



THERMAL PAIRING IN RICHARDSON MODEL

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Content

- Brief introduction of the modified BCS (MBCS)
- Criterion for applicability of MBCS
- BCS, MBCS predictions and exact results
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Lipkin-Nogami method at finite temperature in the static-path approximation

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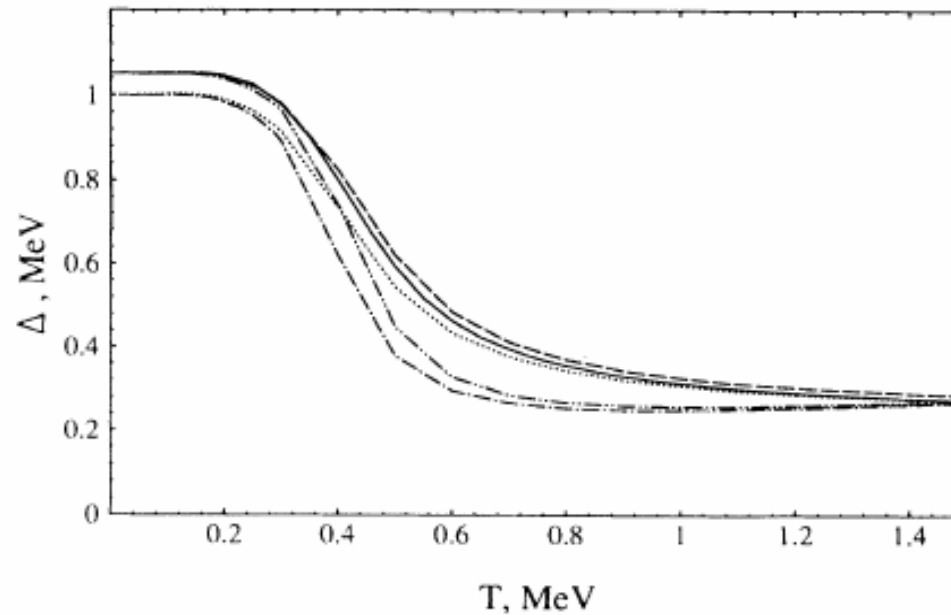


FIG. 4. Pairing gap versus temperature.

- exact
- SPA
- LN method + SPA
- · - · - thermal average within BCS (Moretto 1972)
- · - · - thermal average within BCS + LN corrections

The MBCS

- The BCS (HFB) at $T \neq 0$ ignores the quasiparticle-number fluctuations $\delta N_i = [n_i(1-n_i)]^{1/2}$. Consequence: **superfluid-normal phase transition**
- The MBCS includes δN_i via the secondary Bogolyubov transformation

$$\bar{\alpha}_{jm}^+ = U_j \alpha_{jm}^+ + V_j \alpha_{j\bar{m}} ,$$

$$a_{jm}^+ = \bar{u}_j \bar{\alpha}_{jm}^+ + \bar{v}_j \bar{\alpha}_{j\bar{m}} ,$$

$$\bar{\alpha}_{j\bar{m}} = U_j \alpha_{j\bar{m}} - V_j \alpha_{jm}^+ ,$$

$$a_{j\bar{m}} = \bar{u}_j \bar{\alpha}_{j\bar{m}} - \bar{v}_j \bar{\alpha}_{jm}^+ ,$$

$$U_j = \sqrt{1-n_j} , \quad V_j = \sqrt{n_j} .$$

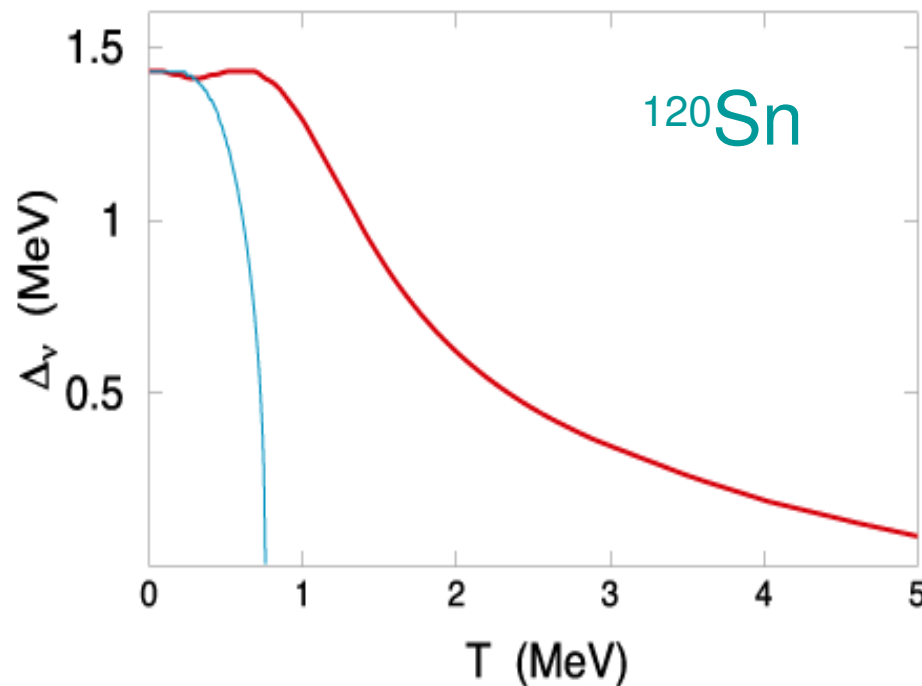
$$\bar{u}_j = u_j \sqrt{1-n_j} + v_j \sqrt{n_j} , \quad \bar{v}_j = v_j \sqrt{1-n_j} - u_j \sqrt{n_j} .$$

$$\bar{\Delta} = G \sum_j \Omega_j \bar{u}_j \bar{v}_j = G \sum_j \Omega_j \left[(1-2n_j) u_j v_j - \sqrt{n_j(1-n_j)} (u_j^2 - v_j^2) \right] ,$$

$$N = 2 \sum_j \Omega_j \bar{v}_j^2 = 2 \sum_j \Omega_j \left[(1-2n_j) v_j^2 + n_j - 2 \sqrt{n_j(1-n_j)} u_j v_j \right] .$$

Thermal-pairing effect

Effect of non-vanishing thermal gap due to statistical fluctuations in realistic nuclei:



Blue line: BCS
Red line: MBCS

Refs:

NDD & V. Zelevinsky,
PRC 64 (2001) 06439;
NDD & A. Arima,
PRC 67 (2003) 014304,
PRC 68 (2003) 014318

We extract the pairing gap in ^{184}W at finite temperature for the first time from the experimental level densities of ^{183}W , ^{184}W , and ^{185}W using the “thermal” odd-even mass difference. We found the quenching of the pairing gap near the critical temperature $T_c = 0.47$ MeV in the BCS calculations. It is shown that the monopole pairing

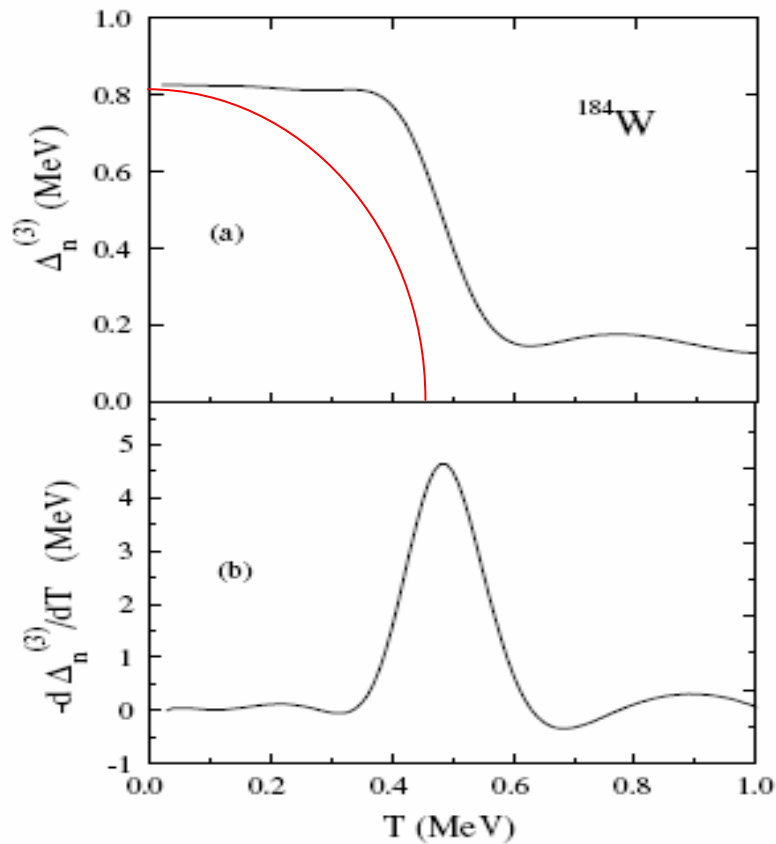


FIG. 3. Thermal pairing gap and variation. (a) The thermal pairing gap (solid line) extracted from the thermal odd-even mass difference Eq. (5) as a function of temperature. (b) The variation of the thermal odd-even mass difference defined by Eq. (5).

that the thermal odd-even mass difference has the following identity:

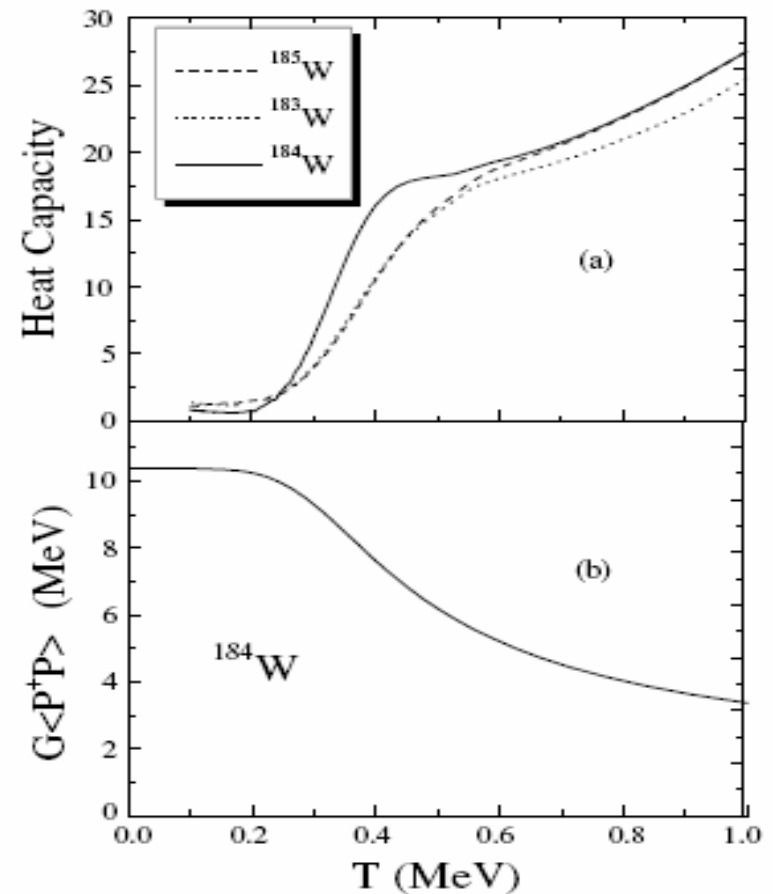
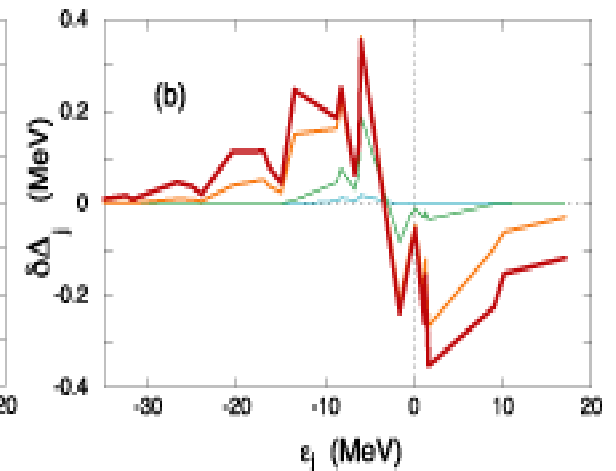
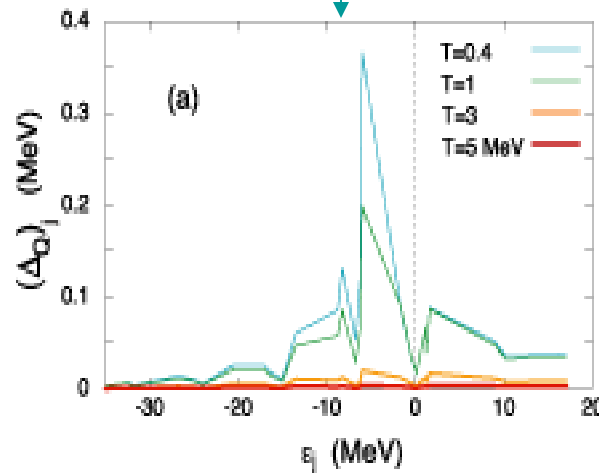
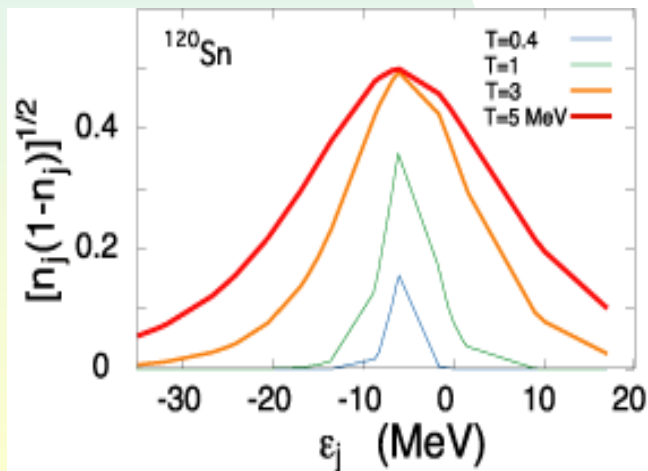


FIG. 4. Calculated heat capacities and pairing gap as a function of temperature T in the SPA+RPA for the monopole pairing model. The upper graph (a) shows the heat capacities where the dash, dotted, and solid lines denote, respectively, those of ^{183}W , ^{184}W , and ^{185}W . The lower graph (b) shows the pairing energy.

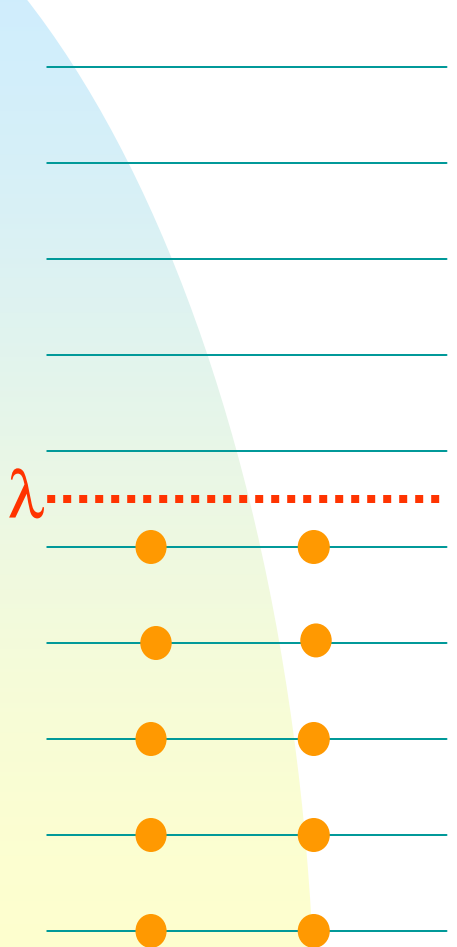
Applicability of MBCS

$$\bar{\Delta} = G \sum_j \Omega_j \bar{u}_j \bar{v}_j = G \sum_j \Omega_j \left[(1 - 2n_j) u_j v_j - \sqrt{n_j (1 - n_j)} (u_j^2 - v_j^2) \right],$$

$$N = 2 \sum_j \Omega_j \bar{v}_j^2 = 2 \sum_j \Omega_j \left[(1 - 2n_j) v_j^2 + n_j - 2 \sqrt{n_j (1 - n_j)} u_j v_j \right].$$



The Richardson model



$$H = \sum_{i=1}^{\Omega} (\varepsilon_i - \lambda) N_i - G \sum_{i,j=1}^{\Omega} P_i^+ P_j ,$$

$$N_i = c_i^+ c_i + c_{-i}^+ c_{-i} ,$$

$$P_i^+ = c_i^+ c_{-i}^+ , \quad P_i = (P_i^+)^+ ,$$

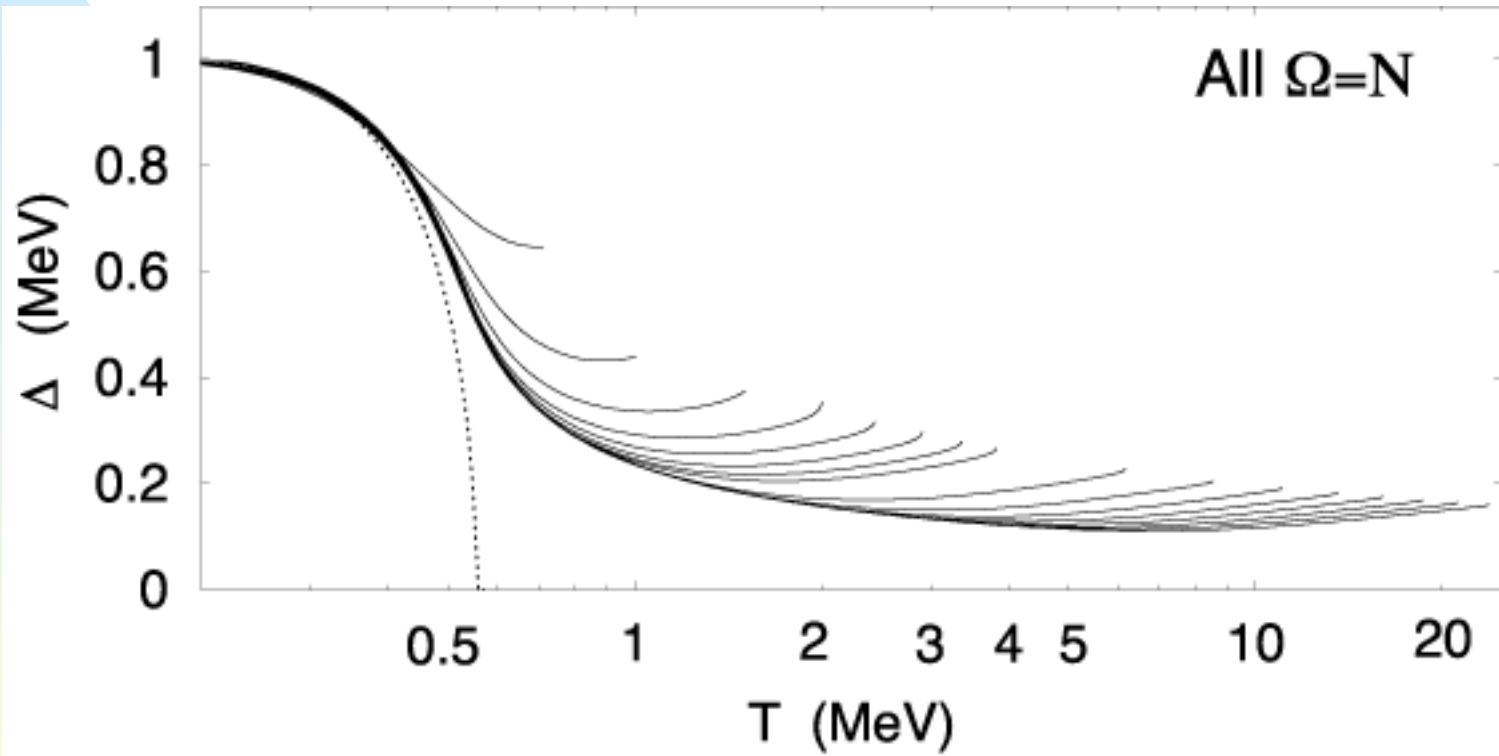
$$\varepsilon_p = \varepsilon(\Omega_h + p) , \quad p = 1, \dots, \Omega - \Omega_h ,$$

$$\varepsilon_h = \varepsilon(\Omega_h - h + 1) , \quad h = 1, \dots, \Omega_h .$$

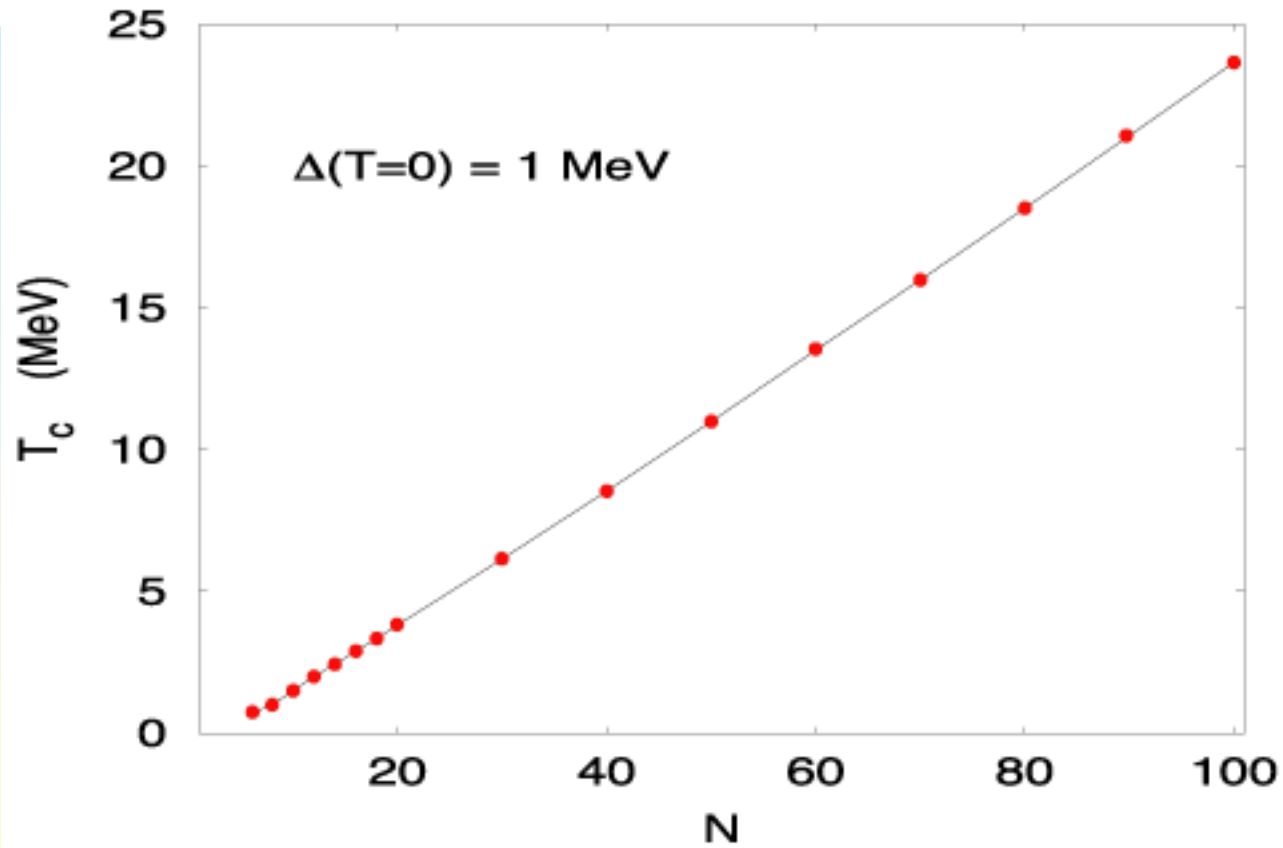
Exact solutions:

A. Volya, B.A. Brown, V. Zelevinsky, PLB 509 (2001) 37

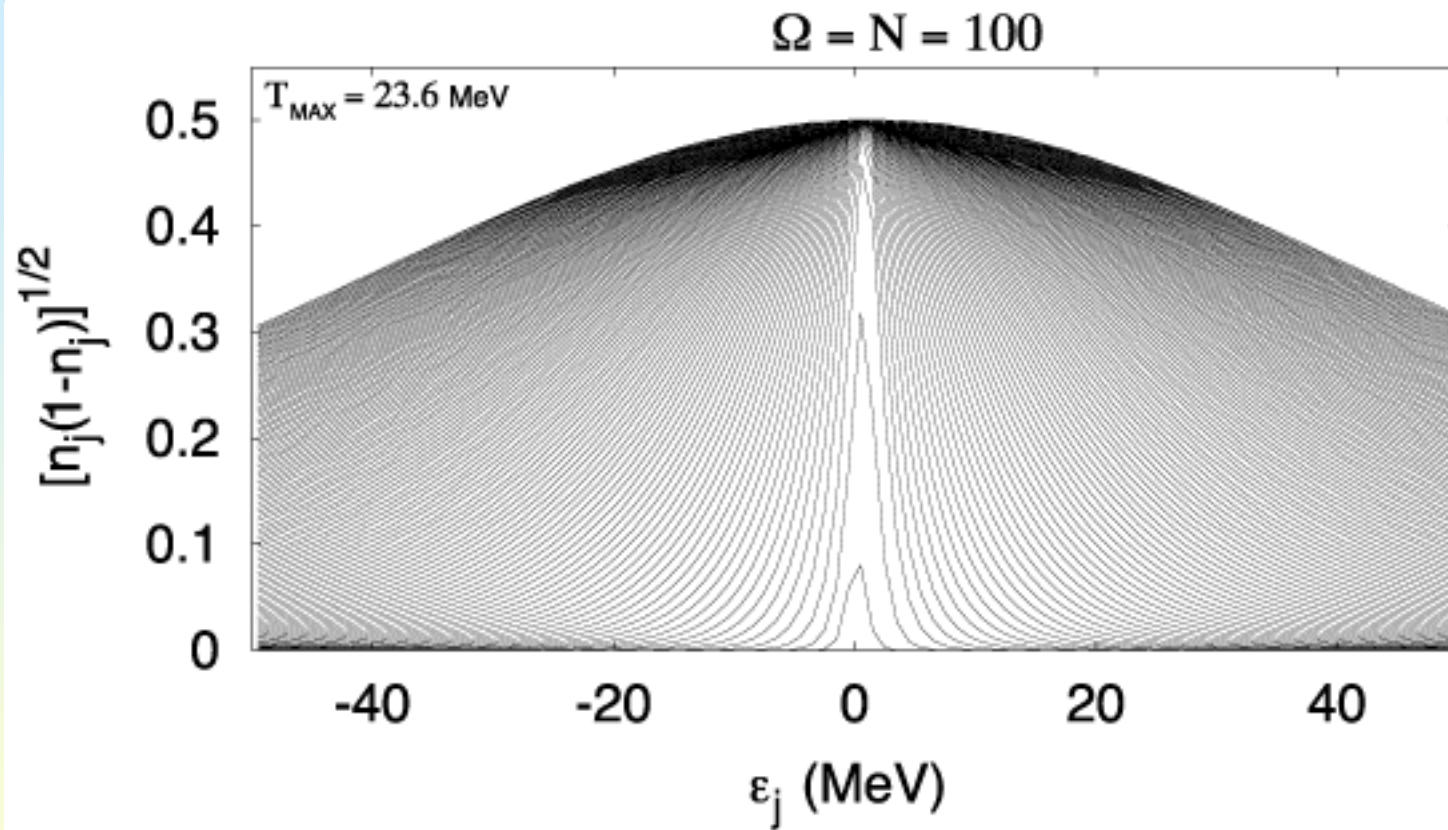
$$\Omega = N$$



Limiting temperature

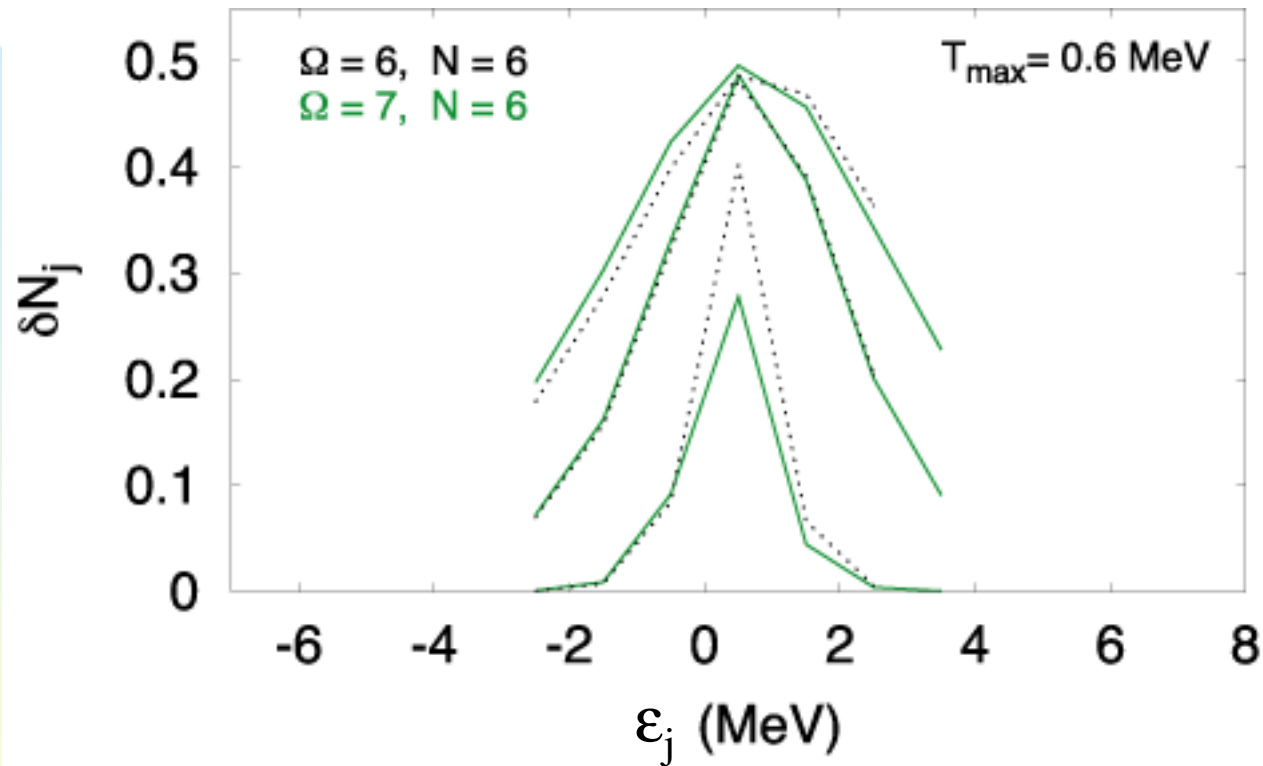


Quasiparticle number fluctuations δN_j



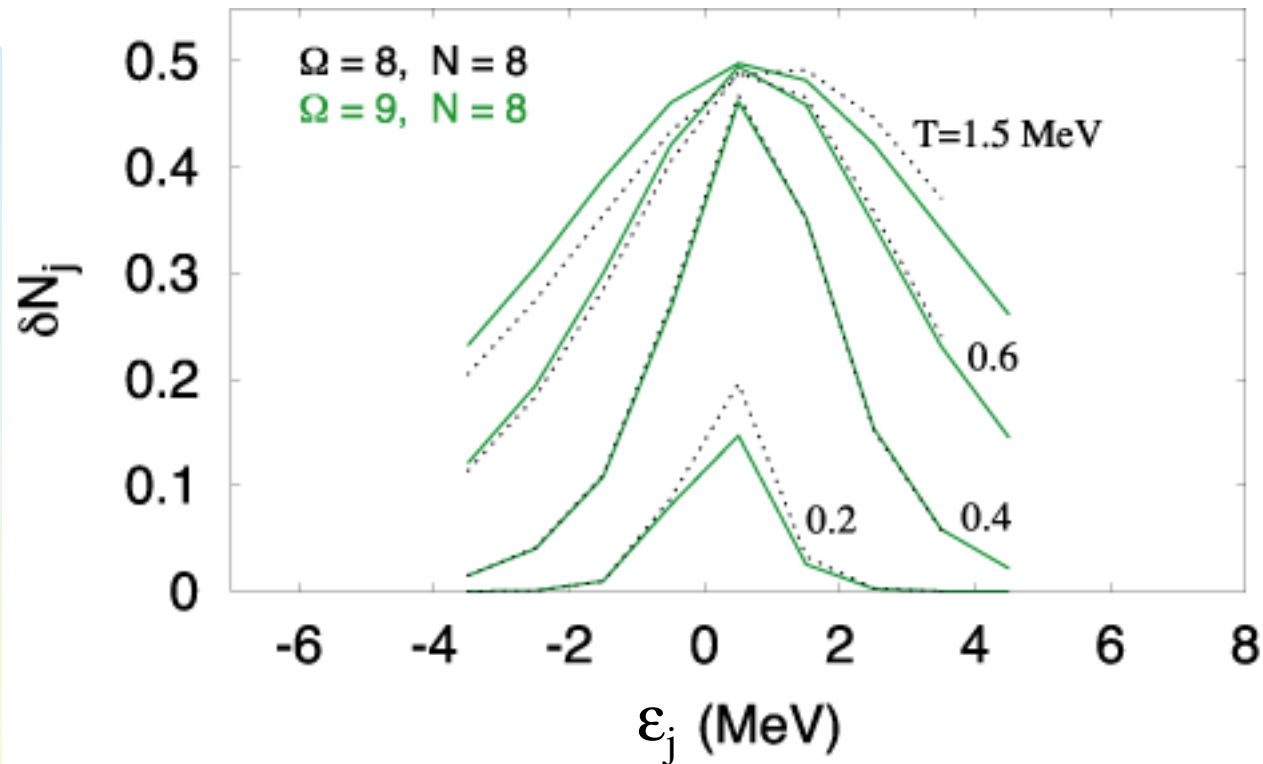
$\Delta T = 0.2 \text{ MeV}$

Enlarged space: $\Omega = N+1$



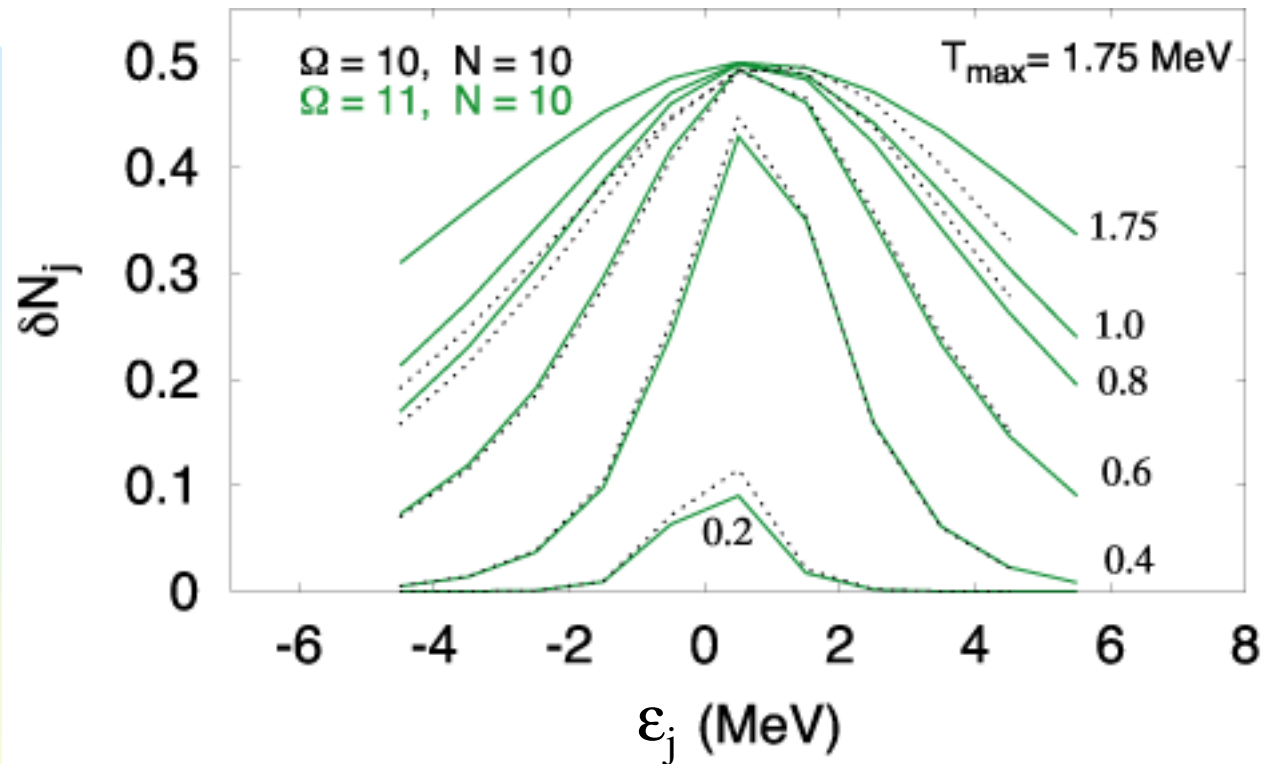
$G = 0.4$ MeV

Enlarged space: $\Omega = N+1$



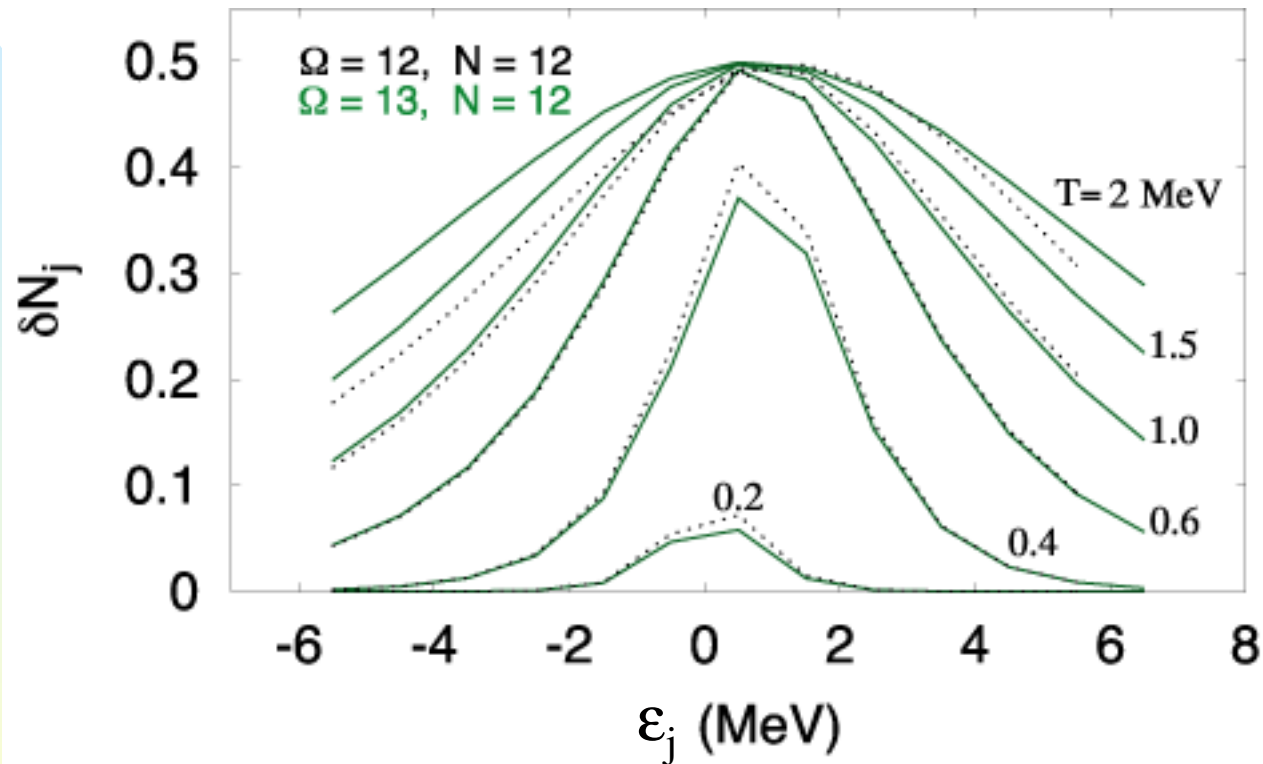
$G = 0.4$ MeV

Enlarged space: $\Omega = N+1$



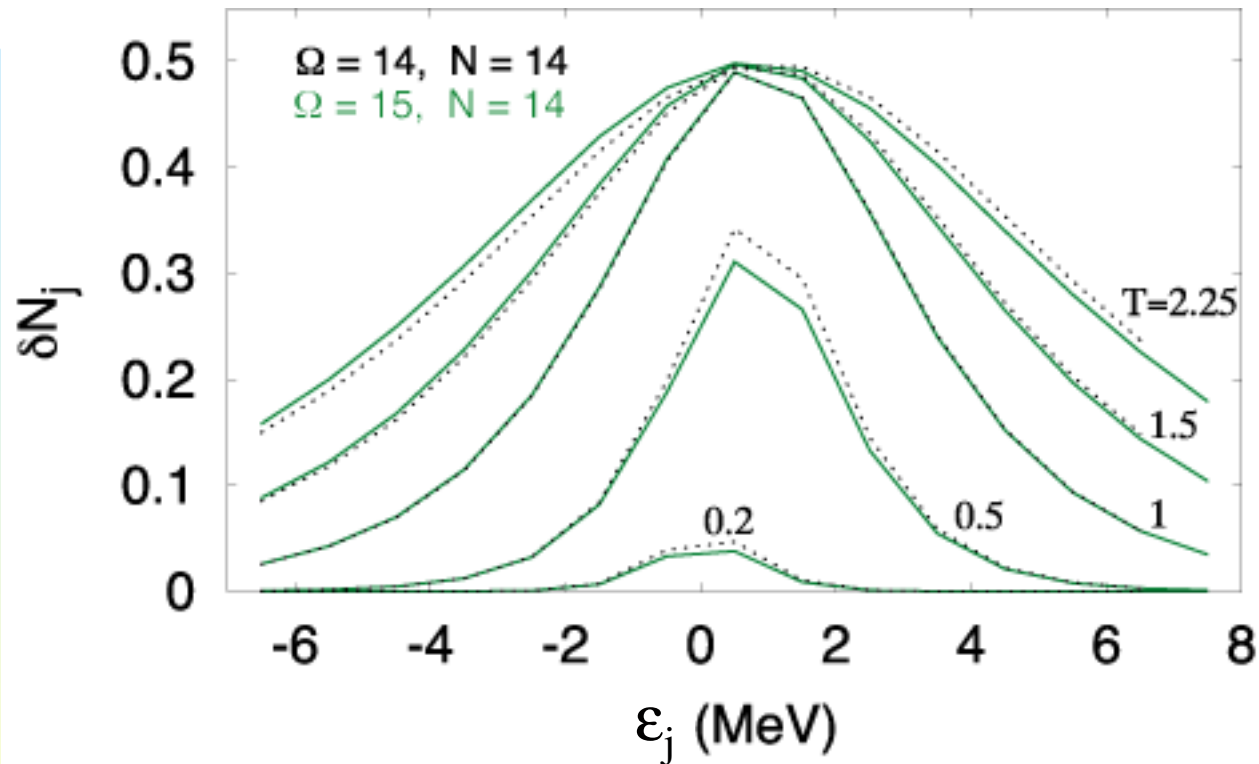
$G = 0.4$ MeV

Enlarged space: $\Omega = N+1$

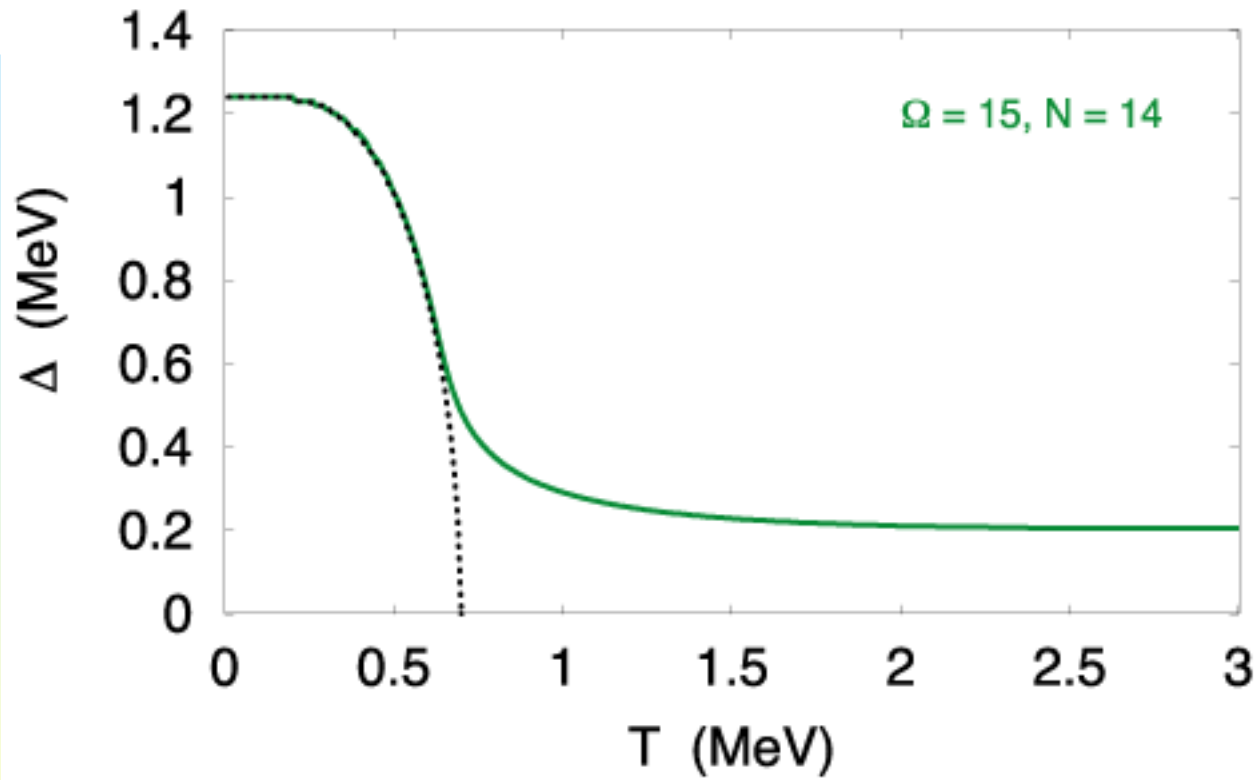


$G = 0.4 \text{ MeV}$

Enlarged space: $\Omega = N+1$

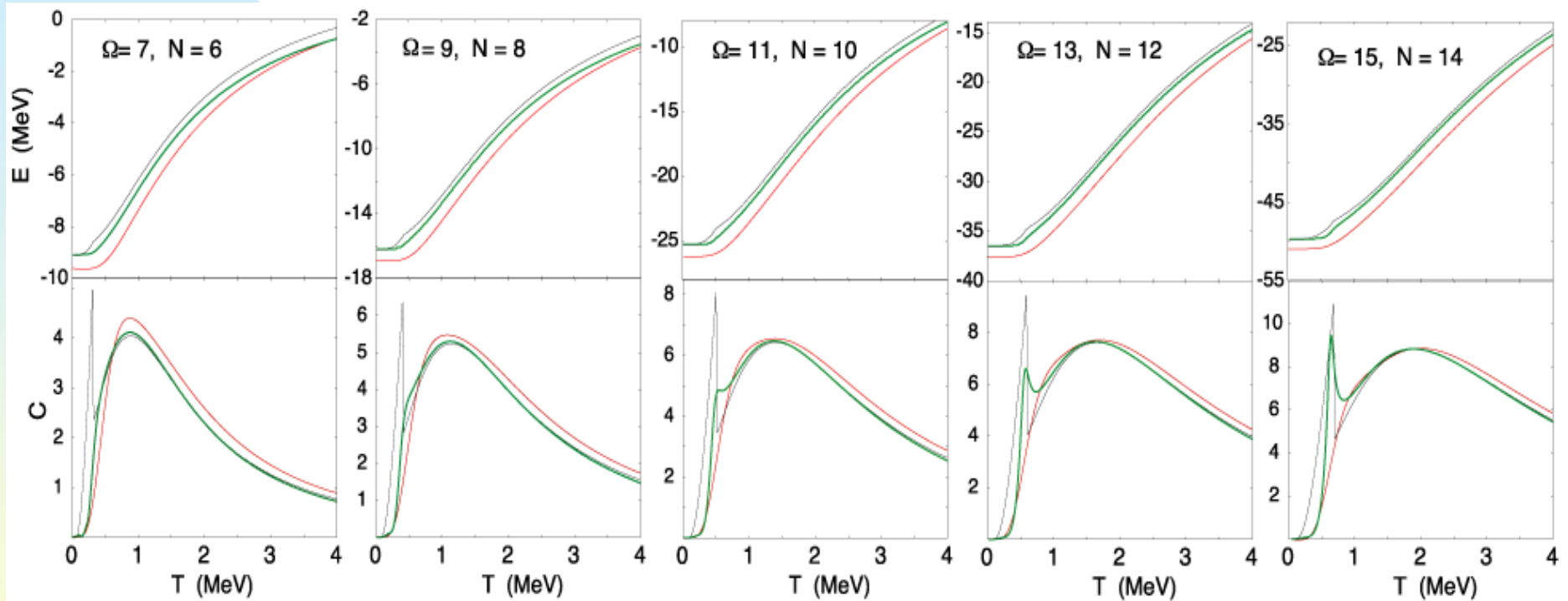


$G = 0.4$ MeV



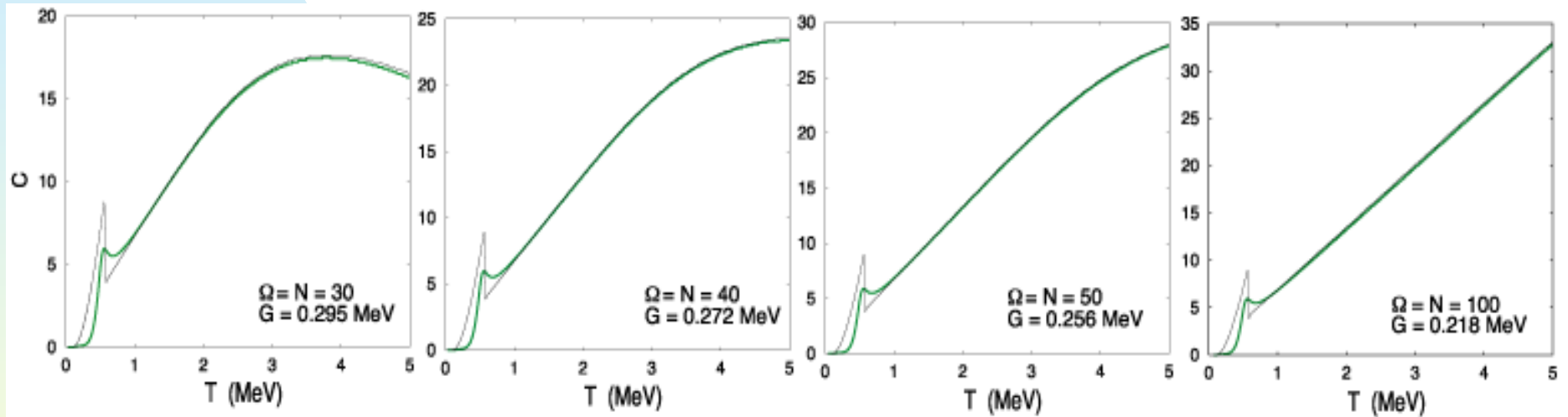
$G = 0.4$ MeV

$$\Omega = N+1$$



$$G = 0.4 \text{ MeV}$$

Large $\Omega = N$



Conclusions

- The MBCS can be applied for light systems provided the configuration space is sufficient large.
- Within the MBCS the thermal pairing gap does not vanish at T_c of BCS, but decreases monotonously with increasing T (*microscopically shown*).
- Quasiparticle-number fluctuations smooth out the superfluid-normal phase transition especially in light systems (*microscopically shown*).