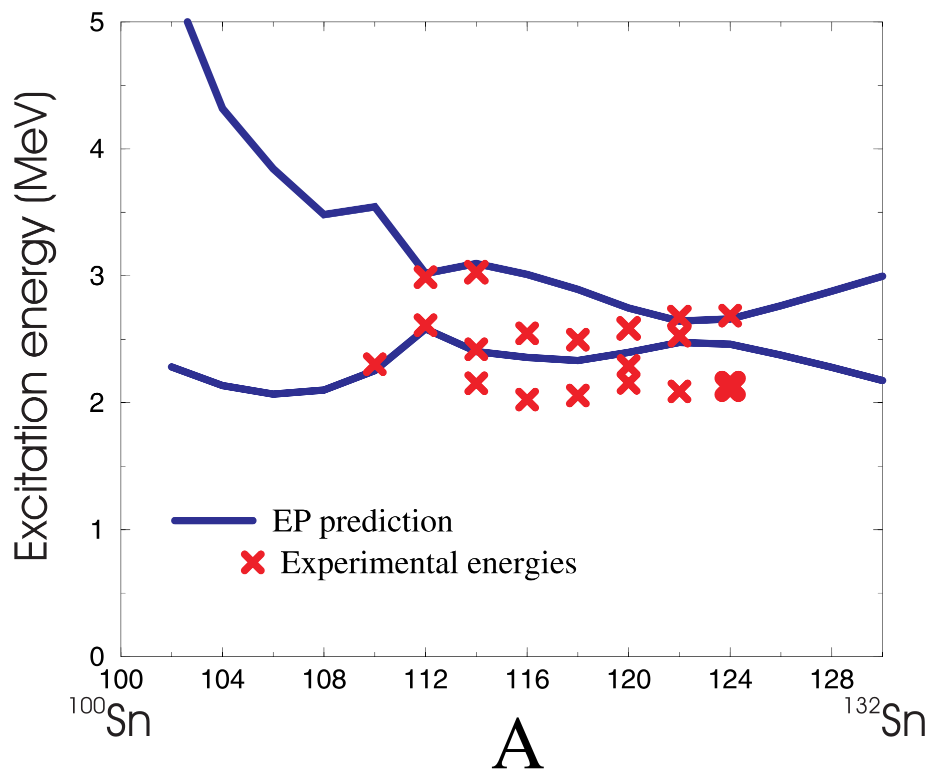
Contour plot of invariant correlational entropy showing a phase diagram as a function of T=1 pairing ($\lambda_{T=1}$) and T=0 pairing ($\lambda_{T=0}$); three plots indicate phase diagram as a function of non-pairing matrix elements (λ_{np}) . Realistic case is $\lambda_{T=1}=\lambda_{T=0}=\lambda_{np}=1$

Pair vibrations in realistic cases

- A number of J=0 states are pair-vibrations: "worsened" copies of the ground state.
- J=0 states are less effected by other interactions
- These states lie above the 2 Δ



J=0 states in Sn isotopes



Pairing and other residual interactions. Why pairing correlations survive?

- Nucleon-nucleon interaction is short range.
- Many non-pairing components of interaction preserve seniority (do not effect pairing).
- Finite nuclear size can be important.
- Pairing correlations can be enhanced in special regions, such as near oblate ⇔ prolate shape change.

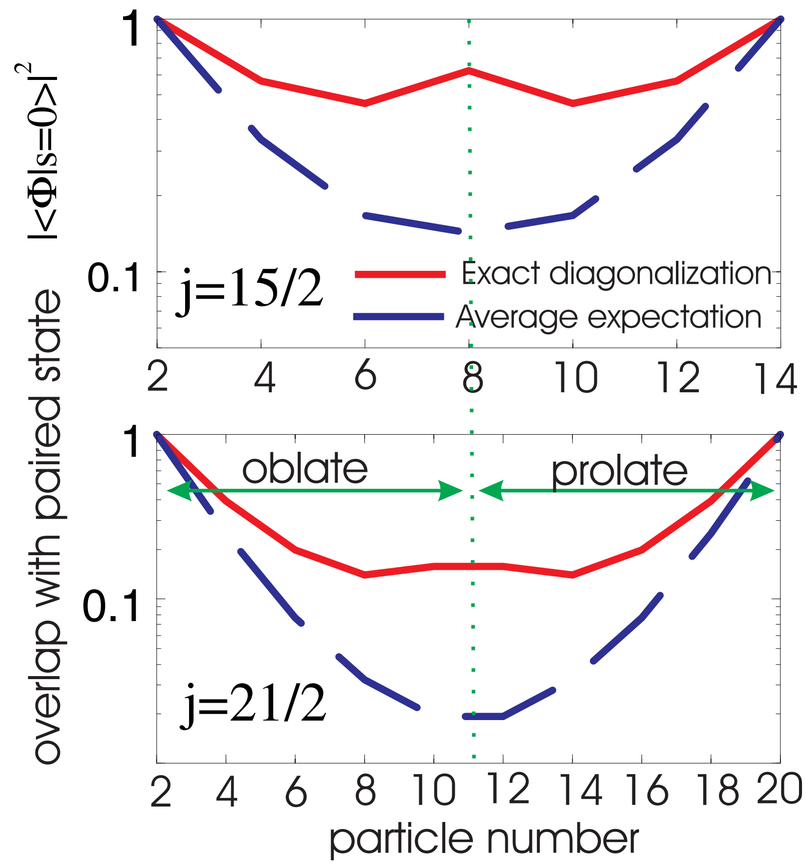
How deformation can enhance paring. Quadrupole-quadrupole interaction on single j-level

$$H = -\frac{\xi}{2} Q \cdot Q$$

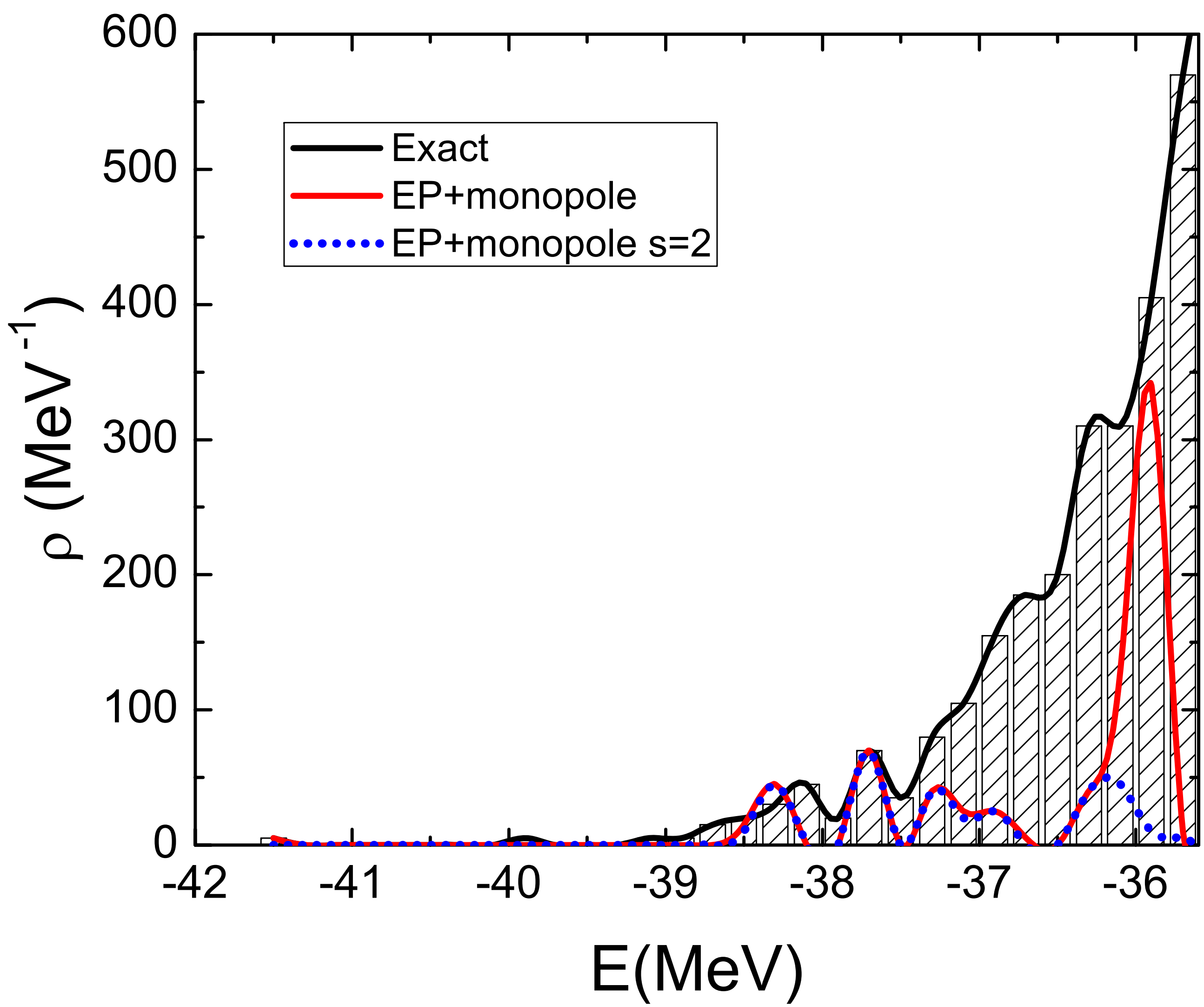
- Exchange terms result in attractive pairing interaction

$$G = \xi/(2\Omega)$$

- Terms scale as $1/\Omega$ where $\Omega = 2j+1$
- Pairing is the strongest particle-particle interactions



^{106}Sn Full Calculation and Pairing

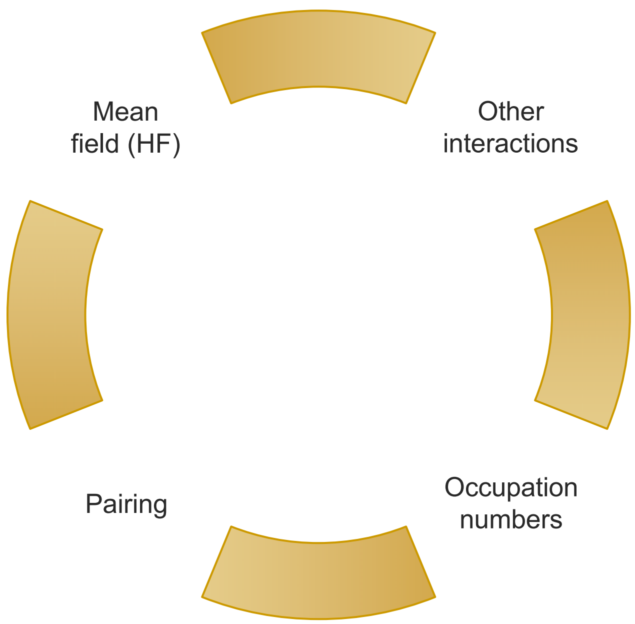


Using pairing in treating nuclear many-body problem

- Perturbative calculations starting from pairing [1]
- Hartree-Fock plus exact pairing
- Pairing and RPA for other interactions

(1) A. Volya, B.A. Brown, V. Zelevinsky, Phys. Lett. **B509** (2001) 37.

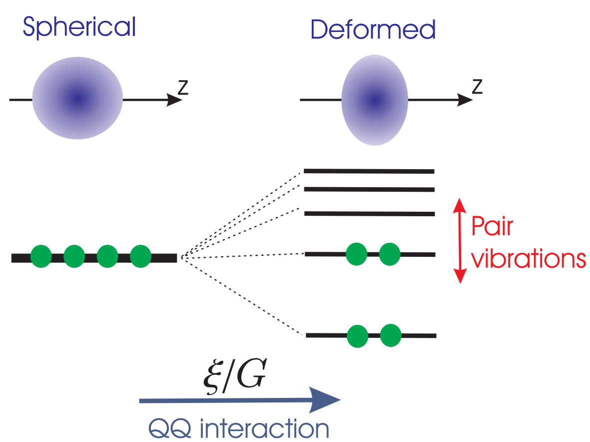
Hartree-Fock +EP

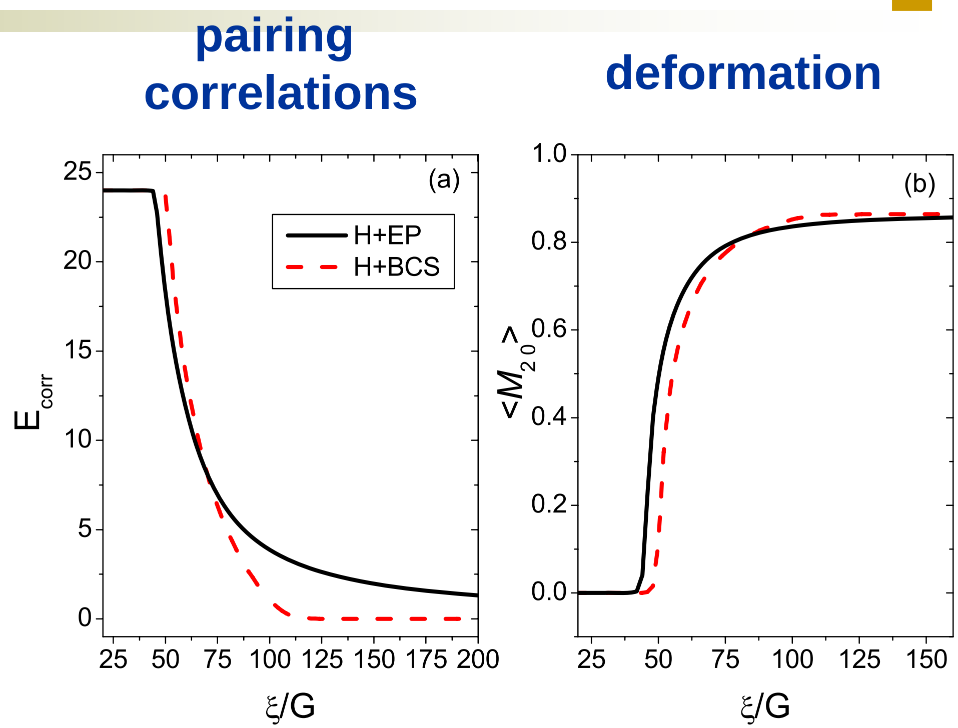


Pairing plus quadrupole model

- Hamiltonian $H = -GP^{\dagger}P - \frac{\xi}{2}Q \cdot Q$ on a single j-level
- Quadrupole creates deformed mean field
  $$\epsilon_m = -3\xi m^2 \langle Q \rangle$$
- Particles are distributed with pairing
- Self consistent mean-field
  $$\langle Q \rangle = \sum_m \left[3m^2 - j(j+1)\right] n_m$$

Hartree+EP on a single-j level



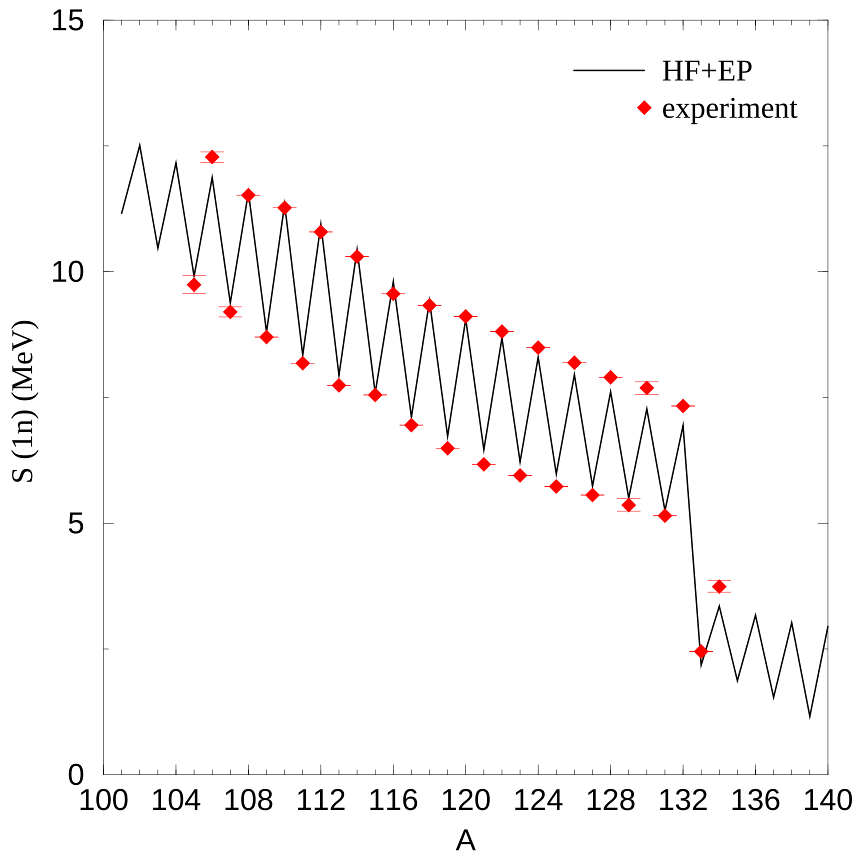


Features of exact solution

- Phase transition is smooth and extended.
- Pairing correlation survive in deformed state

Hartree Fock +EP calculation: Realistic case, tin isotopes.

- We use Skyrme HF (SKX[(1)])
- Pairing matrix elements from G-matrix calculations [(2)]



(1) B.A. Brown, Phys Rev. C **58**, (1998) 220.
(2) A. Hold, *et.al*., Nucl. Phys. **A634**, (1998) 41.

Nuclear pairing and Coriolis effects in proton emitters

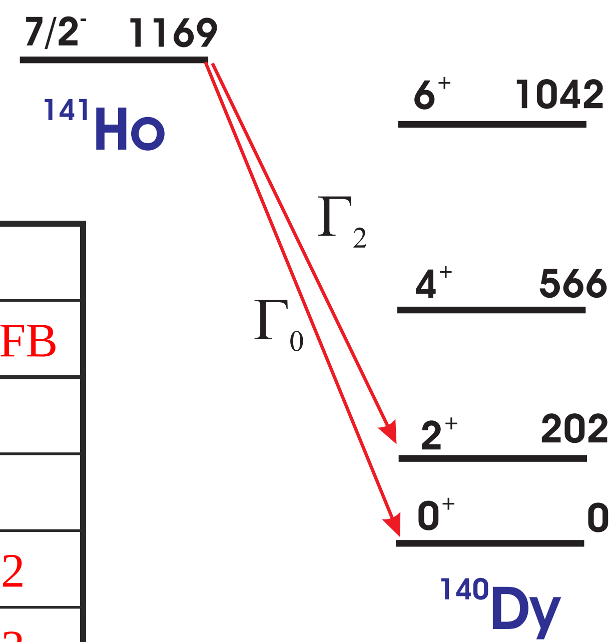
Motivations

Why proton emitters?

- •Remarkable experimental progress
- •Sensitive tool for structure physics
- •Development of new theoretical tools
- •Addressing old questions

Questions of interest:

- •Single particle motion in deformed field
- •Rotation of a superfluid nucleus
- •Kinematical recoil effects (Coriolis forces)

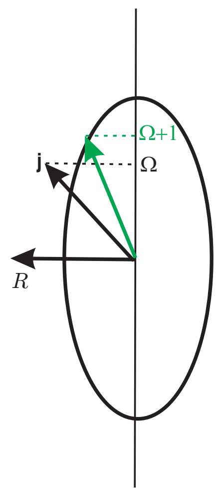


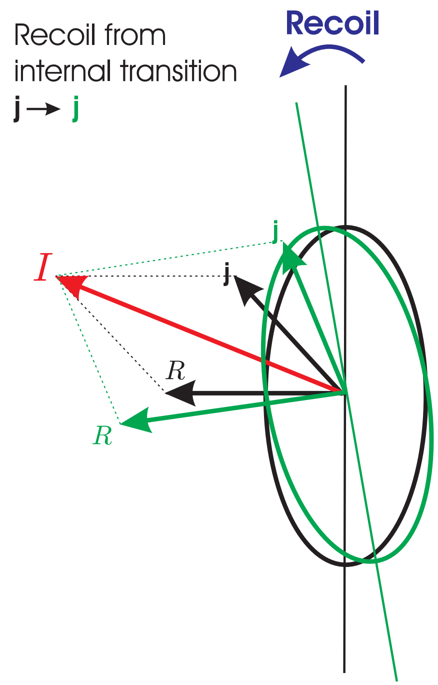
Coriolis effect and attenuation problem

	Γ_0 (10^{-20} MeV)		Γ_2/Γ_0 (%)	
	Rotor	RHFB	P+Rotor	RHFB
Experiment	10.9		0.71	
Adiabatic	15.0		0.73	
Coriolis	1.4	5.9	1.8	1.2
Coriolis+pairing	1.7	7.0	1.7	0.3

Coriolis effect and HFB in non-inertial frame

Adiabatic treatment





Full Picture

- Kinematic Coupling
- Pairing Spectroscopic factors
- Recoil of a superfluid "drop"

$$H = H_{\text{rotor}} + H_{\text{intr}}$$

Rotating Hartree-Fock-Bogoliubov (RHFB) mean field

Elementary excitations $a^{\dagger}_{\Omega}\ a^{\dagger}_{\Omega+1}a_{\Omega}$ for $\Omega \rightarrow \Omega + 1$

Find quasiparticles $\beta^{\dagger}_{i} = \sum_{\Omega}\left(u^{i}_{\Omega}a^{\dagger}_{\Omega} + v^{i}_{\Omega}a_{\Omega}\right)$

and stationary mean field $[\beta_i H] = \langle\epsilon_i\rangle\beta_i$

Broken symmetries: Particle number $\langle\Delta_i\rangle = \sum_{\Omega>0} G u^{i}_{\Omega}v^{i}_{\Omega}$

Deformation alignment, K-symmetry $\langle j_+\rangle \neq 0$, $\langle j_3^2\rangle \neq 0$

Understanding Coriolis attenuation problem

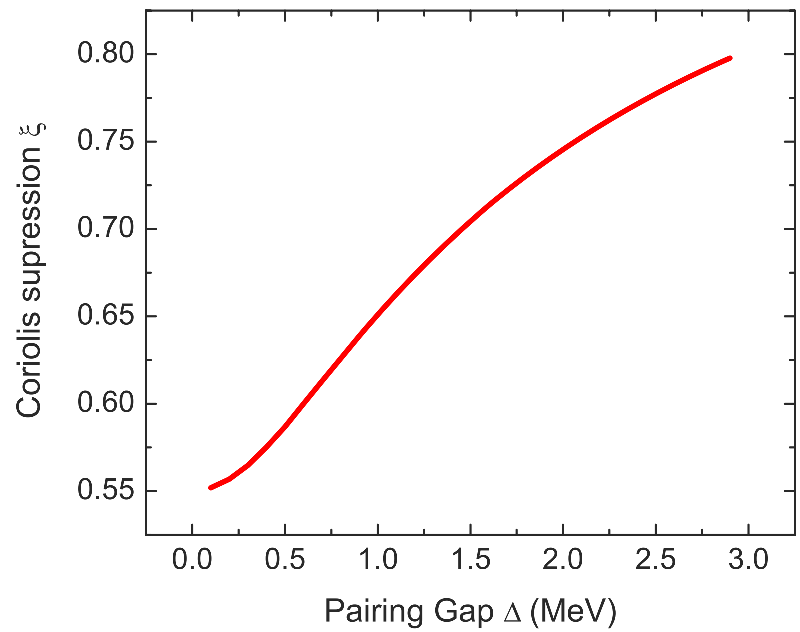
response of superfluid mean field to recoil

Traditional Coriolis term $$H_{cor} = -\frac{\hbar}{2\mathcal{L}}\left(I_+ j_- + I_- j_+\right)$$

RHFB MF Coriolis term $$H_{cor} = -\frac{\hbar}{2\mathcal{L}}\left[\left(I_- - \langle j_- \rangle\right) j_- + \left(I_+ - \langle j_+ \rangle\right) j_+\right]$$

Attenuation factor $$\xi = 1 - \frac{\langle j_+ \rangle}{\langle I_+ \rangle}$$

Applications to ^{141}Ho



Further steps

- **Self consistent moment of inertia**
- **Larger valence space**
- **From HFB to HF+EP**

*P.Ring and P.Schuck, Then nuclear many-body problem, (Springer-Verlag, Berlin Heidelberg, 2000)

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