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# Viscosity of hot nuclei

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**Abstract.** The ratio  $\eta/s$  of shear viscosity to entropy density is extracted from the experimental systematics of the giant dipole resonances in copper, tin, and lead regions at finite temperature  $T$ . These empirical results are then compared with the predictions by several independent models as well as with almost model-independent estimations. Based on these results, it is concluded that the ratio  $\eta/s$  in medium and heavy nuclei decreases with increasing temperature  $T$  to reach  $(1.3 - 4) \times \hbar/(4\pi k_B)$  at  $T = 5$  MeV.

## 1. Introduction

The recent experimental data from the Relativistic Heavy Ion Collider (RHIC) [1] at the Brookhaven National Laboratory and the Large Hadron Collider (LHC) [2] at CERN have revealed that the matter formed in ultrarelativistic heavy-ion collisions is a nearly perfect fluid with extremely low viscosity. This has driven the attention to the calculation of the ratio  $\eta/s$  of shear viscosity  $\eta$  to entropy density  $s$ . Using the string theory, Kovtun, Son and Starinets conjectured a universal value  $\hbar/(4\pi k_B)$  for this ratio as the lowest bound for all fluids (the KSS bound) [3]. No fluid that violates this lower bound has ever been found experimentally [4]. For finite nuclei, the recent calculations by Auerbach and Shlomo estimated the values of  $\eta/s$  within (4 – 19) and (2.5 – 12.5) times the KSS bound for heavy and light nuclei, respectively [5]. These authors used the Fermi liquid drop model (FLDM) [6], applied to the damping of giant collective vibrations. The obtained shear viscosity  $\eta$  increases with temperature  $T$  up to almost  $T \sim 10$  MeV, then slows down to reach a maximum at  $T \sim 13$  MeV. Consequently, within the region  $0 \leq T \leq 5$  MeV, where giant resonances exist, the damping width predicted by the FLDM increases with  $T$  almost quadratically, which is roughly proportional to  $\eta$  [6, 7]. However, the experimental systematics of the width of the giant dipole resonance (GDR) in heavy nuclei formed after heavy-ion fusions and/or inelastic scattering of light projectiles on heavy targets have shown that the GDR width increases with  $T$  only within  $1 \leq T \leq 2.5$  MeV. Below  $T \sim 1$  MeV, it remains nearly constant, whereas at  $T > 3 - 4$  MeV the width seems to saturate [8, 9, 10, 11, 12, 13, 14]. The entropy in Ref. [5] has been calculated by using a linear temperature dependence in the Fermi gas formula  $S = 2aT$  with a temperature-independent level density parameter  $a$ . This approximation too is rather poor for finite nuclei. Therefore, although by dividing two quantities that increase with  $T$ , the obtained result in Ref. [5] for the ratio  $\eta/s$  does decrease qualitatively to a value within 1 order of the KSS bound, a refined quantitative prediction for this ratio still remains a challenge. The present work proposes to extract the shear viscosity  $\eta$ , the entropy density  $s$ , and the ratio  $\eta/s$  directly from the most

recent and accurate experimental systematics of the GDR widths in hot nuclei. The extracted empirical values are then confronted with theoretical predictions by four models, which have been developed to describe the temperature dependence of the GDR width, namely, the phonon-damping model (PDM) [15, 16, 17], two thermal-shape fluctuation models (TSFM), as well as the FLDM mentioned above. Finally, the high- $T$  limit of  $\eta/s$  is deduced.

## 2. Formalism

The shear viscosity  $\eta$  is expressed in terms of the correlation function of the shear stress tensors  $T_{xy}(t, \mathbf{x})$  through the Green - Kubo formula [18]. By using the fluctuation-dissipation theorem (FDT), which relates the correlation function to the absorption cross section  $\sigma(\omega)$  [19, 20], the shear viscosity  $\eta(T)$  can be calculated from the latter as

$$\eta(T) = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle = \frac{\sigma(\omega = 0, T)}{C}, \quad (1)$$

where  $C$  is a normalization constant. The photoabsorption cross section of the GDR is described by the Breit-Wigner distribution as [21]  $\sigma_{GDR}(\omega) = \sigma_{int}(GDR) \Gamma / \{2\pi[(\omega - E_{GDR})^2 + (\Gamma/2)^2]\}$ , with the integrated cross section  $\sigma_{int}(GDR) \simeq (1 + k)S$  ( $k \simeq 0.5 - 0.7$ ), where  $S = 60NZ/A$  (MeV mb) is the Thomas-Reiche-Kuhn sum rule,  $\Gamma$  is the GDR full width at half maximum (FWHM), and  $E_{GDR}$  is the energy location of the GDR peak (GDR energy). From Eqs. (1) and the expression of  $\sigma_{GDR}(\omega)$ , one finds the shear viscosity at temperature  $T$  as

$$\eta(T) = \frac{\sigma_{int}(GDR)}{\pi C} \frac{\Gamma(T)/2}{E_{GDR}(T)^2 + [\Gamma(T)/2]^2}. \quad (2)$$

which can be applied to any transport process with measurable resonance scattering cross section. By determining the normalization constant  $C$  to reproduce the value  $\eta(0)$  at  $T = 0$ , Eq. (2) is rewritten as

$$\eta(T) = \eta(0) \frac{\Gamma(T)}{\Gamma(0)} \frac{E_{GDR}(0)^2 + [\Gamma(0)/2]^2}{E_{GDR}(T)^2 + [\Gamma(T)/2]^2}. \quad (3)$$

The FLDM predicted the value  $\eta(0)$  in the range of  $(0.5 - 2.5)u$  with  $u = 10^{-23}$  Mev s fm $^{-3}$  [5]. Fitting data of giant resonances at  $T = 0$  [7] found  $\eta(0) = 1.0u$ , whereas by fitting nuclear fission [22], one found  $0.6u \leq \eta(0) \leq 1.2u$ . The present work adopts  $\eta(0) = 1.0_{-0.4}^{+0.2}u$  in agreement with these values.

The entropy density is calculated as  $s = \rho S/A$  with the nuclear density  $\rho = 0.16$  fm $^{-3}$ , and  $S = S_F + S_B$ , where  $S_F$  and  $S_B$  are the entropies of the quasiparticle and phonon fields in the PDM Hamiltonian, respectively [See Eq. (1) of Ref. [15]]. The entropy  $S_\alpha$  ( $\alpha = F, B$ ) is given in units of Boltzmann constant  $k_B$  as

$$S_\alpha = - \sum_j N_j [p_j \ln p_j \pm (1 \mp p_j) \ln(1 \mp p_j)], \quad (4)$$

where  $p_j = n_j$  are the quasiparticle occupation numbers ( $\alpha = F$ ) or phonon occupation numbers  $p_j = \nu_j$  ( $\alpha = B$ ), the upper (lower) sign is for quasiparticles (phonons),  $N_j = 2j + 1$  and 1 for  $\alpha = F$  and  $B$ , respectively. For  $\alpha = F$ , the index  $j$  denotes the single-particle energy level, corresponding to the orbital angular momentum  $j$ , whereas for  $\alpha = B$ , it corresponds to that of GDR phonon. In general,  $n_j$  has the shape of a Fermi-Dirac distribution,  $n_j^{FD} = [\exp(E_j/T) + 1]^{-1}$ , smoothed with a Breit-Wigner kernel, whose width is equal to the quasiparticle damping with the quasiparticle energy  $E_j = \sqrt{(\epsilon_j - \lambda)^2 + \Delta(T)^2}$  [See Eq. (2) of [17]]. Here  $\epsilon_j$ ,  $\lambda$ , and  $\Delta(T)$  are the (neutron or proton) single-particle energy, chemical

potential, and pairing gap, respectively. However, because the quasiparticle (single-particle) damping is negligible for heavy nuclei [16], it is neglected in the present calculations of entropy  $S_F$  for the sake of simplicity, assuming  $n_j = n_j^{FD}$ . Regarding the GDR phonon occupation numbers, for  $E_{GDR} \gg T$ , it is well approximated with the Bose-Einstein distribution as  $\nu_{GDR} = [\exp(E_{GDR}/T) - 1]^{-1}$ .

For the theoretical description of the GDR width  $\Gamma(T)$ , the prediction by the PDM is adopted, which employs the single-particle energies obtained within the Woods-Saxon potentials for  $^{63}\text{Cu}$ ,  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  nuclei. In the presence of strong thermal fluctuations, the pairing gap  $\Delta(T)$  of a finite nucleus does not collapse at the critical temperature  $T_c$ , corresponding to the superfluid-normal phase transition predicted by the BCS theory for infinite systems, but decreases monotonically as  $T$  increases [23, 24, 25]. The PDM employs  $E_j$  and  $\Delta(T)$  by solving the modified BCS equations that include thermal fluctuations of quasiparticle numbers [24]. The GDR width  $\Gamma(T)$  is found as the sum of the quantal and thermal widths. The former is caused by coupling of the GDR vibration (phonon) to noncollective particle-hole ( $ph$ ) configurations with the factors  $(1 - n_p - n_h)$ , whereas the latter arises due to coupling of the GDR phonon to  $pp$  and  $hh$  configurations including the factors  $(n_s - n_{s'})$  with  $(s, s') = (h, h')$  or  $(p, p')$ . As the result, the quantal width decreases slightly, whereas the thermal width increases sharply with increasing  $T$  and saturates at  $T \geq 4 - 5$  MeV in tin and lead isotopes [15]. Thermal pairing gap  $\Delta(T)$  causes the GDR width in  $^{120}\text{Sn}$  to remain nearly constant or even slightly decrease at  $T \leq 1$  MeV [17] in agreement with the data of Ref. [12]. For  $^{63}\text{Cu}$ , the effect of pairing on the GDR width is small, so it is not included in the calculations of GDR width within the PDM, but the finite temperature BCS pairing with blocking by the odd proton is taken into account for the entropy to ensure its vanishing value at low  $T$ .

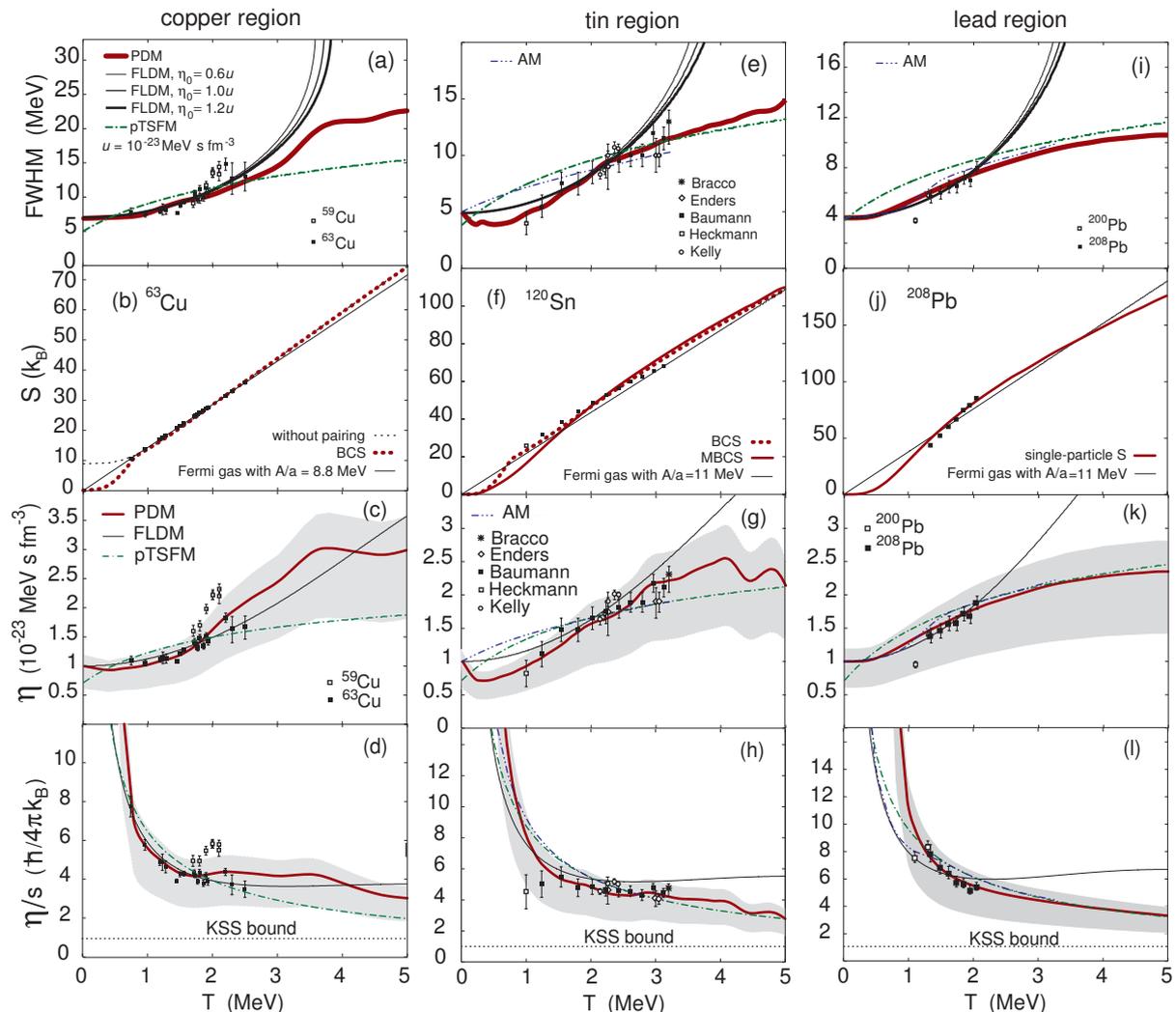
### 3. Analysis of numerical results

Shown in Fig. 1 are the GDR width, entropy  $S$ , shear viscosity  $\eta$ , and ratio  $\eta/s$  as functions of  $T$  for copper, tin, and lead regions as predicted by the PDM, the adiabatic thermal shape fluctuation model (AM) [26], the phenomenological thermal shape fluctuation model (pTSFM) [27], and the FLDM, as well as the corresponding results of the empirical extraction from the GDR experimental systematics.

The PDM predictions for the GDR width best fit the experimental systematics for all three nuclei  $^{63}\text{Cu}$ ,  $^{120}\text{Sn}$ , and  $^{208}\text{Pb}$ . The AM fails to describe the GDR width at low  $T$  for  $^{120}\text{Sn}$  because thermal pairing was not included in the AM calculations, while it slightly overestimates the width for  $^{208}\text{Pb}$ . (The AM prediction for GDR width in  $^{63}\text{Cu}$  is not available.) The predictions by the pTSFM are qualitatively similar to those by the AM, although to achieve this agreement, the pTSFM needs to use  $\Gamma(0) = 5$  MeV for  $^{63}\text{Cu}$  and 3.8 MeV for  $^{120}\text{Sn}$ , i.e., substantially smaller than the experimental values of around 7 and 4.9 MeV, respectively. The widths obtained within the FLDM fit the data fairly well up to  $T \simeq 2.5$  MeV. However, they do not saturate at high  $T$  but increase sharply with  $T$  and break down at  $T_c < 4$  MeV.

As for the entropies, the good agreement between the results of microscopic calculations and the empirical extraction indicates that the level-density parameter for  $^{63}\text{Cu}$ , within the temperature interval  $0.7 < T < 2.5$  MeV, can be considered to be temperature-independent and equal to  $a = 63/8.8 \simeq 7.16$  MeV $^{-1}$ , whereas, for  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$ , the level-density parameter varies significantly with  $T$  [10]. The Fermi-gas entropy with a constant level-density parameter  $a$  best fits the microscopic and empirical results with  $A/a = 8.8$  MeV for  $^{63}\text{Cu}$ , and 11 MeV for  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$ .

The predictions of  $\eta$  by the PDM have the best overall agreement with the empirical results for all three nuclei  $^{63}\text{Cu}$ ,  $^{120}\text{Sn}$ , and  $^{208}\text{Pb}$ . The PDM produces an increase in  $\eta(T)$  with  $T$  up to 33.5 MeV and a saturation in  $\eta(T)$  within  $(2 - 3)u$  at higher  $T$ . The ratio  $\eta/s$  decreases sharply with increasing  $T$  up to  $T \sim 1.5$  MeV, starting from which the decrease gradually slows down



**Figure 1.** (Color online) Shear viscosity  $\eta$  (a), entropy  $S$  (b), and the ratio  $\eta/s$  (c) as functions of  $T$  for tin region. The empirical values are extracted from the experimental systematics for for copper ( $\text{Cu}^{59}$  and  $\text{Cu}^{63}$  [8]), tin (by Bracco *et al.* [9], Enders *et al.* [10], Baumann *et al.* [11], Heckmann *et al.* [12], and Kelly *et al.* [13]), and lead ( $\text{Pb}^{208}$  [11] and  $\text{Pb}^{200}$  [14]) regions.

to reach (2 - 3) KSS units at  $T = 5$  MeV. The FLDM has a similar trend as that of the PDM up to  $T \sim 2 - 3$  MeV, but at higher  $T$  ( $T > 3$  MeV for  $^{120}\text{Sn}$  or 2 MeV for  $^{208}\text{Pb}$ ), it produces an increase in both  $\eta$  and  $\eta/s$  with  $T$ . At  $T = 5$  MeV, the FLDM model predicts the ratio  $\eta/s$  within (3.7 - 6.5) KSS units, which are roughly 1.5 timestwice larger than the PDM predictions. The AM and pTSFM show a similar trend to that predicted by the PDM. However, to obtain such similarity,  $\eta(0)$  in the pTSFM calculations has to be reduced to  $0.72u$  instead of  $1u$ . They all overestimate  $\eta$  at  $T < 1.5$  MeV.

A model-independent estimation for the high- $T$  limit of  $\eta$  can be obtained under the assumption of GDR width saturation,  $\Gamma_{max} \simeq 3\Gamma(0) \simeq 0.9E_{GDR}$  at  $T \simeq 5 - 6$  MeV. By using this assumption, one finds  $\eta_{max} \simeq 2.551 \times \eta(0)$  from Eq. (3). The high- $T$  limit of the entropy density  $s$  is obtained by noticing that  $S_F \rightarrow 2\Omega \ln 2$  at  $T \rightarrow \infty$  because  $n_j \rightarrow 1/2$  where  $\Omega = \sum_j (j + 1/2)$  for the spherical single-particle basis or sum of all doubly degenerate levels for the deformed basis. The particle-number conservation requires that  $A = \Omega$  since all single-particle occupation

numbers are equal to  $1/2$ . This leads to the high- $T$  limit of entropy density  $s$ :  $s_{max} = 2\rho \ln 2 \simeq 0.222 (k_B)$ . Dividing  $\eta_{max}$  by  $s_{max}$  yields the high- $T$  limit (or lowest bound) for  $\eta/s$  in finite nuclei

$$\left(\frac{\eta}{s}\right)_{min} \simeq 2.2_{-0.9}^{+0.4} \text{ (KSS units) ,} \quad (5)$$

#### 4. Conclusion

In conclusion, by using the Green-Kubo relation and the fluctuation-dissipation theorem, the ratio  $\eta/s$  has been extracted from the experimental systematics of the GDR widths in copper, tin and lead regions at  $T \neq 0$ , and compared with the theoretical predictions by the PDM, AM, pTSM, and FLDM. At  $T = 5$  MeV, the values of  $\eta/s$  predicted by the PDM reach  $3_{-1.2}^{+0.63}$ ,  $2.8_{-1.1}^{+0.5}$ ,  $3.3_{-1.3}^{+0.7}$  KSS units for  $^{63}\text{Cu}$ ,  $^{120}\text{Sn}$ , and  $^{208}\text{Pb}$ , respectively. Combining these results with the model-independent estimation for the high- $T$  limit of  $\eta/s$ , which is  $2.2_{-0.9}^{+0.4}$  KSS units, one can conclude that the value of  $\eta/s$  for medium and heavy nuclei at  $T = 5$  MeV is in between (1.3 - 4.0) KSS units, which is about (3 - 5) times smaller (and of much less uncertainty) than the value between (4 - 19) KSS units predicted by the FLDM for heavy nuclei, where the same lower value  $\eta(0) = 0.6u$  was used. This estimation also indicates that nucleons inside a hot nucleus at  $T = 5$  MeV has nearly the same ratio  $\eta/s$  as that of QGP, around (2 - 3) KSS units, at  $T > 170$  MeV discovered at RHIC and LHC.

The numerical calculations were carried out using the FORTRAN IMSL Library by Visual Numerics on the RIKEN Integrated Cluster of Clusters (RICC) system.

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